Sets and Numbers 5

SINGER MATHEMATICS PROGRAM

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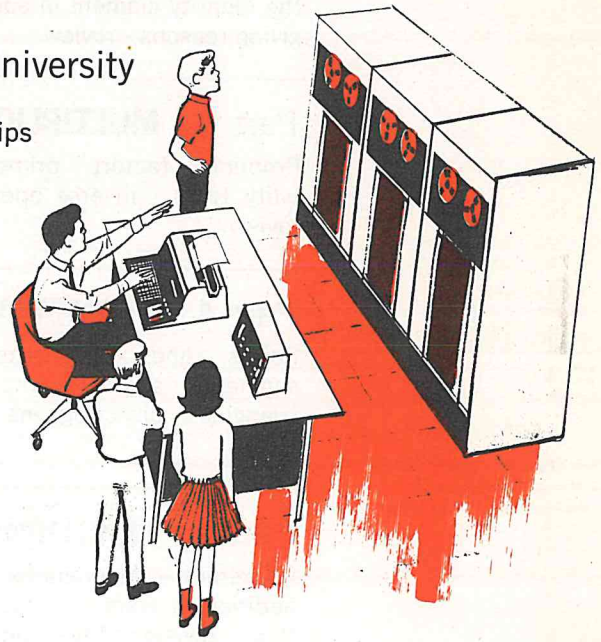
PATRICK SUPPES / Stanford University

Edited under the direction of Gussie Phillips



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Part 1 Sets

A **set** is any collection of things. We use braces to show sets.

$\{1, 2\}$ $\{\ \}$

The numbers 1 and 2 are **members** of the set $\{1, 2\}$.

The set with no members is **the empty set.**

"$1 \in \{1, 2\}$" is read "One is a member of the set whose members are one and two."

1. Tell which sentences are true and which are false.

   a. 1 is a member of $\{0, 1, 2\}$.

   b. 2 is a member of $\{1, 3, 4\}$.

   c. Tom is a member of $\{\text{Sally, Tom}\}$.

   d. $5 \in \{1, 3, 5, 7\}$

   e. $7 \in \{1, 3, 5, 7\}$

   f. $8 \in \{1, 3, 5, 7\}$

2. Many sets are easily described without listing the members. What are the members of each of the following sets?

   a. The set of odd numbers greater than 0 and less than 5.

   b. The set of the first two presidents of the United States.

   c. The set of odd numbers greater than 0 and less than 10.

   d. The set of your brothers.

   e. The set of your sisters.

   f. The set of even numbers greater than 3 and less than 4.

   g. The set of live elephants now in this room.

3. Which of the sets in Exercise 2 name the empty set?

4. The symbol used to show that a thing belongs to a set is ■.

Equal and Equivalent Sets

Sets are **equal** when they have the same members or elements.

$\{\text{Tom, Mary}\} = \{\text{Mary, Tom}\}$ $\quad \{\text{Tom, Mary}\} \neq \{\text{Tom, Jane}\}$

Sets are **equivalent** when they have the same number of members.

$\{\text{Tom, Mary, Jane}\} \approx \{1, 2, 3\}$ $\quad \{\text{Tom, Mary, Jane}\} \not\approx \{1, 2\}$

1. Choose $=$ or $\neq$.

a. $\{a, b\} \bullet \{c, a\}$

b. $\{1, 2, 3\} \bullet \{3, 2, 1\}$

c. $\{d, e\} \bullet \{d, e, f\}$

d. $\{\text{Betty}\} \bullet \{\text{Ann}\}$

e. $\{r, s\} \bullet \{s, r\}$

f. $\{\ \} \bullet \{\ \}$

g. $\{t\} \bullet \{a\}$

h. $\{a, c\} \bullet \{c, a\}$

i. $\{d, r, t\} \bullet \{t, r, d\}$

j. $\{n, r\} \bullet \{s, r\}$

2. Choose $\approx$ or $\not\approx$.

a. $\{m\} \bullet \{n\}$

b. $\{t, w\} \bullet \{1, 2\}$

c. $\{\text{John}\} \bullet \{\text{Jim}\}$

d. $\{\ \} \bullet \{\ \}$

e. $\{a, b\} \bullet \{a, b, c\}$

f. $\{34, 60\} \bullet \{35, 60\}$

g. $\{7, 8, 9\} \bullet \{9, 8\}$

h. $\{45\} \bullet \{54, 45\}$

i. $\{17, 18\} \bullet \{19, 20\}$

j. $\{\text{Joe}\} \bullet \{\text{David, Ed}\}$

3. Are all equal sets equivalent sets?

4. Are all equivalent sets equal sets?

5. The symbol used to show that two sets are equivalent is ▪ .

6. The symbol used to show that two sets are equal is ▪ .

Operations on Sets

Addition, subtraction, multiplication, and division are operations on numbers.

We also have operations on sets. The operations on sets that we shall study are **union, intersection,** and **difference.**

The union symbol "∪" is our symbol for showing that we are putting sets together. We list the common members only once in the **answer set.**

$\{\text{April, May}\} \cup \{\text{May, June}\} = \{\text{April, May, June}\}$

$\{1, 2\} \cup \{2, 3, 4\} \cup \{3, 4\} = \{1, 2, 3, 4\}$

1. Find each answer set.

a. $\{1, 2, 3\} \cup \{4, 5\} =$ ■

b. $\{a, b\} \cup \{b\} \cup \{a\} =$ ■

c. $\{\text{Mary}\} \cup \{\text{Kay}\} \cup \{\ \} =$ ■

d. $\{2, 4, 6\} \cup \{0, 2, 4\} \cup \{0, 2, 4, 8\} =$ ■

e. $\{5, 10, 15\} \cup \{10, 20, 30\} \cup \{15, 30\} =$ ■

f. $\{3, 6, 9\} \cup \{2, 4, 6, 8\} \cup \{4, 5, 6\} =$ ■

g. $\{\text{Bill, Jack}\} \cup \{\ \} \cup \{\text{Jack, Roy}\} =$ ■

2. True or false?

a. $3 \in \{1, 2\} \cup \{3, 4, 5\}$

b. $4 \in \{1, 3\} \cup \{5, 7\}$

c. $\text{Tom} \in \{\text{Ted}\} \cup \{\text{Tom}\}$

d. $\text{Jill} \in \{\text{Mary}\} \cup \{\text{Joan}\}$

3. Two equal sets each have three members. The union of these two equal sets is a set that has ■ members.

The intersection symbol "$\cap$" is our symbol for showing that we are finding the members common to both sets.

$\{4, 5, 6, 7\} \cap \{9, 8, 7, 4\} = \{4, 7\}$

$\{\text{May, June}\} \cap \{\text{July}\} = \{\ \}$

Find each answer set.

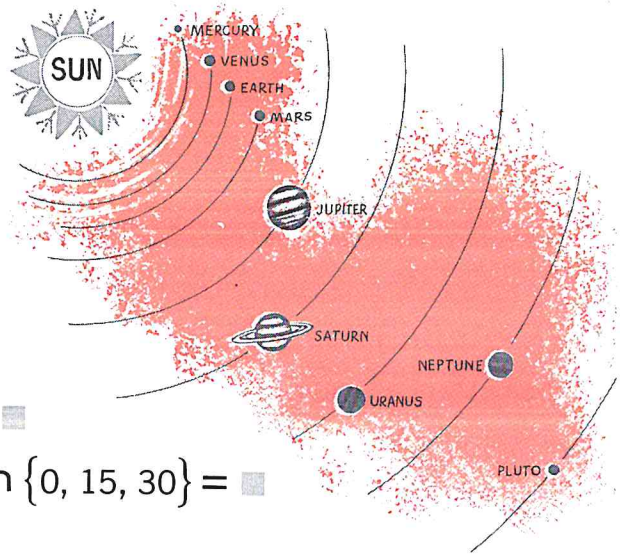
1. a. $\{\text{Venus, Mars, Saturn}\} \cap \{\text{Jupiter, Pluto, Neptune}\} =$ ▢

   b. $\{\text{Marco Polo, Richard Byrd}\} \cap \{\text{Richard Byrd}\} =$ ▢

   c. $\{4, 5, 6\} \cap \{5, 6\} =$ ▢

   d. $\{a, b, c\} \cap \{b, a, c\} =$ ▢

   e. $\{0, 1, 2, 3, 4\} \cap \{5, 6, 7\} =$ ▢



2. a. $\{\text{Ada}\} \cap \{\text{Tina, Jill}\} \cap \{\text{Jill}\} =$ ▢

   b. $\{q, r, s, t\} \cap \{\ \} \cap \{r, s\} =$ ▢

   c. $\{2, 4, 6, 8\} \cap \{1, 3, 5\} \cap \{\ \} =$ ▢

   d. $\{0, 5, 10, 15\} \cap \{0, 10, 20, 30\} \cap \{0, 15, 30\} =$ ▢

3. Two equal sets each have four members. Their intersection is a set that has ▢ members.

4. Two sets have no members in common. Their intersection is the ▢ set.

5. True or false?

   a. $3 \in \{1, 2\} \cap \{3, 4, 5\}$

   b. $2 \in \{1, 2\} \cap \{1, 2, 3\}$

   c. $8 \in \{6\} \cap \{7, 8\}$

   d. $4 \in \{2, 4\} \cap \{0, 4, 8\}$

Another operation on sets is finding the difference of two sets. The difference symbol "–" is our symbol for showing that we are taking away a set of things.

$\{\text{April, May, June}\} - \{\text{June}\} = \{\text{April, May}\}$

Sometimes there are elements in the second set that are not in the first set. We can take away only elements that are in both sets.

$\{\text{April, May, June}\} - \{\text{June, July}\} = \{\text{April, May}\}$

$\{\text{June, July}\} - \{\text{April, May, June}\} = \{\text{July}\}$

Find each answer set.

1. a. $\{\text{June, July, August, September}\} - \{\text{September, October}\} =$ ■
   b. $\{\text{September, October}\} - \{\text{June, July, August, September}\} =$ ■
   c. $\{3, 4, 5\} - \{5, 6, 7\} =$ ■
   d. $\{5, 6, 7\} - \{3, 4, 5\} =$ ■
   e. $\{\text{Jill, Fran, Kay}\} - \{\text{Jill}\} =$ ■
   f. $\{\text{Jill}\} - \{\text{Jill, Fran, Kay}\} =$ ■

2. a. $\{j, k, l, m\} - \{k, l\} =$ ■
   b. $\{s, h, m, p\} - \{s\} =$ ■
   c. $\{59, 58, 57, 56\} - \{56\} =$ ■
   d. $\{\text{Karl, Hugh}\} - \{\text{Hugh}\} =$ ■
   e. $\{l, h, x, i, z\} - \{l, x, z\} =$ ■
   f. $\{0, 4, 7, 10\} - \{0, 1, 2\} =$ ■
   g. $\{0, 1, 2, 3\} - \{1, 2, 3\} =$ ■
   h. $\{\text{Ida, Kay}\} - \{\text{Kay, Ann}\} =$ ■

3. True or False?
   a. $1 \in \{1, 2, 3\} - \{2, 3\}$
   b. $3 \in \{1, 2, 3\} - \{2, 3\}$
   c. $4 \in \{2, 4, 6\} - \{2\}$
   d. $a \in \{a, b, c\} - \{a\}$

Disjoint Sets and Overlapping Sets

Disjoint sets have no members in common.
$\{\text{Jean, Mary}\}$ $\{\text{Cindy}\}$

Overlapping sets have at least one member in common.
$\{\text{Ben, Leon, Roy}\}$ $\{\text{Leon, Tim}\}$

1. Is each pair of sets disjoint or overlapping?

a. $\{a, b, c\}$ $\{c, d, e, f\}$

b. $\{a, b, c\}$ $\{b, c\}$

c. $\{0, 1, 2, 3\}$ $\{5, 10, 15\}$

d. $\{\text{Glen, Walt}\}$ $\{\text{Nick, Glen}\}$

e. $\{0, 2, 4, 6, 8\}$ $\{2, 3, 4\}$

f. $\{\text{Jean, Joan}\}$ $\{\text{Jenny}\}$

g. $\{7, 4, 3\}$ $\{1, 3, 4, 7, 11\}$

h. $\{10, 20, 30, 40, 50\}$ $\{0\}$

i. $\{\ \}$ $\{59, 60, 61, 62\}$

j. $\{\text{Ohio, Illinois}\}$ $\{\text{Iowa, Maine}\}$

Find each answer set.

2. a. $\{2, 3, 4, 5\} \cap \{2, 3\} =$ ▢

b. $\{\text{Alex}\} \cap \{\text{Russ}\} =$ ▢

c. $\{\text{Liz, Dot, Ada}\} \cap \{\text{Ada}\} =$ ▢

3. a. $\{a, b, c\} \cup \{d, e, f\} =$ ▢

b. $\{\text{Jerry}\} \cup \{\text{Nina, Tony}\} =$ ▢

c. $\{3, 4, 8\} \cup \{0, 5\} =$ ▢

4. a. $\{1, 2, 3\} - \{4, 5, 6\} =$ ▢

b. $\{\text{John, Lynn}\} - \{\text{Josie}\} =$ ▢

c. $\{69, 70\} - \{70\} =$ ▢

5. a. $\{j, t, m\} \cap \{k\} =$ ▢

b. $\{2, 4, 6, 8\} \cap \{1, 3, 5, 7\} =$ ▢

c. $\{x, y, z\} \cap \{i, j\} =$ ▢

6. The intersection of two disjoint sets is the ▢ set.

7. The intersection of the empty set and any other set is the ▢ set.

8. The union of the empty set and any other set is the ▢ set.

Subsets

A set is a **proper subset** of a larger set if all its members are members of the larger set.

The set $\{a, b, c\}$ is a subset of the set $\{a, b, c, d\}$.

The symbol "$\subset$" is used to show that one set is a proper subset of another.

$$\{a, b, c\} \subset \{a, b, c, d\}$$

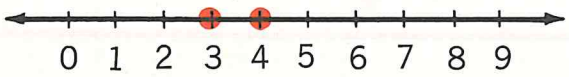
True or false?

1. a. $\{a, b\} \subset \{a, c, d\}$
   b. $\{\ \} \subset \{\text{Tom}\}$
   c. $\{r, s\} \subset \{r, t, v, w\}$

2. a. $\{\text{Jack}\} \subset \{\text{Bill, Jack}\}$
   b. $\{3, 8\} \subset \{3, 6, 9, 12\}$
   c. $\{3, 4\} \subset \{3, 4\}$

A number may be **less than** another number.

$3 < 4$



A number may be **greater than** another number.

$4 > 3$

Choose $<$ or $>$.

3. a. $16 + 1 \bigcirc 18$
   b. $3 + 4 \bigcirc 8$
   c. $81 \bigcirc 18 + 60$

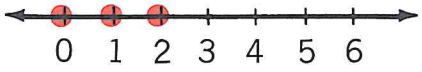
4. a. $246 \bigcirc 251$
   b. $389 \bigcirc 390$
   c. $405 \bigcirc 396$

5. True or false?
   a. The empty set is a subset of every non-empty set.
   b. The union of two sets is always a subset of each of the two sets.

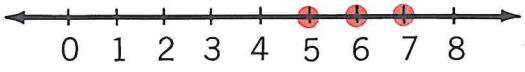
Sets of Numbers

The set of all **whole numbers** is the set $\{0, 1, 2, 3, 4, \ldots\}$.

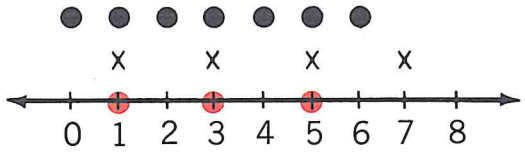
The set of all whole numbers n such that $n < 3$ is $\{0, 1, 2\}$.



The set of all whole numbers n such that $4 < n < 8$ is $\{5, 6, 7\}$.
This is the set of whole numbers such that $n > 4$ and $n < 8$.



The set of all whole numbers such that $n < 7$ and n is odd is $\{1, 3, 5\}$.
Here we are actually finding the intersection of the set of whole numbers less than seven and the set of odd numbers.



Find each set.

1. **a.** The set of all whole numbers n such that $n < 4$.
   b. The set of all whole numbers n such that $n < 5$.
   c. The set of all whole numbers n such that $6 < n < 10$.
   d. The set of all whole numbers n such that $101 < n < 105$.

2. **a.** The set of all whole numbers such that $n < 9$ and n is even.
   b. The set of all whole numbers such that $n < 5$ and n is odd.
   c. The set of all whole numbers such that $n < 8$ and n is odd.
   d. The set of all whole numbers such that $3 < n < 11$ and n is even.

In place of:

The set of all whole numbers n such that $1 < n < 4$ is $\{2, 3\}$.

We write:

$\{n: 1 < n < 4\} = \{2, 3\}$.

1. List the members of each set.

a. $\{n: 10 < n < 15\}$

b. $\{n: 102 < n < 107\}$

c. $\{n: 204 < n < 209\}$

d. $\{n: 198 < n < 202\}$

e. $\{n: 459 < n < 463\}$

f. $\{n: 899 < n < 901\}$

2. Perform each operation and write each answer set.

$$\{n: 0 < n < 3\} \cup \{n: 5 < n < 8\} = \{1, 2\} \cup \{6, 7\}$$
$$= \{1, 2, 6, 7\}$$

a. $\{n: 1 < n < 4\} \cup \{n: 20 < n < 23\} =$ ▪

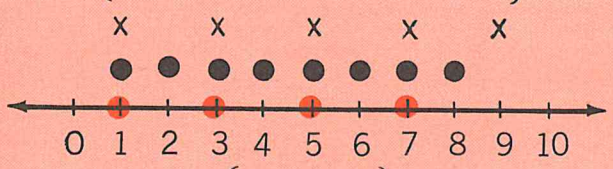
b. $\{n: 10 < n < 12\} \cup \{n: 28 < n < 31\} =$ ▪

c. $\{n: 5 < n < 9\} \cup \{n: 7 < n < 11\} =$ ▪

d. $\{n: 1 < n < 5\} \cup \{n: 3 < n < 7\} =$ ▪

3. Draw a number line for each problem and show the answer set.

$\{n: 0 < n < 9 \text{ and } n \text{ is odd}\}$



$\{1, 3, 5, 7\}$

a. $\{n: 1 < n < 8 \text{ and } n \text{ is even}\}$

b. $\{n: 3 < n < 10 \text{ and } n \text{ is odd}\}$

c. $\{n: 3 < n < 5 \text{ and } n \text{ is odd}\}$

d. $\{n: 2 < n < 8 \text{ and } n \text{ is even}\}$

Sets of Points

Every geometric figure is simply a set of points. The line segment AB whose endpoints are A and B is the set of all points on the segment. Instead of writing "line segment AB," we shall write $\overline{AB}$. $\overline{AB}$ is a set of points.

1. Match each set description to a figure.

   a. $\overline{AB} \cup \overline{BC}$

   b. $\overline{AB} \cup \overline{BC} \cup \overline{AC}$

   c. $\overline{AB} \cup \overline{BC} \cup \overline{BD}$

   d. $\overline{AB} \cup \overline{AC} \cup \overline{AD}$

   e. $\overline{AB} \cup \overline{AC} \cup \overline{BC} \cup \overline{CD}$

   f. $\overline{AB} \cup \overline{BC} \cup \overline{CD} \cup \overline{AD}$

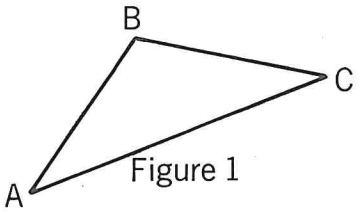


Figure 1

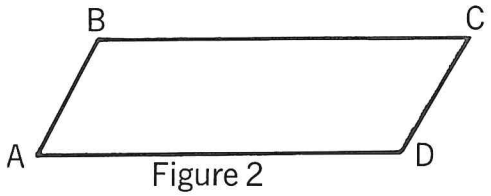


Figure 2

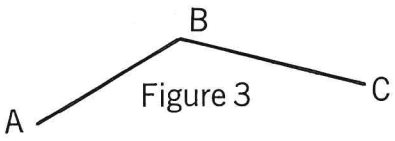


Figure 3

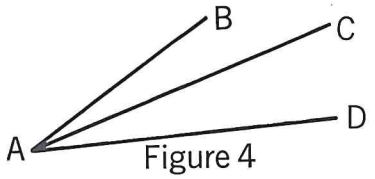


Figure 4

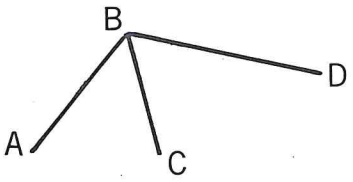


Figure 5

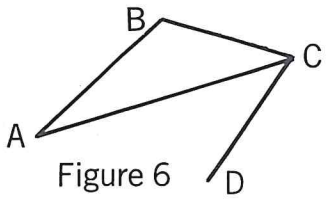


Figure 6

2. Draw a figure to match each set description.

   a. $\overline{RS} \cup \overline{ST}$

   b. $\overline{RS} \cup \overline{RT} \cup \overline{ST}$

   c. $\overline{RS} \cup \overline{RT} \cup \overline{TW}$

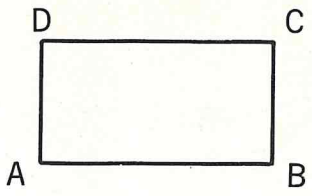
   d. $\overline{RS} \cup \overline{ST} \cup \overline{TW} \cup \overline{WS}$

   e. $\overline{RS} \cup \overline{ST} \cup \overline{TW}$

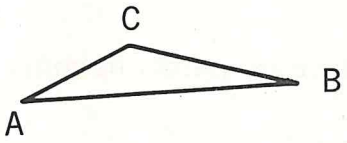
   f. $\overline{RS} \cup \overline{ST} \cup \overline{TW} \cup \overline{WS} \cup \overline{RW}$

1. Write a set description for each figure.

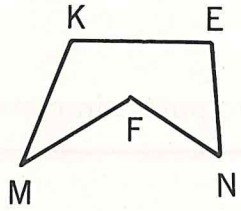
a.



b.



c.



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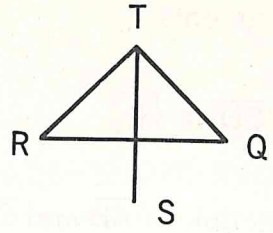

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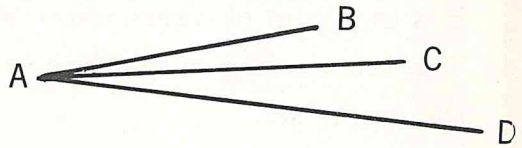

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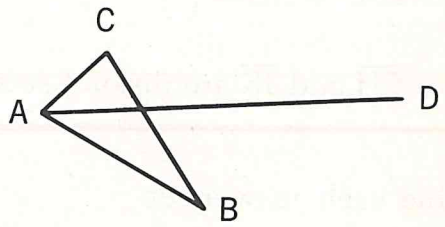
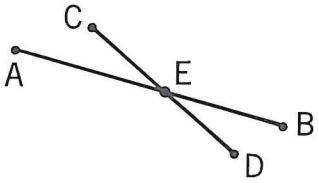


2. a. Line segment GH has endpoints G and ■.
   b. Line segment GH is a set of ■.

3. a. Triangle PQR is the union of sets $\overline{PQ}$, $\overline{QR}$, and ■.
   b. Triangle PQR is a set of ■.

4. Find each answer set.
   a. {Utah, Texas, Ohio, Idaho, Iowa} ∪ {Ohio, Texas, Iowa} = ■
   b. {Pat, Kit} ∪ {Pat, Kit, Helen} = ■
   c. {1, 2, 3} ∪ {4, 5} = ■
   d. {a, b, c} ∪ {c, b, a} = ■
   e. {10, 20, 30, 40} ∪ {50} = ■

The intersection of two line segments is the set of points common to the two line segments.



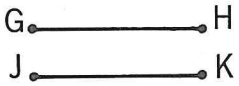
$\overline{AB} \cap \overline{CD} = \{E\}$

The intersection of $\overline{AB}$ and $\overline{CD}$ is the set whose only member is point E.

E is the point of intersection of $\overline{AB}$ and $\overline{CD}$.

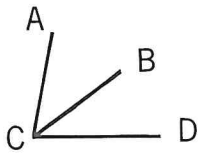
The intersection of two line segments that have no points in common is the empty set.

$\overline{GH} \cap \overline{JK} = \{\ \}$

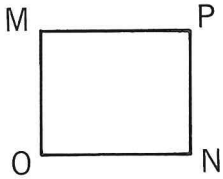


$\overline{GH}$ and $\overline{JK}$ are disjoint sets since they have no points in common.

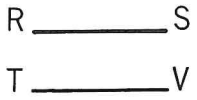
1. Find each answer set.



a. $\overline{CA} \cap \overline{CB} \cap \overline{CD} =$



b. $\overline{MP} \cap \overline{PN} =$



c. $\overline{RS} \cap \overline{TV} =$

2. Match each set description with a drawing.

a. $\overline{AC} \cup \overline{BC}$

b. $\overline{AC} \cap \overline{BD} = \{G\}$

c. $\overline{AC} \cap \overline{DE} = \{\ \}$

Figure 1

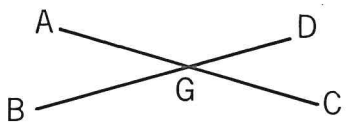


Figure 2

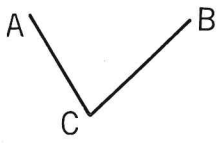
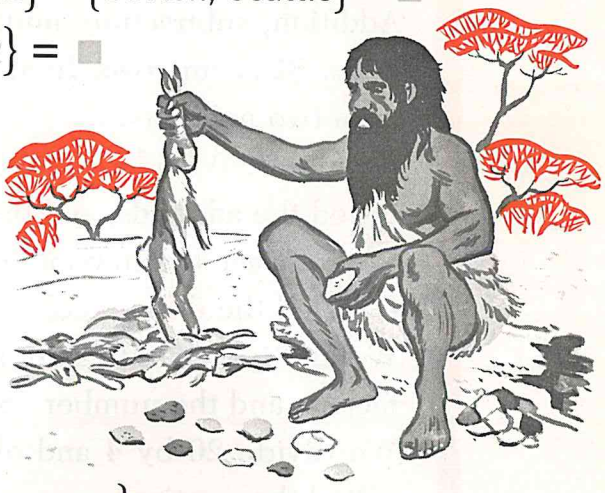


Figure 3

Feedback

Find each answer set.

1. a. $\{\text{Boston, Chicago, San Francisco, Dallas}\} \cup \{\text{Boston, Seattle}\} =$ ■
   b. $\{n: 0 < n < 2\} \cup \{n: 10 < n < 12\} =$ ■
   c. $\{n: n < 5\} \cup \{n: n < 7\} =$ ■



2. a. $\{\text{June, July}\} \cap \{\text{May, June}\} =$ ■
   b. $\{\text{Ohio, Maine, Alaska}\} \cap \{\text{Texas}\} =$ ■
   c. $\{3, 4, 5\} \cap \{n: n < 7\} =$ ■

3. a. $\{\text{California, Vermont, Kentucky}\} - \{\text{Kentucky}\} =$ ■
   b. $\{\text{St. Louis, Madison, Cleveland}\} - \{\text{Cleveland, Denver}\} =$ ■
   c. $\{2, 4, 6, 8\} - \{n: 4 < n < 8\} =$ ■

4. Match each set description with a figure.
   a. $\overline{AB} \cup \overline{BD} \cup \overline{DA}$
   b. $\overline{AB} \cap \overline{CD} = \{E\}$

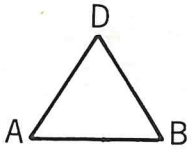


Figure 1

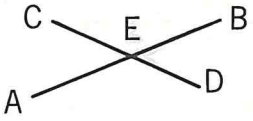


Figure 2

5. True or false?
   a. 1 is a member of $\{1, 3, 5\}$
   b. 5 is a member of $\{4, 6, 8\}$
   c. Hank is a member of $\{\text{Tom, Hank}\}$
   d. $0 \in \{0, 1, 5\}$
   e. $3 \in \{2, 4, 6\}$
   f. $10 \in \{1, 10, 100, 1000\}$

Part 2 Laws of Arithmetic

In arithmetic the main operations on numbers are addition, subtraction, multiplication, and division.

Addition, subtraction, multiplication, and division are all **binary** operations. **Bi** means **two.** In all binary operations we obtain a third number from two numbers.

Examples: We add 4 and 5 and obtain 9. The two numbers 4 and 5 are called the **addends,** and the number 9 is called the **sum.**

We subtract 4 from 6 and obtain 2. The resulting number in subtraction is called the **difference.**

We multiply 4 and 7 and obtain 28. The two numbers 4 and 7 are called **factors,** and the number 28 is called the **product.**

We divide 20 by 4 and obtain 5. The resulting number in division is called the **quotient.**

Choose +, −, ×, or ÷. Give all possible operations.

1. 17 ○ 12 = 5
2. 2 ○ 2 = 4
3. 15 ○ 5 = 3
4. 8 ○ 7 = 56
5. 4 ○ 2 = 2
6. 18 ○ 6 = 3
7. 21 ○ 1 = 21
8. 4 ○ 4 = 16
9. 72 ○ 9 = 8
10. 6 ○ 0 = 6
11. 1 ○ 1 = 1
12. 8 ○ 2 = 6
13. 12 ○ 12 = 144
14. 0 ○ 4 = 0
15. 27 ○ 7 = 20

Complete each operation. In each case tell whether you have found a sum, a difference, a product, or a quotient.

16. 8 + 5 = ■
17. 0 × 8 = ■
18. 59 − 19 = ■
19. 9 ÷ 1 = ■
20. 9 × 6 = ■
21. 41 − 19 = ■
22. 59 + 29 = ■
23. 8 × 7 = ■
24. 36 ÷ 6 = ■
25. 14 − 9 = ■
26. 40 ÷ 8 = ■
27. 38 + 45 = ■

Binary Operations

Complete each problem.

1. $12 + 9 = \blacksquare$
2. $9 \times 8 = \blacksquare$
3. $41 \div 1 = \blacksquare$
4. $7 \times 8 = \blacksquare$
5. $14 + 9 = \blacksquare$
6. $0 \div 1 = \blacksquare$
7. $9 \times 7 = \blacksquare$
8. $36 \div 4 = \blacksquare$
9. $0 + 19 = \blacksquare$
10. $14 - 0 = \blacksquare$
11. $3 \times 9 = \blacksquare$
12. $14 - 6 = \blacksquare$
13. $0 \times 8 = \blacksquare$
14. $9 + 7 = \blacksquare$
15. $8 \times 8 = \blacksquare$

Solve each equation.

16. $7 \times 7 = n$
17. $27 \div 3 = n$
18. $8 + 7 = n$
19. $63 \div 9 = n$
20. $11 - 6 = n$
21. $13 - n = 8$
22. $56 \div n = 8$
23. $9 + n = 14$
24. $9 \times n = 72$
25. $9 + n = 18$
26. $n - 9 = 6$
27. $n \times 8 = 64$
28. $n + 8 = 16$
29. $n \div 6 = 7$
30. $n - 8 = 6$

Choose $=$ or $\neq$.

31. a. $6 + 3 \bigcirc 3 + 6$
    b. $6 - 3 \bigcirc 3 - 6$
    c. $6 \times 3 \bigcirc 3 \times 6$
    d. $6 \div 3 \bigcirc 3 \div 6$

32. a. $(8 + 4) + 2 \bigcirc 8 + (4 + 2)$
    b. $(8 - 4) - 2 \bigcirc 8 - (4 - 2)$
    c. $(8 \times 4) \times 2 \bigcirc 8 \times (4 \times 2)$
    d. $(8 \div 4) \div 2 \bigcirc 8 \div (4 \div 2)$

Two questions may be asked about each operation.

Question 1: Does the order in which we operate on two numbers change the answer?

If the order in which we operate on two numbers does not change the answer, the operation is **commutative.**

Question 2: If we operate on more than two numbers, does the way in which we pair the numbers change the answer?

If the way in which we pair the numbers does not change the answer, the operation is **associative.**

Addition

Find each sum.

1. a. $9 + 4 =$ ▢
   b. $4 + 9 =$ ▢
2. a. $8 + 7 =$ ▢
   b. $7 + 8 =$ ▢
3. a. $67 + 35 =$ ▢
   b. $35 + 67 =$ ▢
4. a. $0 + 93 =$ ▢
   b. $93 + 0 =$ ▢
5. a. $46 + 38 =$ ▢
   b. $38 + 46 =$ ▢
6. a. $27 + 57 =$ ▢
   b. $57 + 27 =$ ▢
7. a. $58 + 19 =$ ▢
   b. $19 + 58 =$ ▢
8. a. $48 + 25 =$ ▢
   b. $25 + 48 =$ ▢
9. a. $36 + 59 =$ ▢
   b. $59 + 36 =$ ▢

The order in which two numbers are added does not change the answer. For any two numbers m and n,

$$m + n = n + m.$$

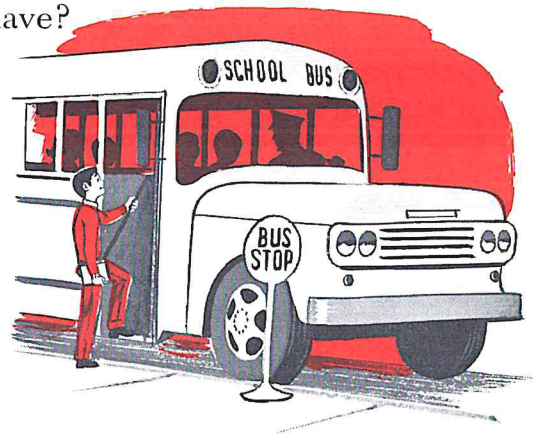
Addition is commutative.

Find each sum.

10. 85 46	11. 96 87	12. 39 56	13. 97 87	14. 38 28
15. 46 85	16. 87 96	17. 56 39	18. 87 97	19. 28 38

20. Mary has 38 books, Joe has 45 books, and Jean has 45 books. If Mary and Joe together have the same number of books that Jean and Don have together, how many books does Don have?

21. Mark can ride to school by two different routes. One route is five miles north and three miles east. The other route is three miles east and five miles north. Are the distances the same?



Copy and complete each problem.

$(4 + 5) + 3 = 9 + 3$
$= 12$

$4 + (5 + 3) = 4 + 8$
$= 12$

1. a. $(7 + 3) + 8 =$ ■
   b. $7 + (3 + 8) =$ ■

2. a. $\frac{1}{6} + \left(\frac{2}{6} + \frac{3}{6}\right) =$ ■
   b. $\left(\frac{1}{6} + \frac{2}{6}\right) + \frac{3}{6} =$ ■

3. a. $8 + (0 + 6) =$ ■
   b. $(8 + 0) + 6 =$ ■

4. a. $\frac{1}{8} + \left(\frac{5}{8} + \frac{3}{8}\right) =$ ■
   b. $\left(\frac{1}{8} + \frac{5}{8}\right) + \frac{3}{8} =$ ■

5. a. $8 + (8 + 7) =$ ■
   b. $(8 + 8) + 7 =$ ■

6. a. $26 + (14 + 19) =$ ■
   b. $(26 + 14) + 19 =$ ■

7. a. $\left(\frac{1}{5} + \frac{3}{5}\right) + \frac{2}{5} =$ ■
   b. $\frac{1}{5} + \left(\frac{3}{5} + \frac{2}{5}\right) =$ ■

8. a. $58 + (29 + 16) =$ ■
   b. $(58 + 29) + 16 =$ ■

9. a. $\frac{1}{12} + \left(\frac{3}{12} + \frac{7}{12}\right) =$ ■
   b. $\left(\frac{1}{12} + \frac{3}{12}\right) + \frac{7}{12} =$ ■

10. a. $55 + (82 + 17) =$ ■
    b. $(55 + 82) + 17 =$ ■

The pairing of numbers does not change the answer in addition.
For any numbers, m, n, and p,

$$(m + n) + p = m + (n + p).$$

Addition is associative.

Find each sum.

11. 846 + 293 + 147

12. 687 + 352 + 438

13. 147 + 293 + 846

14. 352 + 438 + 687

15. 307 + 294 + 348

16. 726 + 184 + 325

17. 362 + 188 + 607

18. 346 + 276 + 525

Subtraction

Choose $=$ or $\neq$.

1. $1 - 4 \bigcirc 4 - 1$
2. $3 - 7 \bigcirc 7 - 3$
3. $0 - 8 \bigcirc 8 - 0$
4. $9 - 9 \bigcirc 9 - 9$
5. $50 - 13 \bigcirc 13 - 50$
6. $76 - 12 \bigcirc 12 - 76$

Does the order in which two numbers are subtracted change the answer?
Kevin has six dollars. He wants to spend four dollars on a birthday present for his sister. He has enough money.
Joe has four dollars. He wants to buy a present for his mother that costs six dollars. He does not have enough money.
It is not the same thing to have six dollars and to spend four dollars as it is to have four dollars and to spend six dollars.

$$6 - 4 \neq 4 - 6$$

For any two numbers m and n, if $m \neq n$,

$$m - n \neq n - m.$$

Subtraction is not commutative.

Find each difference.

7. $\begin{array}{r} 96 \\ -49 \\ \hline \end{array}$	8. $\begin{array}{r} 67 \\ -38 \\ \hline \end{array}$	9. $\begin{array}{r} 96 \\ -58 \\ \hline \end{array}$	10. $\begin{array}{r} 74 \\ -39 \\ \hline \end{array}$	11. $\begin{array}{r} 82 \\ -55 \\ \hline \end{array}$
12. $\begin{array}{r} 75 \\ -69 \\ \hline \end{array}$	13. $\begin{array}{r} 62 \\ -28 \\ \hline \end{array}$	14. $\begin{array}{r} 80 \\ -34 \\ \hline \end{array}$	15. $\begin{array}{r} 404 \\ -167 \\ \hline \end{array}$	16. $\begin{array}{r} 600 \\ -148 \\ \hline \end{array}$
17. $\begin{array}{r} 6843 \\ -2693 \\ \hline \end{array}$	18. $\begin{array}{r} 2602 \\ -1384 \\ \hline \end{array}$	19. $\begin{array}{r} 4392 \\ -1928 \\ \hline \end{array}$	20. $\begin{array}{r} 4200 \\ -1683 \\ \hline \end{array}$	

21. If Sam has three apples, can he eat two? If he has only two apples, can he eat three?

Copy and complete each problem.

$(9 - 3) - 1 = 6 - 1$
$= 5$

$9 - (3 - 1) = 9 - 2$
$= 7$

1. a. $(13 - 7) - 6 =$ ■
   b. $13 - (7 - 6) =$ ■

2. a. $\left(\frac{5}{4} - \frac{2}{4}\right) - \frac{1}{4} =$ ■
   b. $\frac{5}{4} - \left(\frac{2}{4} - \frac{1}{4}\right) =$ ■

3. a. $4 - (3 - 0) =$ ■
   b. $(4 - 3) - 0 =$ ■

4. a. $18 - (18 - 0) =$ ■
   b. $(18 - 18) - 0 =$ ■

5. a. $(38 - 22) - 15 =$ ■
   b. $38 - (22 - 15) =$ ■

6. a. $42 - (20 - 19) =$ ■
   b. $(42 - 20) - 19 =$ ■

The way in which the numbers are paired in subtraction changes the answer. For any three numbers m, n, and p, if $p \neq 0$,

$$(m - n) - p \neq m - (n - p).$$

We must use parentheses to show how to pair the numbers.

Find each sum or difference.

7. $\begin{array}{r} 846 \\ +184 \\ \hline \end{array}$

8. $\begin{array}{r} 293 \\ -145 \\ \hline \end{array}$

9. $\begin{array}{r} 702 \\ -289 \\ \hline \end{array}$

10. $\begin{array}{r} 503 \\ +249 \\ \hline \end{array}$

11. $\begin{array}{r} 6813 \\ +2467 \\ \hline \end{array}$

12. $\begin{array}{r} 8428 \\ -2379 \\ \hline \end{array}$

13. $\begin{array}{r} 1568 \\ +2897 \\ \hline \end{array}$

14. $\begin{array}{r} 7300 \\ -1683 \\ \hline \end{array}$

15. There are 31 days in January, 28 days in February, and 31 days in March. How many days are there all together in January, February, and March?

16. Joan saved 268 pennies. She spent 29 of them for a comb. How many pennies did she have left?

Multiplication

Find each product.

1. a. $4 \times 8 =$ ▢
   b. $8 \times 4 =$ ▢

2. a. $7 \times 0 =$ ▢
   b. $0 \times 7 =$ ▢

3. a. $8 \times 5 =$ ▢
   b. $5 \times 8 =$ ▢

4. a. $9 \times 10 =$ ▢
   b. $10 \times 9 =$ ▢

5. a. $1 \times 6 =$ ▢
   b. $6 \times 1 =$ ▢

6. a. $7 \times 9 =$ ▢
   b. $9 \times 7 =$ ▢

The order in which two numbers are multiplied does not change the answer. For any two numbers m and n,

$$m \times n = n \times m.$$

Multiplication is commutative.

Find each product.

7. a. $8 \times 9 =$ ▢
   b. $6 \times 0 =$ ▢
   c. $7 \times 3 =$ ▢
   d. $4 \times 4 =$ ▢
   e. $9 \times 5 =$ ▢
   f. $8 \times 7 =$ ▢
   g. $9 \times 9 =$ ▢

8. a. $6 \times 9 =$ ▢
   b. $8 \times 8 =$ ▢
   c. $9 \times 4 =$ ▢
   d. $7 \times 6 =$ ▢
   e. $5 \times 5 =$ ▢
   f. $0 \times 8 =$ ▢
   g. $7 \times 8 =$ ▢

9. a. $3 \times 9 =$ ▢
   b. $8 \times 6 =$ ▢
   c. $0 \times 5 =$ ▢
   d. $10 \times 6 =$ ▢
   e. $6 \times 6 =$ ▢
   f. $4 \times 10 =$ ▢
   g. $7 \times 5 =$ ▢

10. Mrs. Gates baked 3 pans of cupcakes today. If each pan contained 8 cupcakes, how many cupcakes did she bake?

11. Miss Thomas made 10 pairs of gloves one day. How many gloves did she make that day?

12. Miss Park worked 7 hours each day for 5 days one week. How many hours did she work that week?

Copy and complete each problem.

$(4 \times 2) \times 3 = 8 \times 3$
$= 24$

$4 \times (2 \times 3) = 4 \times 6$
$= 24$

1. a. $(2 \times 3) \times 6 =$ ■
   b. $2 \times (3 \times 6) =$ ■

2. a. $(5 \times 2) \times 4 =$ ■
   b. $5 \times (2 \times 4) =$ ■

3. a. $4 \times (3 \times 3) =$ ■
   b. $(4 \times 3) \times 3 =$ ■

4. a. $8 \times (1 \times 3) =$ ■
   b. $(8 \times 1) \times 3 =$ ■

5. a. $(0 \times 4) \times 7 =$ ■
   b. $0 \times (4 \times 7) =$ ■

6. a. $(5 \times 2) \times 2 =$ ■
   b. $5 \times (2 \times 2) =$ ■

7. a. $10 \times (0 \times 6) =$ ■
   b. $(10 \times 0) \times 6 =$ ■

8. a. $1 \times (8 \times 4) =$ ■
   b. $(1 \times 8) \times 4 =$ ■

The way in which numbers are paired in multiplication does not change the answer.
For any three numbers m, n, and p,

$$(m \times n) \times p = m \times (n \times p).$$

Multiplication is associative.

Find each product.

9. $2 \times 5 \times 3 =$ ■
10. $5 \times 0 \times 6 =$ ■
11. $2 \times 5 \times 4 =$ ■
12. $3 \times 2 \times 9 =$ ■
13. $9 \times 6 \times 0 =$ ■
14. $4 \times 2 \times 3 =$ ■
15. $5 \times 2 \times 5 =$ ■
16. $0 \times 8 \times 4 =$ ■
17. $2 \times 2 \times 2 =$ ■
18. $2 \times 3 \times 5 =$ ■

19. Does the way in which numbers are paired in multiplication change the answer?

Division

Find each quotient.

1. a. $4 \div 1 =$ ▢
   b. $1 \div 4 =$ ▢

2. a. $1 \div 7 =$ ▢
   b. $7 \div 1 =$ ▢

3. a. $6 \div 1 =$ ▢
   b. $1 \div 6 =$ ▢

4. a. $2 \div 1 =$ ▢
   b. $1 \div 2 =$ ▢

5. a. $10 \div 1 =$ ▢
   b. $1 \div 10 =$ ▢

6. a. $3 \div 1 =$ ▢
   b. $1 \div 3 =$ ▢

The order in which two numbers are divided changes the answer.
For any two numbers m and n, if $m \neq n$, and $m \neq 0$ and $n \neq 0$,

$$m \div n \neq n \div m.$$

Division is not commutative.

Find each quotient.

7. a. $80 \div 10 =$ ▢
   b. $40 \div 8 =$ ▢
   c. $24 \div 3 =$ ▢
   d. $45 \div 9 =$ ▢
   e. $70 \div 7 =$ ▢
   f. $72 \div 9 =$ ▢
   g. $36 \div 6 =$ ▢

8. a. $42 \div 6 =$ ▢
   b. $56 \div 7 =$ ▢
   c. $81 \div 9 =$ ▢
   d. $24 \div 6 =$ ▢
   e. $25 \div 5 =$ ▢
   f. $18 \div 2 =$ ▢
   g. $30 \div 5 =$ ▢

9. a. $64 \div 8 =$ ▢
   b. $36 \div 4 =$ ▢
   c. $35 \div 5 =$ ▢
   d. $49 \div 7 =$ ▢
   e. $27 \div 3 =$ ▢
   f. $60 \div 10 =$ ▢
   g. $48 \div 8 =$ ▢

10. Miss Rossi packed 6 books in each carton. If she packed 24 books, how many cartons did she need?

11. Is it the same to divide 24 cookies equally among 8 children as it is to divide 8 cookies equally among 24 children?

12. Sandy paid thirty-two cents for eight ballons. How much did she pay for each one?

13. The area of a rectangle is thirty-six square units. The width is four units. What is the length?

Copy and complete each problem.

$(8 \div 4) \div 2 = 2 \div 2 = 1$ $\qquad$ $8 \div (4 \div 2) = 8 \div 2 = 4$

1. a. $(16 \div 4) \div 2 =$ ■
   b. $16 \div (4 \div 2) =$ ■

2. a. $15 \div (5 \div 1) =$ ■
   b. $(15 \div 5) \div 1 =$ ■

3. a. $(32 \div 4) \div 4 =$ ■
   b. $32 \div (4 \div 4) =$ ■

4. a. $54 \div (6 \div 3) =$ ■
   b. $(54 \div 6) \div 3 =$ ■

5. a. $8 \div (2 \div 2) =$ ■
   b. $(8 \div 2) \div 2 =$ ■

6. a. $(24 \div 6) \div 2 =$ ■
   b. $24 \div (6 \div 2) =$ ■

The way in which numbers are paired in division changes the answer.
For any three non-zero numbers m, n, and p, if $p \neq 1$,

$$(m \div n) \div p \neq m \div (n \div p).$$

Division is not associative.
We must use parentheses to show how to pair the numbers.

Find each sum, difference, product, or quotient.

7. $82 - 14 =$ ■
8. $16 + 85 =$ ■
9. $8 \times 7 =$ ■
10. $45 \div 9 =$ ■
11. $17 - 8 =$ ■
12. $47 + 23 =$ ■
13. $0 \times 3 =$ ■
14. $34 - 28 =$ ■
15. $81 \div 9 =$ ■
16. $8 \times 8 =$ ■
17. $38 - 19 =$ ■
18. $72 \div 8 =$ ■
19. $46 + 25 =$ ■
20. $9 + 7 =$ ■
21. $80 \div 10 =$ ■

Choose $=$ or $\neq$.

22. $1 \div 12$ ● $12 \div 1$
23. 5×17 ● 17×5
24. $6 \div 6$ ● $6 \div 6$
25. $8 + (9 + 3)$ ● $(8 + 9) + 3$
26. $(18 \div 6) \div 3$ ● $18 \div (6 \div 3)$
27. $21 \div (7 \div 1)$ ● $(21 \div 7) \div 1$

28. Is division associative?

We can perform several binary operations. Parentheses show how to group the numbers. Complete each problem.

$$9 \div (2 + 1) = 9 \div 3$$
$$= 3$$

1. $4 \times (3 + 5) =$ ▢
2. $12 \div (2 + 4) =$ ▢
3. $(4 + 3) \times 5 =$ ▢
4. $(3 + 5) \times 4 =$ ▢
5. $18 \div (3 + 6) =$ ▢
6. $(4 \times 3) + (4 \times 5) =$ ▢
7. $(12 \div 2) + (12 \div 4) =$ ▢
8. $(4 \times 5) + (3 \times 5) =$ ▢
9. $(3 \times 4) + (5 \times 4) =$ ▢
10. $(18 \div 3) + (18 \div 6) =$ ▢

Complete each problem.

11. $h = 5 \times (4 + 3)$
    $j = (5 \times 4) + (5 \times 3)$
12. $h = 8 \times (3 + 7)$
    $j = (8 \times 3) + (8 \times 7)$
13. $h = 2 \times (5 + 2)$
    $j = (2 \times 5) + (2 \times 2)$
14. $h = 3 \times (6 + 4)$
    $j = (3 \times 6) + (3 \times 4)$
15. Does h equal j in problems 11-14?

Sometimes the same numbers and the same operations performed in two different ways give the same answer.
For any numbers m, n, and p,

$$m \times (n + p) = (m \times n) + (m \times p).$$

This is the **distributive law for multiplication over addition.**

Copy and complete each problem.

16. $5 \times (8 + 2) =$ ▢
17. ▢ $\times (2 + 3) = 20$
18. ▢ $\times (4 + 7) = 22$
19. $5 \times (6 +$ ▢ $) = 35$
20. $4 \times (10 + 2) = (4 \times$ ▢ $) + (4 \times 2)$
21. $5 \times (7 + 3) = ($ ▢ $\times 7) + ($ ▢ $\times 3)$
22. $6 \times (3 + 6) = ($ ▢ $\times 3) + ($ ▢ $\times 6)$
23. $7 \times (4 + 5) =$ ▢

Find f and g in each problem.

1. $f = (3 + 5) \times 7$
   $g = (3 \times 7) + (5 \times 7)$

2. $f = (4 + 3) \times 5$
   $g = (4 \times 5) + (3 \times 5)$

3. $f = (9 + 0) \times 8$
   $g = (9 \times 8) + (0 \times 8)$

4. $f = (3 + 5) \times 4$
   $g = (3 \times 4) + (5 \times 4)$

5. Does f equal g in each problem?

There are two forms of the distributive law for multiplication over addition.

For any numbers m, n, and p,

$m \times (n + p) = (m \times n) + (m \times p)$ and $(n + p) \times m = (n \times m) + (p \times m)$.

Copy and complete each problem.

6. $(6 + 3) \times 5 = (6 \times \square) + (3 \times \square)$
7. $(4 + 6) \times 3 = (\square \times 3) + (6 \times 3)$
8. $(5 + 4) \times 2 = \square$
9. $(9 + 1) \times \square = 30$

Find q and r in each problem.

10. $q = 5 \times (3 - 2)$
    $r = (5 \times 3) - (5 \times 2)$

11. $q = 4 \times (6 - 2)$
    $r = (4 \times 6) - (4 \times 2)$

12. $q = 7 \times (4 - 2)$
    $r = (7 \times 4) - (7 \times 2)$

13. $q = 8 \times (7 - 2)$
    $r = (8 \times 7) - (8 \times 2)$

14. Does q equal r in each problem?

For any numbers m, n, and p,

$$m \times (n - p) = (m \times n) - (m \times p).$$

This is the **distributive law for multiplication over subtraction.**

Find q and r in each problem.

1. $q = (8 + 4) \div 2$
   $r = (8 \div 2) + (4 \div 2)$
2. $q = (6 + 9) \div 3$
   $r = (6 \div 3) + (9 \div 3)$
3. $q = (10 + 5) \div 5$
   $r = (10 \div 5) + (5 \div 5)$
4. $q = (20 + 8) \div 4$
   $r = (20 \div 4) + (8 \div 4)$

5. Does r equal q in each problem?

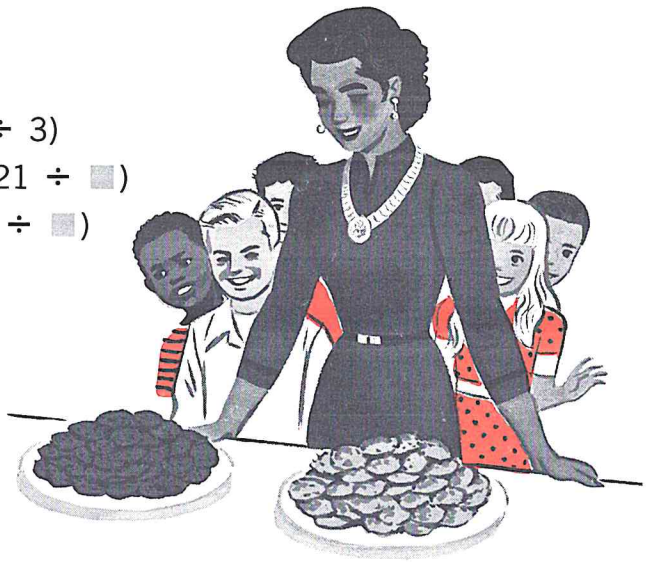
For any numbers m, n, and p, if $p \neq 0$,

$$(m + n) \div p = (m \div p) + (n \div p).$$

This is the **distributive law for division over addition.**

Copy and complete each problem.

6. $(18 + 9) \div 3 = (\square \div 3) + (9 \div 3)$
7. $(14 + 21) \div 7 = (14 \div \square) + (21 \div \square)$
8. $(6 + 10) \div 2 = (6 \div \square) + (10 \div \square)$
9. $(5 + 20) \div 5 = \square$
10. $(8 + 8) \div 4 = \square$
11. $(12 + 6) \div 3 = \square$
12. $(6 + \square) \div 3 = 3$
13. $(\square + 14) \div 7 = 3$
14. $(9 + 6) \div \square = 3$



15. Mrs. Gomez bought two dozen chocolate cookies and three dozen oatmeal cookies to divide equally among the six children at the party. How many cookies did each child get?

16. During 5 days Judy read 35 pages of one book and 45 pages of a second book. If she read the same number of pages each day, how many pages did she read each day?

Find r and q in each problem.

1. $r = 12 \div (4 + 2)$
   $q = (12 \div 4) + (12 \div 2)$

2. $r = 18 \div (6 + 3)$
   $q = (18 \div 6) + (18 \div 3)$

3. $r = 12 \div (1 + 3)$
   $q = (12 \div 1) + (12 \div 3)$

4. $r = 30 \div (5 + 5)$
   $q = (30 \div 5) + (30 \div 5)$

5. Does r equal q in any of the problems?

There is only one form of the distributive law for division.
For any numbers m, n, and p,

$$(n + p) \div m = (n \div m) + (p \div m),$$

but usually,

$$m \div (n + p) \neq (m \div n) + (m \div p).$$

Find a and b in each problem.

6. $a = 5 + (3 \times 2)$
   $b = (5 + 3) \times (5 + 2)$

7. $a = 4 + (3 \times 2)$
   $b = (4 + 3) \times (4 + 2)$

8. Does a equal b in either problem?
9. Is addition distributive over multiplication?

Choose $=$ or $\neq$.

10. $(6 + 4) \times 2 \bigcirc (6 \times 2) + (4 \times 2)$
11. $(12 + 8) \div 4 \bigcirc (12 \div 4) + (8 \div 4)$
12. $5 \times (8 - 3) \bigcirc (5 \times 8) - (5 \times 3)$
13. $4 + (6 \times 2) \bigcirc (4 + 6) \times (4 + 2)$
14. $3 \times (4 + 6) \bigcirc (3 \times 4) + (3 \times 6)$
15. $8 \div (4 + 4) \bigcirc (8 \div 4) + (8 \div 4)$
16. $(6 + 9) \div 3 \bigcirc (6 \div 3) + (9 \div 3)$
17. $40 \div (5 + 5) \bigcirc (40 \div 5) + (40 \div 5)$
18. $7 \times (6 + 2) \bigcirc (7 \times 6) + (7 \times 2)$

The Identity Element in Addition

If we add **zero** to any number, the sum is that number.

$0 + 1 = 1$ $\quad$ $2 + 0 = 2$

$1 + 0 = 1$ $\quad$ $0 + 2 = 2$

For any number n,

$n + 0 = n$ and $0 + n = n$.

Zero is the **identity element** in addition.

Find each sum.

1. $0 + 5 =$ ▢
2. $5 + 0 =$ ▢
3. $19 + 0 =$ ▢
4. $185 + 0 =$ ▢
5. $0 + \frac{1}{2} =$ ▢
6. $0 + a =$ ▢

Find each missing addend.

7. $0 +$ ▢ $= 7$
8. ▢ $+ 15 = 15$
9. ▢ $+ 0 = 0$
10. $x +$ ▢ $= x$
11. $18 +$ ▢ $= 18$
12. ▢ $+ 73 = 73$
13. $17 +$ ▢ $= 17$
14. $\frac{1}{3} +$ ▢ $= \frac{1}{3}$
15. ▢ $+ 0 = a$

If we subtract zero from any number, the difference is that number.

$3 - 0 = 3$ $\quad$ $5 - 0 = 5$

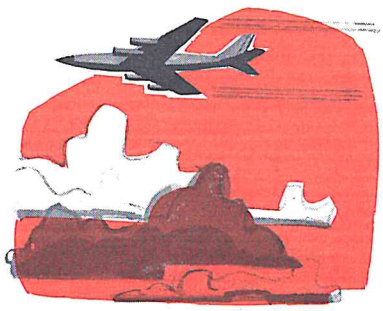
For any number n,

$n - 0 = n$.

Find each difference.

16. $6 - 0 =$ ▢
17. $28 - 0 =$ ▢
18. $48 - 0 =$ ▢
19. $341 - 0 =$ ▢
20. $\frac{1}{4} - 0 =$ ▢
21. $a - 0 =$ ▢

22. A plane is flying at an altitude of 30,000 feet. For the next hour it neither gains nor loses altitude. What is its altitude at the end of the hour?



The Identity Element in Multiplication

If we multiply **one** by any other number, the product is that number.

$1 \times 1 = 1$ $\quad 3 \times 1 = 3$ $\quad 24 \times 1 = 24$

For any number n,

$1 \times n = n$ and $n \times 1 = n$.

One is the **identity element** in multiplication.

Find each product.

1. $6 \times 1 =$ ■
2. $1 \times 8 =$ ■
3. $92 \times 1 =$ ■
4. $63 \times 1 =$ ■
5. $100 \times 1 =$ ■
6. $1 \times 203 =$ ■

Find each missing factor.

7. ■ $\times 6 = 6$
8. $1 \times$ ■ $= 18$
9. $1 \times$ ■ $= 35$
10. ■ $\times 15 = 15$
11. $8 \times$ ■ $= 8$
12. $17 \times$ ■ $= 17$
13. ■ $\times 1 = \frac{1}{2}$
14. $m \times$ ■ $= m$
15. ■ $\times a = a$

If we divide any number by one, the quotient is that number.

$6 \div 1 = 6$ $\quad 31 \div 1 = 31$ $\quad 103 \div 1 = 103$

For any number n,

$n \div 1 = n$.

If we divide any number other than zero by itself, the quotient is one.

$8 \div 8 = 1$ $\quad 46 \div 46 = 1$ $\quad 157 \div 157 = 1$

For any number n, if $n \neq 0$,

$n \div n = 1$.

Find each missing number.

16. ■ $\div 1 = 14$
17. $23 \div$ ■ $= 23$
18. $67 \div 1 =$ ■
19. $16 \div 16 =$ ■
20. $74 \div$ ■ $= 74$
21. $m \div$ ■ $= m$
22. ■ $\div 29 = 1$
23. $85 \div 85 =$ ■
24. $43 \div$ ■ $= 1$
25. ■ $\div 24 = 1$
26. ■ $\div a = 1$
27. $a \div a =$ ■

Giving Reasons

Give a reason why each equation is true.

This equation is true.	Reason:
$17 \times 4 = 4 \times 17$	multiplication is commutative
$8 - 0 = 8$	$n - 0 = n$
$42 \times 1 = 42$	$n \times 1 = n$
$18 + 30 = 30 + 18$	addition is commutative
$(20 + 5) \div 5 = (20 \div 5) + (5 \div 5)$	the distributive law for division
$6 \times (4 \times 2) = (6 \times 4) \times 2$	multiplication is associative
$16 + 0 = 16$	$n + 0 = n$
$6 \div 6 = 1$	$n \div n = 1$
$(9 + 4) + 2 = 9 + (4 + 2)$	addition is associative
$8 \div 1 = 8$	$n \div 1 = n$

1. $(4 \times 2) \times 5 = 4 \times 2 \times 5$
2. $9 \times 7 = 7 \times 9$
3. $17 + 4 = 4 + 17$
4. $(5 + 7) + 2 = 2 + (5 + 7)$
5. $8 \times 1 = 8$
6. $(5 + 2) \times 4 = 4 \times (5 + 2)$
7. $0 + 284 = 284$
8. $(2 \times 4) \times 3 = 2 \times (4 \times 3)$
9. $38 - 0 = 38$
10. $(9 + 3) + 7 = 9 + (3 + 7)$
11. $(8 + 0) \times 17 = 8 \times 17$
12. $12 \div 12 = 1$
13. $0 + 45 = 45$
14. $(8 + 4) + 2 = (4 + 8) + 2$
15. $(4 \times 5) \times 7 = 7 \times (4 \times 5)$
16. $(10 - 2) \times 8 = 8 \times (10 - 2)$
17. $(2 \times 3) \times 6 = (3 \times 2) \times 6$
18. $(7 - 5) + 3 = 3 + (7 - 5)$
19. $m \times (n + p) = (n + p) \times m$
20. $8 \div 1 = 8$
21. $(6 + 3) \times 8 = (6 \times 8) + (3 \times 8)$
22. $(6 + 3) \times 8 = 8 \times (6 + 3)$
23. $8 \times (6 + 3) = (8 \times 6) + (8 \times 3)$
24. $(6 + 9) \div 3 = (6 \div 3) + (9 \div 3)$
25. $(12 + 15) \div 3 = (12 \div 3) + (15 \div 3)$
26. $(3 + 4) \times 7 = (3 \times 7) + (4 \times 7)$

27. What is the identity element in addition?

28. What is the identity element in multiplication?

Feedback

Find each answer set.

1. a. $\{1, 2, 3\} \cup \{4, 5, 6\} =$ ■
   b. $\{\text{Jean, Mary}\} \cup \{\text{Kate}\} =$ ■
   c. $\{4, 8, 12\} \cup \{2, 4, 6, 8, 10, 12\} =$ ■

2. a. $\{\text{Illinois, Indiana}\} \cap \{\text{New York}\} =$ ■
   b. $\{2, 4, 6, 8, 10\} \cap \{4, 8, 12, 16\} =$ ■
   c. $\{1, 3, 5\} \cap \{1, 4, 7, 10\} =$ ■

3. a. $\{2, 4, 6, 8\} - \{4, 8\} =$ ■
   b. $\{1, 3, 5, 7\} - \{1, 2, 3\} =$ ■
   c. $\{\text{California, Alaska}\} - \{\text{Texas, California}\} =$ ■

True or false?

4. a. $\{a, b\} \approx \{1, 2\}$
   b. $\{a, b\} = \{1, 2\}$
   c. $a \in \{a, b\}$

5. a. $\{1, 3, 5\} \subset \{1, 3, 5, 7\}$
   b. $\{n: 1 < n < 3\} = \{1, 2, 3\}$
   c. $18 + 2 > 19$

6. a. $6 + n = n + 6$
   b. $38 \div 38 = 0$
   c. $24 \div 1 = 1 \div 24$

7. a. $4 \times (5 + 3) = (4 \times 5) + (4 \times 3)$
   b. $\left(\frac{1}{8} + \frac{2}{8}\right) + \frac{4}{8} = \frac{1}{8} + \left(\frac{2}{8} + \frac{4}{8}\right)$
   c. $(4 \times 2) \times 6 = 4 \times (2 \times 6)$

Part 3 Multiplication and Division

We can use the distributive law of multiplication over addition to multiply 7×28.

$$\begin{aligned} 7 \times 28 &= 7 \times (20 + 8) \\ &= (7 \times 20) + (7 \times 8) \\ &= 140 + 56 \\ &= 196 \end{aligned}$$

$$\begin{array}{r} 28 \\ \times\ 7 \\ \hline 56 \\ 140 \\ \hline 196 \end{array}$$

Find each product.

1. a. $6 \times 10 =$ ▢
   b. $10 \times 8 =$ ▢
   c. $7 \times 10 =$ ▢
   d. $4 \times 10 =$ ▢
   e. $10 \times 5 =$ ▢

2. a. $4 \times 20 = (4 \times 2) \times 10 =$ ▢
   b. $3 \times 40 = (3 \times 4) \times 10 =$ ▢
   c. $20 \times 4 = 4 \times 20 =$ ▢
   d. $40 \times 3 = 3 \times 40 =$ ▢
   e. $5 \times 20 = (5 \times 2) \times 10 =$ ▢

Use the distributive law to find each answer.

3. $6 \times 20 =$ ▢
4. $8 \times 74 =$ ▢
5. $4 \times 92 =$ ▢
6. $9 \times 57 =$ ▢
7. $3 \times 82 =$ ▢
8. $5 \times 99 =$ ▢
9. $6 \times 48 =$ ▢
10. $2 \times 76 =$ ▢
11. $4 \times 30 =$ ▢
12. $7 \times 63 =$ ▢
13. $9 \times 50 =$ ▢
14. $3 \times 29 =$ ▢
15. $5 \times 64 =$ ▢
16. $8 \times 49 =$ ▢
17. $6 \times 31 =$ ▢

Find each product.

18. $\begin{array}{r} 54 \\ \times\ 7 \\ \hline \end{array}$
19. $\begin{array}{r} 35 \\ \times\ 6 \\ \hline \end{array}$
20. $\begin{array}{r} 70 \\ \times\ 9 \\ \hline \end{array}$
21. $\begin{array}{r} 43 \\ \times\ 8 \\ \hline \end{array}$
22. $\begin{array}{r} 67 \\ \times\ 5 \\ \hline \end{array}$

23. $\begin{array}{r} 29 \\ \times\ 4 \\ \hline \end{array}$
24. $\begin{array}{r} 67 \\ \times\ 7 \\ \hline \end{array}$
25. $\begin{array}{r} 95 \\ \times\ 9 \\ \hline \end{array}$
26. $\begin{array}{r} 30 \\ \times\ 8 \\ \hline \end{array}$
27. $\begin{array}{r} 22 \\ \times\ 6 \\ \hline \end{array}$

Find each product.

1. $2 \times 57 =$ ■
2. $6 \times 94 =$ ■
3. $4 \times 67 =$ ■
4. $3 \times 17 =$ ■
5. $7 \times 89 =$ ■
6. $3 \times 48 =$ ■
7. $7 \times 55 =$ ■
8. $2 \times 96 =$ ■
9. $8 \times 81 =$ ■
10. $4 \times 75 =$ ■
11. $6 \times 69 =$ ■
12. $9 \times 34 =$ ■
13. $3 \times 64 =$ ■
14. $2 \times 36 =$ ■
15. $5 \times 42 =$ ■

16. Mr. Jacobs owns three hobby shops in different parts of the city. If he ordered these things for each store, how many of each item did he order all together?

a. 16 woodburning sets
b. 25 stamp albums
c. 72 model airplane kits
d. 34 books on paper folding
e. 46 chemistry sets
f. 68 jewelry-making kits
g. 85 boxes of water colors
h. 18 weaving sets

17. Mr. Martinez set up six food booths at the county fair. If he stocked these items in each booth, how many of each item did he need to order all together?

a. 12 jars of pickles
b. 7 jars of mustard
c. 24 heads of lettuce
d. 15 jars of mayonnaise
e. 30 cartons of hamburger buns
f. 96 packages of hot dogs
g. 82 pounds of hamburger
h. 46 tomatoes
i. 14 cartons of hot dog buns
j. 36 onions

18. Mr. Shannon raises flowers for a florist. He has seven rows of each kind of flower. The figures below tell the average number of flowers he gets from each row. How many flowers of each kind does he get from the seven rows all together?

a. 28 tulips
b. 90 violets
c. 96 carnations
d. 42 daisies
e. 52 lilies
f. 84 roses

We can multiply hundreds and thousands.

$$\begin{array}{r} 354 \\ \times\ 9 \\ \hline 36 \\ 450 \\ 2700 \\ \hline 3186 \end{array} \qquad \begin{array}{r} 4867 \\ \times\ 3 \\ \hline 21 \\ 180 \\ 2400 \\ 12000 \\ \hline 14601 \end{array}$$

Copy each problem and multiply as shown above.

1. 428 × 4	**2.** 235 × 7	**3.** 681 × 9	**4.** 574 × 3	**5.** 389 × 8
6. 853 × 3	**7.** 746 × 7	**8.** 937 × 8	**9.** 780 × 9	**10.** 618 × 5
11. 3876 × 5	**12.** 1249 × 4	**13.** 8765 × 9	**14.** 4294 × 7	**15.** 7685 × 6
16. 5284 × 8	**17.** 3645 × 2	**18.** 8764 × 6	**19.** 7351 × 9	**20.** 9876 × 5

21. Mr. Gibson shipped 2150 bolts in each box. How many bolts did he ship in 6 boxes?

22. One day Mr. Welch shipped 645 pencils in one box and 327 pencils in another box. How many pencils did he ship that day?

23. During the week Mr. Welch shipped 6 boxes each containing 645 pencils and 6 boxes each containing 327 pencils. How many pencils did he ship all together?

We can multiply by a two-digit number.

$$\begin{array}{r} 43 \\ \times 68 \\ \hline 24 \\ 320 \\ 180 \\ 2400 \\ \hline 2924 \end{array}$$

because 8 ones × 3 ones = 24 ones
because 8 ones × 4 tens = 32 tens
because 6 tens × 3 ones = 18 tens
because 6 tens × 4 tens = 24 hundreds

Copy and complete each problem. Give a reason for each step.

1. $\begin{array}{r} 57 \\ \times 65 \\ \hline \blacksquare \\ 250 \\ 420 \\ \hline 3000 \\ \hline 3705 \end{array}$

2. $\begin{array}{r} 39 \\ \times 48 \\ \hline 72 \\ \blacksquare \\ 360 \\ \hline 1200 \\ \hline 1872 \end{array}$

3. $\begin{array}{r} 62 \\ \times 73 \\ \hline 6 \\ 180 \\ 140 \\ \hline 4200 \\ \hline \blacksquare \end{array}$

4. $\begin{array}{r} 86 \\ \times 54 \\ \hline 24 \\ 320 \\ \blacksquare \\ \hline 4000 \\ \hline 4644 \end{array}$

5. $\begin{array}{r} 57 \\ \times 70 \\ \hline 0 \\ \blacksquare \\ 490 \\ \hline 3500 \\ \hline 3990 \end{array}$

Find each product.

6. $\begin{array}{r} 45 \\ \times 37 \\ \hline \end{array}$

7. $\begin{array}{r} 94 \\ \times 23 \\ \hline \end{array}$

8. $\begin{array}{r} 62 \\ \times 73 \\ \hline \end{array}$

9. $\begin{array}{r} 26 \\ \times 49 \\ \hline \end{array}$

10. $\begin{array}{r} 61 \\ \times 33 \\ \hline \end{array}$

11. $\begin{array}{r} 49 \\ \times 78 \\ \hline \end{array}$

12. $\begin{array}{r} 54 \\ \times 36 \\ \hline \end{array}$

13. $\begin{array}{r} 27 \\ \times 62 \\ \hline \end{array}$

14. $\begin{array}{r} 94 \\ \times 31 \\ \hline \end{array}$

15. $\begin{array}{r} 68 \\ \times 15 \\ \hline \end{array}$

16. $\begin{array}{r} 73 \\ \times 56 \\ \hline \end{array}$

17. $\begin{array}{r} 49 \\ \times 28 \\ \hline \end{array}$

18. $\begin{array}{r} 35 \\ \times 16 \\ \hline \end{array}$

19. $\begin{array}{r} 57 \\ \times 40 \\ \hline \end{array}$

20. $\begin{array}{r} 97 \\ \times 52 \\ \hline \end{array}$

21. Dale has 25 boxes of shells with 12 shells in each box. How many shells does he have all together?

Instead of this:

$$\begin{array}{r} 28 \\ \times\ 7 \\ \hline 56 \\ 140 \\ \hline 196 \end{array}$$

We may do this:

Step 1 Think: 7×8 ones $= 56$ ones.
Write: "6" in the ones' place and remember **5 tens.**

$$\begin{array}{r} {\scriptstyle 5} \\ 28 \\ \times\ 7 \\ \hline 6 \end{array}$$

Step 2 Think: 7×2 tens $= 14$ tens
14 tens $+ 5$ tens $= 19$ tens
Write: "9" in the tens' place and "1" in the hundreds' place.

$$\begin{array}{r} {\scriptstyle 5} \\ 28 \\ \times\ 7 \\ \hline 196 \end{array}$$

Copy and complete each problem. Use the new method shown above.

1. $\begin{array}{r} 46 \\ \times\ 7 \\ \hline \end{array}$	2. $\begin{array}{r} 39 \\ \times\ 4 \\ \hline \end{array}$	3. $\begin{array}{r} 83 \\ \times\ 6 \\ \hline \end{array}$	4. $\begin{array}{r} 52 \\ \times\ 8 \\ \hline \end{array}$	5. $\begin{array}{r} 76 \\ \times\ 3 \\ \hline \end{array}$
6. $\begin{array}{r} 69 \\ \times\ 8 \\ \hline \end{array}$	7. $\begin{array}{r} 57 \\ \times\ 6 \\ \hline \end{array}$	8. $\begin{array}{r} 48 \\ \times\ 2 \\ \hline \end{array}$	9. $\begin{array}{r} 34 \\ \times\ 7 \\ \hline \end{array}$	10. $\begin{array}{r} 41 \\ \times\ 9 \\ \hline \end{array}$
11. $\begin{array}{r} 95 \\ \times\ 7 \\ \hline \end{array}$	12. $\begin{array}{r} 58 \\ \times\ 3 \\ \hline \end{array}$	13. $\begin{array}{r} 74 \\ \times\ 8 \\ \hline \end{array}$	14. $\begin{array}{r} 85 \\ \times\ 5 \\ \hline \end{array}$	15. $\begin{array}{r} 49 \\ \times\ 6 \\ \hline \end{array}$

16. a. A television factory received an order for 75 television sets. Each of these sets has three speakers. How many speakers were needed to build all 75 sets?

b. Each of the sets has 7 dials. How many dials were needed to make these sets?

c. Each set has 10 tubes. How many tubes were used in the sets?

Instead of this:

$$\begin{array}{r} 354 \\ \times\ 9 \\ \hline 36 \\ 450 \\ 2700 \\ \hline 3186 \end{array}$$

We may write:

$$\begin{array}{r} {\scriptstyle 4\ 3} \\ 354 \\ \times\ 9 \\ \hline 3186 \end{array}$$

9×4 ones $= 36$.
Write "6" in the ones' place and remember **3 tens.**
9×5 tens $= 45$ tens. 45 tens $+$ **3 tens** $= 48$ tens.
Write "8" in the tens' place and remember **4 hundreds.**
9×3 hundreds $= 27$ hundreds.
27 hundreds $+$ **4 hundreds** $= 31$ hundreds.
Write "1" in the hundreds' place and "3" in the thousands' place.

Copy each problem and multiply the new way shown above.

1. 583 × 4	2. 924 × 7	3. 632 × 9	4. 845 × 6	5. 487 × 3
6. 263 × 8	7. 876 × 5	8. 447 × 8	9. 380 × 6	10. 532 × 9
11. 651 × 7	12. 924 × 4	13. 2463 × 2	14. 8437 × 6	15. 9861 × 3
16. 3728 × 9	17. 6482 × 5	18. 7628 × 7	19. 7296 × 4	20. 5384 × 8
21. 2768 × 6	22. 9705 × 9	23. 6834 × 7	24. 8416 × 5	25. 4680 × 6

We can find a shorter way to multiply by a two-digit number.

Instead of:

$$\begin{array}{r} 43 \\ \times 68 \\ \hline 24 \\ 320 \\ 180 \\ 2400 \\ \hline 2924 \end{array}$$

We write:

$$\begin{array}{r} 43 \\ \times 68 \\ \hline 344 \\ 2580 \\ \hline 2924 \end{array}$$

8 ones × 3 ones = 24.
Write "4" in the ones' place and remember **2 tens.**

8 ones × 4 tens = 32 tens.
32 tens + **2 tens** = 34 tens.

6 tens × 3 ones = 180.
Write 80 and remember **1 hundred.**

6 tens × 4 tens = 24 hundreds.
24 hundreds + **1 hundred** = 25 hundreds.

Copy each problem and find each product using the method shown above.

1. $\begin{array}{r} 24 \\ \times 46 \\ \hline \end{array}$	**2.** $\begin{array}{r} 83 \\ \times 37 \\ \hline \end{array}$	**3.** $\begin{array}{r} 92 \\ \times 17 \\ \hline \end{array}$	**4.** $\begin{array}{r} 74 \\ \times 55 \\ \hline \end{array}$	**5.** $\begin{array}{r} 70 \\ \times 62 \\ \hline \end{array}$
6. $\begin{array}{r} 88 \\ \times 59 \\ \hline \end{array}$	**7.** $\begin{array}{r} 72 \\ \times 30 \\ \hline \end{array}$	**8.** $\begin{array}{r} 29 \\ \times 46 \\ \hline \end{array}$	**9.** $\begin{array}{r} 67 \\ \times 38 \\ \hline \end{array}$	**10.** $\begin{array}{r} 95 \\ \times 84 \\ \hline \end{array}$
11. $\begin{array}{r} 63 \\ \times 67 \\ \hline \end{array}$	**12.** $\begin{array}{r} 19 \\ \times 84 \\ \hline \end{array}$	**13.** $\begin{array}{r} 37 \\ \times 29 \\ \hline \end{array}$	**14.** $\begin{array}{r} 94 \\ \times 46 \\ \hline \end{array}$	**15.** $\begin{array}{r} 23 \\ \times 41 \\ \hline \end{array}$
16. $\begin{array}{r} 32 \\ \times 84 \\ \hline \end{array}$	**17.** $\begin{array}{r} 45 \\ \times 63 \\ \hline \end{array}$	**18.** $\begin{array}{r} 86 \\ \times 59 \\ \hline \end{array}$	**19.** $\begin{array}{r} 47 \\ \times 37 \\ \hline \end{array}$	**20.** $\begin{array}{r} 63 \\ \times 58 \\ \hline \end{array}$

21. Mrs. Connors works 35 hours each week. How many hours does she work in 50 weeks?

22. Mr. Connors works 40 hours each week. How many hours does he work in 50 weeks?

23. Billy works 16 hours each week. How many hours does he work in 50 weeks?

To multiply a three-digit number by a two-digit number:

First: Multiply all three digits by the number of ones in the two-digit number.

$$\begin{array}{r} 428 \\ \times 37 \\ \hline 2996 \end{array}$$ because $7 \times 428 = 2996$

Second: Multiply all three digits by the number of tens in the two-digit number.

$$\begin{array}{r} 428 \\ \times 37 \\ \hline 2996 \\ 12840 \end{array}$$ because $30 \times 428 = 12840$

Third: Add.

$$\begin{array}{r} 428 \\ \times 37 \\ \hline 2996 \\ 12840 \\ \hline 15836 \end{array}$$

We use the distributive law of multiplication over addition to multiply 428 by 37. We think of 37 as $30 + 7$ and multiply 428 by 7 and by 30.

Copy and complete each problem as shown above.

1. $\begin{array}{r} 457 \\ \times 83 \\ \hline \end{array}$	2. $\begin{array}{r} 726 \\ \times 45 \\ \hline \end{array}$	3. $\begin{array}{r} 628 \\ \times 39 \\ \hline \end{array}$	4. $\begin{array}{r} 837 \\ \times 58 \\ \hline \end{array}$	5. $\begin{array}{r} 925 \\ \times 27 \\ \hline \end{array}$
6. $\begin{array}{r} 833 \\ \times 26 \\ \hline \end{array}$	7. $\begin{array}{r} 635 \\ \times 82 \\ \hline \end{array}$	8. $\begin{array}{r} 943 \\ \times 57 \\ \hline \end{array}$	9. $\begin{array}{r} 724 \\ \times 65 \\ \hline \end{array}$	10. $\begin{array}{r} 524 \\ \times 88 \\ \hline \end{array}$
11. $\begin{array}{r} 627 \\ \times 54 \\ \hline \end{array}$	12. $\begin{array}{r} 321 \\ \times 46 \\ \hline \end{array}$	13. $\begin{array}{r} 878 \\ \times 33 \\ \hline \end{array}$	14. $\begin{array}{r} 930 \\ \times 41 \\ \hline \end{array}$	15. $\begin{array}{r} 568 \\ \times 60 \\ \hline \end{array}$
16. $\begin{array}{r} 194 \\ \times 28 \\ \hline \end{array}$	17. $\begin{array}{r} 582 \\ \times 74 \\ \hline \end{array}$	18. $\begin{array}{r} 685 \\ \times 95 \\ \hline \end{array}$	19. $\begin{array}{r} 744 \\ \times 83 \\ \hline \end{array}$	20. $\begin{array}{r} 916 \\ \times 38 \\ \hline \end{array}$

We can solve some problems by using either addition or multiplication.

John paid $1.98 for each of three books. What was the total cost of the books?

$$\begin{array}{r} \$1.98 \\ 1.98 \\ +\ 1.98 \\ \hline \$5.94 \end{array} \qquad \begin{array}{r} \$1.98 \\ \times\ \ 3 \\ \hline \$5.94 \end{array}$$

In the problems below use both addition and multiplication to find the total amount spent by each person.

1. Mrs. Andrews spent $1.34 for each of four steaks.
2. Mr. Thomas paid $4.83 for each of six gallons of paint.
3. Mr. Bellini spent $1.45 for each of five gallons of ice cream.
4. Mrs. Graham spent $3.95 for each of three shirts for her son.
5. Mr. Schultz paid $18.67 for each of four new tires.

The store is having a sale. Find the total amounts of money spent on each kind of item. Use either addition or multiplication.

6. 7 towel sets at $8.32 each
7. 6 table cloths at $6.28 each
8. 4 blankets at $12.74 each
9. 9 pillows at $9.37 each
10. 2 quilts at $24.85 each
11. 8 sheets at $1.89 each
12. 8 napkins at $1.10 each
13. 5 beach towels at $3.13 each
14. 4 irons at $14.86 each
15. 2 ice crushers at $9.88 each
16. 8 can openers at $11.37 each
17. 7 toasters at $22.43 each
18. 5 waffle irons at $28.79 each
19. 3 sewing machines at $87.50 each
20. 6 blenders at $26.74 each
21. 9 mixers at $9.27 each

Factors

If a whole number divides another whole number with no remainder, we say the first number is a **factor** of the second number.

3 divides 15 five times, so 3 is a factor of 15.

When 20 is divided by 3 there is a remainder of 2, so 3 is not a factor of 20.

Solve each equation and answer each question.

1. a. $14 \div 7 = \square$
   b. Is 7 a factor of 14?
2. a. $48 \div 6 = \square$
   b. Is 6 a factor of 48?
3. a. $37 \div 8 = \square$
   b. Is 8 a factor of 37?
4. a. $82 \div 10 = \square$
   b. Is 10 a factor of 82?
5. a. $72 \div 9 = \square$
   b. Is 9 a factor of 72?
6. a. $64 \div 8 = \square$
   b. Is 8 a factor of 64?

7. Rename each number as a product of two of its factors.

a. 21	d. 35	g. 63	j. 60
b. 28	e. 45	h. 54	k. 50
c. 32	f. 36	i. 12	l. 11

Every number n has the factor 1. $n \div 1 = n$

Every number n, $n \neq 0$, is a factor of itself. $n \div n = 1$

A number with exactly two different factors is a **prime number.**

The factors of 17 are 1 and 17. 17 is prime.

The factors of 6 are 1, 2, 3, and 6. 6 is not prime.

The only factor of 1 is 1. 1 is not prime.

The two factors of a prime number are 1 and the number itself.

1. Give all the factors of each of these numbers.

a. 27	e. 2	i. 28	m. 3	q. 13
b. 12	f. 14	j. 18	n. 20	r. 9
c. 21	g. 8	k. 7	o. 11	s. 10
d. 15	h. 19	l. 16	p. 24	t. 23

2. Which of the numbers in problem 1 are prime numbers?

The first four prime numbers are 2, 3, 5, and 7.
If a number less than 100 can not be divided by 2, 3, 5, or 7, then that number is prime.

2 divides 42. — 42 is not prime.
43 can not be divided by 2, 3, 5, or 7. — 43 is prime.

3. Which of these numbers are prime numbers?

a. 29	d. 41	g. 87	j. 68	m. 51
b. 30	e. 52	h. 19	k. 31	n. 47
c. 32	f. 35	i. 49	l. 53	o. 37

4. Express each of these numbers as the sum of two prime numbers.

a. 8	c. 16	e. 20	g. 24
b. 14	d. 18	f. 22	h. 32

A **prime factor** is a factor that has no factors except itself and one.

2 and 3 are prime factors of 6.
3 and 4 are not prime factors of 12 because 4 has the factor 2.

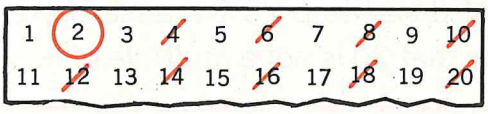
Choose **are** or **are not**.

5. 6 and 2 ○ prime factors of 12.
6. 3 and 5 ○ prime factors of 15.
7. 7 and 3 ○ prime factors of 21.
8. 2, 3, and 2 ○ prime factors of 12.
9. 4, 2, and 2 ○ prime factors of 8.
10. 6, 2, and 2 ○ prime factors of 24.

1. a. Copy a table like the one at the right.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   b. Circle the "2" and indicate with a mark as shown every number that has the factor 2.



   c. Circle the "3" and indicate with a mark every number that has the factor 3.
   d. Circle the "5" and indicate every number that has the factor 5.
   e. Circle the "7" and indicate every number that has the factor 7.
   f. Circle all the numerals you have not marked through.
   g. What numbers have their numerals circled?
   h. Are all these numbers prime numbers?

2. Using only prime numbers and 1 complete these "magic" squares so that the numbers in each row and in each column total the same. You may use the same number more than once.

   a.

7		5
17	11	13

   b.

1	29	13
23		17

3. Can you find an even number greater than 4 which cannot be expressed as the sum of two primes?
4. Can you find an odd number greater than 3 that cannot be expressed as the sum of two primes?

Multiples

If a number can be divided by another number with no remainder, we say the first number is **divisible by** the second number or that it is a **multiple** of the second number.

$15 \div 3 = 5$ We say that 15 is divisible by 3 because there is no remainder. We know that 15 is a multiple of 3.

$20 \div 3 = 6\frac{2}{3}$ We say that 20 is not divisible by 3 because there is a remainder. We know that 20 is not a multiple of 3.

Solve each equation and answer each question.

1. a. $32 \div 4 =$ ▒ b. Is 32 a multiple of 4?
2. a. $25 \div 6 =$ ▒ b. Is 25 a multiple of 6?
3. a. $19 \div 3 =$ ▒ b. Is 19 a multiple of 3?
4. a. $17 \div 1 =$ ▒ b. Is 17 a multiple of 1?
5. a. $28 \div 7 =$ ▒ b. Is 28 a multiple of 7?

6. Which of these numbers are multiples of 7?

a. 66	c. 60	e. 56
b. 49	d. 77	f. 15

7. Which of these numbers are multiples of 5?

a. 60	c. 90	e. 22
b. 48	d. 75	f. 0

8. Give the set whose members are the first ten multiples of each of these numbers.

The set of the first ten multiples of 2 = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18}

a. 5	c. 3	e. 8	g. 1
b. 7	d. 6	f. 4	h. 9

1. Find the set whose members are the first twenty multiples of 2.

2. Which of these numbers are divisible by 2?

a. 47	c. 56	e. 92	g. 99	i. 82
b. 222	d. 374	f. 50	h. 41	j. 34

3. Give a rule for identifying multiples of 2.

4. Add the digits of each number to find n. Is n divisible by 3?

a. 54	5 + 4 = n	d. 261	2 + 6 + 1 = n
b. 83	8 + 3 = n	e. 119	1 + 1 + 9 = n
c. 56	5 + 6 = n	f. 639	6 + 3 + 9 = n

5. Which of these numbers are divisible by 3?

a. 54	c. 56	e. 119
b. 83	d. 261	f. 639

6. A number is divisible by 3 if the sum of its digits is divisible by ■.

7. Give the first ten multiples of 10.

8. Give a rule for identifying multiples of 10.

9. Give the first twenty multiples of 5.

10. Which of these numbers are divisible by 5?

a. 94	c. 25	e. 42	g. 65
b. 553	d. 771	f. 80	h. 57

11. Give a rule for identifying multiples of 5.

12. Add the digits of each number to find n. Is n divisible by 9?

a. 99	9 + 9 = n	d. 108	1 + 0 + 8 = n
b. 64	6 + 4 = n	e. 585	5 + 8 + 5 = n
c. 96	9 + 6 = n	f. 595	5 + 9 + 5 = n

13. Which of these numbers are divisible by 9?

a. 99	c. 96	e. 585
b. 64	d. 108	f. 595

14. A number is divisible by 9 if the sum of its digits is divisible by ■.

Division and multiplication are **inverse** operations.

We know in multiplication that: factor times factor equals product.

To find a quotient is the same problem as to find a missing factor.

$16 \div 2 = \square$ is the same problem as $\square \times 2 = 16$

(product; missing factor; known factor)

The divisor is the known factor.

$16 \div 2 = 8$

(dividend; divisor; quotient)

Find each quotient.

1. $16 \div 8 = \square$
2. $9 \div 3 = \square$
3. $24 \div 6 = \square$
4. $32 \div 4 = \square$
5. $45 \div 5 = \square$
6. $36 \div 9 = \square$
7. $64 \div 8 = \square$
8. $28 \div 4 = \square$
9. $63 \div 9 = \square$
10. $21 \div 3 = \square$
11. $6 \div 6 = \square$
12. $56 \div 8 = \square$

Copy and complete each problem.

13. $13 \div 6 = 2\frac{1}{6}$
14. $8 \div 3 = \square$
15. $22 \div 5 = \square$
16. $45 \div 7 = \square$
17. $33 \div 8 = \square$
18. $19 \div 9 = \square$
19. $59 \div 7 = \square$
20. $26 \div 8 = \square$
21. $27 \div 4 = \square$
22. $9 \div 5 = \square$
23. $65 \div 8 = \square$
24. $14 \div 3 = \square$

Divide as in the examples.

$$3\overline{)10}\quad 3 \qquad \underline{9} \qquad 1$$

because 3 × 3 ones = 9

$$8\overline{)16}\quad 2 \qquad \underline{16}$$

because 8 × 2 ones = 16

25. $7\overline{)14}$
26. $8\overline{)72}$
27. $6\overline{)54}$
28. $9\overline{)76}$
29. $5\overline{)35}$
30. $8\overline{)35}$
31. $4\overline{)19}$
32. $2\overline{)17}$
33. $6\overline{)50}$
34. $3\overline{)26}$

1. Complete each problem.

a. $4 \times 9 < 39 < 4 \times 10$

b. $4 \times \_\_ < 26 < 4 \times \_\_$

c. $4 \times \_\_ < 17 < 4 \times \_\_$

d. $4 \times \_\_ < 30 < 4 \times \_\_$

e. $4 \times \_\_ < 23 < 4 \times \_\_$

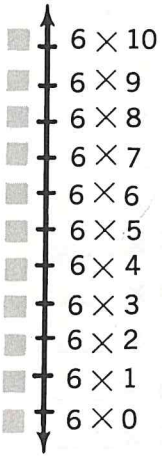
f. $4 \times \_\_ < 10 < 4 \times \_\_$

g. $4 \times \_\_ < 5 < 4 \times \_\_$

h. $4 \times \_\_ < 13 < 4 \times \_\_$

Copy and complete each number line.

2.



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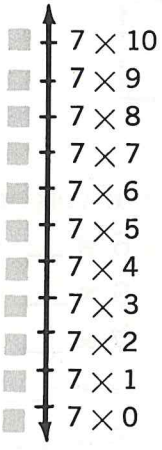

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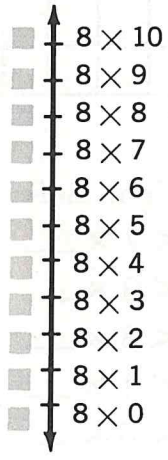

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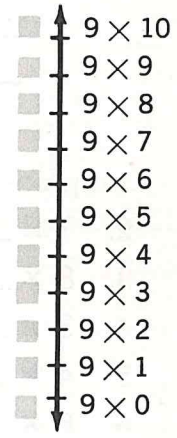


Complete each problem.

6. a. $6 \times \_\_ < 14 < 6 \times \_\_$

b. $6 \times \_\_ < 56 < 6 \times \_\_$

c. $6 \times \_\_ < 45 < 6 \times \_\_$

d. $6 \times \_\_ < 21 < 6 \times \_\_$

e. $6 \times \_\_ < 35 < 6 \times \_\_$

f. $6 \times \_\_ < 50 < 6 \times \_\_$

g. $6 \times \_\_ < 25 < 6 \times \_\_$

h. $6 \times \_\_ < 37 < 6 \times \_\_$

7. a. $7 \times \_\_ < 11 < 7 \times \_\_$

b. $7 \times \_\_ < 41 < 7 \times \_\_$

c. $7 \times \_\_ < 58 < 7 \times \_\_$

d. $7 \times \_\_ < 45 < 7 \times \_\_$

e. $7 \times \_\_ < 29 < 7 \times \_\_$

f. $7 \times \_\_ < 24 < 7 \times \_\_$

g. $7 \times \_\_ < 67 < 7 \times \_\_$

h. $7 \times \_\_ < 53 < 7 \times \_\_$

1. A multiplication table can also be used as a division table. Copy and complete this table.

× / ÷	1	2	3	4	5	6	7	8	9
1									
2									
3				12					
4									
5									
6									
7									
8									
9									

This table shows that if

$3 \times 4 = 12$,

then

$12 \div 3 = 4$

and

$12 \div 4 = 3$.

Complete each problem. Refer to the table if you need to.

2. $7 \times \_\_ < 52 < 7 \times \_\_$

3. $8 \times \_\_ < 55 < 8 \times \_\_$

4. $3 \times \_\_ < 22 < 3 \times \_\_$

5. $4 \times \_\_ < 35 < 4 \times \_\_$

6. $6 \times \_\_ < 44 < 6 \times \_\_$

7. $7 \times \_\_ < 65 < 7 \times \_\_$

8. $9 \times \_\_ < 64 < 9 \times \_\_$

9. $8 \times \_\_ < 60 < 8 \times \_\_$

10. $3 \times \_\_ < 26 < 3 \times \_\_$

11. $2 \times \_\_ < 17 < 2 \times \_\_$

12. $5 \times \_\_ < 42 < 5 \times \_\_$

13. $6 \times \_\_ < 40 < 6 \times \_\_$

14. $9 \times \_\_ < 85 < 9 \times \_\_$

15. $7 \times \_\_ < 30 < 7 \times \_\_$

16. $4 \times \_\_ < 27 < 4 \times \_\_$

17. $5 \times \_\_ < 26 < 5 \times \_\_$

18. $3 \times \_\_ < 19 < 3 \times \_\_$

19. $7 \times \_\_ < 36 < 7 \times \_\_$

20. $2 \times \_\_ < 15 < 2 \times \_\_$

21. $4 \times \_\_ < 19 < 4 \times \_\_$

22. $6 \times \_\_ < 28 < 6 \times \_\_$

23. $8 \times \_\_ < 65 < 8 \times \_\_$

24. Bill has 47 pennies. He wants to wrap 10 pennies in a roll.

a. Does he have enough pennies to make 4 rolls?

b. Does he have enough pennies to make 5 rolls?

If we estimate, we can divide tens and hundreds in the same way.

$$\begin{array}{r} 3 \\ 7\overline{)21} \\ \underline{21} \end{array}$$ because 7 × 3 ones = 21

We may think of 210 as 21 tens.

$$\begin{array}{r} 30 \\ 7\overline{)210} \\ \underline{210} \end{array}$$ because 7 × 3 tens = 210

We may think of 2100 as 21 hundreds.

$$\begin{array}{r} 300 \\ 7\overline{)2100} \\ \underline{2100} \end{array}$$ because 7 × 3 hundreds = 2100

Complete each problem.

1. a. 810 = ■ tens
   b. 320 = ■ tens
   c. 420 = ■ tens
   d. 640 = ■ tens
   e. 310 = ■ tens
   f. 580 = ■ tens

2. a. 7200 = ■ hundreds
   b. 4800 = ■ hundreds
   c. 3900 = ■ hundreds
   d. 5600 = ■ hundreds
   e. 5200 = ■ hundreds
   f. 1700 = ■ hundreds

Find each quotient.

3. a. $6\overline{)4200}$ b. $5\overline{)350}$ c. $2\overline{)1400}$ d. $8\overline{)6400}$

4. a. $3\overline{)180}$ b. $7\overline{)490}$ c. $6\overline{)3000}$ d. $4\overline{)280}$

5. a. $8\overline{)5600}$ b. $3\overline{)150}$ c. $8\overline{)7200}$ d. $7\overline{)210}$

6. a. $5\overline{)400}$ b. $9\overline{)8100}$ c. $4\overline{)2400}$ d. $9\overline{)360}$

In some problems we divide both the tens and the ones. We can estimate before we divide.

$342 \div 8$ or $8\overline{)342}$

We think of 342 as 34 tens and 2 ones.

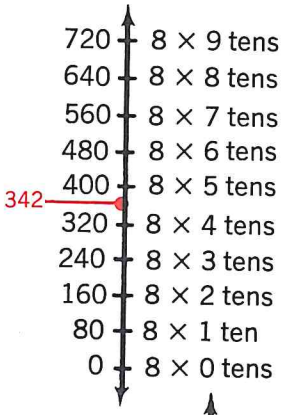
We know that:

$8 \times 4 \text{ tens} < 34 \text{ tens} < 8 \times 5 \text{ tens}$

$$\begin{array}{r} 4 \\ 8\overline{)342} \\ 320 \\ \hline 22 \end{array}$$

because $8 \times 4 \text{ tens} = 320$

because $342 - 320 = 22$



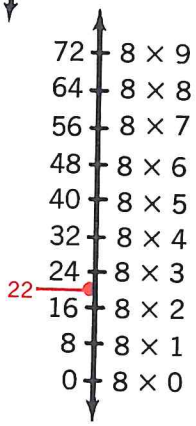
We think of 22 as 22 ones.

$8 \times 2 < 22 < 8 \times 3$

$$\begin{array}{r} 42 \\ 8\overline{)342} \\ 320 \\ \hline 22 \\ 16 \\ \hline 6 \end{array}$$

because $8 \times 2 \text{ ones} = 16$

because $22 - 16 = 6$



Copy and complete each problem.

1.

$$\begin{array}{r} 5\square \\ 4\overline{)234} \\ 200 \\ \hline 34 \\ 32 \\ \hline 2 \end{array}$$

because $4 \times 5\ \square = 200$

because $4 \times \square = 32$

2.

$$\begin{array}{r} 98 \\ 7\overline{)691} \\ 630 \\ \hline 61 \\ \square \\ \hline 5 \end{array}$$

because $\square$

because $\square$

3.

$$\begin{array}{r} \square 6 \\ 6\overline{)276} \\ 240 \\ \hline 36 \\ \square \\ \hline \end{array}$$

because $\square$

because $\square$

4.

$$\begin{array}{r} \square 1 \\ 5\overline{)408} \\ 400 \\ \hline 8 \\ 5 \\ \square \end{array}$$

because $\square$

because $\square$

Copy and complete each problem.

1.
$$\begin{array}{r} \square 3 \\ 6\overline{)259} \\ \underline{240} \\ 19 \\ \underline{18} \\ 1 \end{array}$$
because $6 \times \square = 240$

because $6 \times \square = 18$

2.
$$\begin{array}{r} 3\square \\ 8\overline{)273} \\ \underline{240} \\ 33 \\ \underline{32} \\ 1 \end{array}$$
because $8 \times \square = 240$

because $8 \times \square = 32$

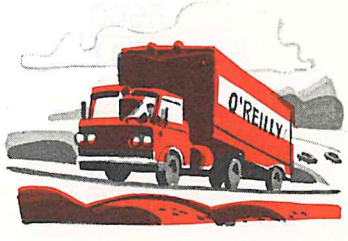
Complete each problem.

3.	**a.** $2\overline{)164}$	**b.** $3\overline{)218}$	**c.** $6\overline{)543}$	**d.** $6\overline{)329}$
4.	**a.** $3\overline{)194}$	**b.** $9\overline{)547}$	**c.** $5\overline{)328}$	**d.** $2\overline{)146}$
5.	**a.** $5\overline{)325}$	**b.** $2\overline{)138}$	**c.** $8\overline{)636}$	**d.** $9\overline{)764}$
6.	**a.** $4\overline{)385}$	**b.** $5\overline{)486}$	**c.** $3\overline{)147}$	**d.** $6\overline{)470}$
7.	**a.** $7\overline{)438}$	**b.** $4\overline{)275}$	**c.** $7\overline{)526}$	**d.** $5\overline{)239}$
8.	**a.** $9\overline{)666}$	**b.** $7\overline{)635}$	**c.** $4\overline{)118}$	**d.** $8\overline{)524}$
9.	**a.** $3\overline{)158}$	**b.** $8\overline{)743}$	**c.** $5\overline{)178}$	**d.** $3\overline{)249}$
10.	**a.** $4\overline{)328}$	**b.** $7\overline{)313}$	**c.** $6\overline{)289}$	**d.** $4\overline{)160}$

11. Bill collects pictures of airplanes. He puts 8 pictures on each page of his album. How many pages will 224 pictures fill?

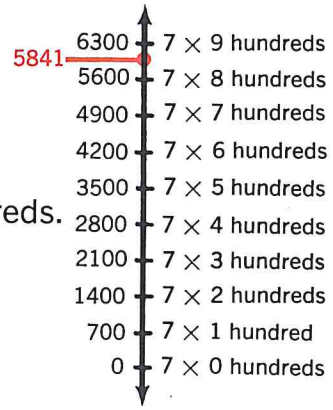
12. Mr. Talley packages 6 apples in each tray. How many trays will he need if he has 156 apples?

13. Mr. O'Reilly's truck uses one gallon of gasoline to travel 7 miles. In traveling 483 miles, how many gallons of gasoline did the truck use?



In some problems we divide hundreds, tens, and ones. We can estimate in the same way before we divide.

$5841 \div 7$ **or** $7\overline{)5841}$


5841
6300 7 × 9 hundreds
5600 7 × 8 hundreds
4900 7 × 7 hundreds
4200 7 × 6 hundreds
3500 7 × 5 hundreds
2800 7 × 4 hundreds
2100 7 × 3 hundreds
1400 7 × 2 hundreds
700 7 × 1 hundred
0 7 × 0 hundreds

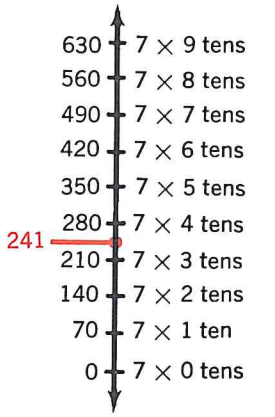
We think of 5841 as 58 hundreds and 41.

7×8 hundreds $<$ 58 hundreds $<$ 7×9 hundreds.

$$\begin{array}{r} 8 \\ 7\overline{)5841} \\ 5600 \\ \hline 241 \end{array}$$ because 7×8 hundreds $= 5600$

We think of 241 as 24 tens and 1.

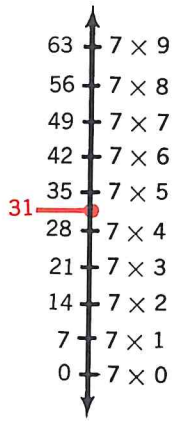
7×3 tens $<$ 24 tens $<$ 7×4 tens



$$\begin{array}{r} 83 \\ 7\overline{)5841} \\ 5600 \\ \hline 241 \\ 210 \\ \hline 31 \end{array}$$ because 7×3 tens $= 210$

We think of 31 as 31 ones.

$7 \times 4 < 31 < 7 \times 5$



$$\begin{array}{r} 834 \\ 7\overline{)5841} \\ 5600 \\ \hline 241 \\ 210 \\ \hline 31 \\ 28 \\ \hline 3 \end{array}$$

Copy and complete each problem.

1.
$$\begin{array}{r} \square 85 \\ 8\overline{)5486} \\ 4800 \\ \hline 686 \\ 640 \\ \hline 46 \\ \square \\ \hline \square \end{array}$$

2.
$$\begin{array}{r} 6\square 4 \\ 4\overline{)2536} \\ 2400 \\ \hline 136 \\ 120 \\ \hline 16 \\ \square \\ \hline \end{array}$$

3.
$$\begin{array}{r} \square 3\square \\ 2\overline{)1265} \\ 1200 \\ \hline 65 \\ \square \\ \hline 5 \\ \square \\ \hline 1 \end{array}$$

4.
$$\begin{array}{r} 5\square 8 \\ 3\overline{)1794} \\ \square \\ \hline 294 \\ 270 \\ \hline 24 \\ \square \\ \hline \end{array}$$

5. $7\overline{)6278}$ **6.** $9\overline{)2485}$ **7.** $5\overline{)3729}$ **8.** $8\overline{)7246}$

9. $6\overline{)4070}$ **10.** $5\overline{)4280}$ **11.** $2\overline{)1964}$ **12.** $6\overline{)3260}$

13. $3\overline{)2695}$ **14.** $6\overline{)3784}$ **15.** $8\overline{)4366}$ **16.** $9\overline{)8462}$

17. $4\overline{)3276}$ **18.** $7\overline{)6354}$ **19.** $6\overline{)3485}$ **20.** $5\overline{)4357}$

21. Mrs. Bowden invited friends for a pancake breakfast. She made 63 pancakes. If each of the 9 people ate the same number of pancakes, how many did each one eat?

22. Mr. Chang makes Chinese fortune cookies. He puts 8 cookies into each bag. One day he made 4288 cookies. How many bags of cookies did he have?

23. Mrs. Sanchez has a restaurant that serves Mexican food. She made 582 tacos one week. Three tacos come with each order. How many orders of tacos did she sell that week?

Feedback

Find each product.

1. $\begin{array}{r} 39 \\ \times\ 8 \\ \hline \end{array}$

2. $\begin{array}{r} 84 \\ \times\ 6 \\ \hline \end{array}$

3. $\begin{array}{r} 859 \\ \times\ 7 \\ \hline \end{array}$

4. $\begin{array}{r} 956 \\ \times\ 9 \\ \hline \end{array}$

5. $\begin{array}{r} 84 \\ \times 35 \\ \hline \end{array}$

6. $\begin{array}{r} 96 \\ \times 67 \\ \hline \end{array}$

7. $\begin{array}{r} 308 \\ \times\ 85 \\ \hline \end{array}$

8. $\begin{array}{r} 437 \\ \times\ 94 \\ \hline \end{array}$

Solve each problem.

9. $6\overline{)495}$

10. $5\overline{)682}$

11. $3\overline{)843}$

12. $8\overline{)921}$

13. $4\overline{)8321}$

14. $6\overline{)4208}$

15. $9\overline{)1926}$

16. $7\overline{)8130}$

17. Which of these numbers are prime numbers?

a. 1
b. 2
c. 17
d. 51
e. 31
f. 15
g. 23
h. 13

18. Which of these numbers are multiples of 8?

a. 0
b. 8
c. 25
d. 32
e. 48
f. 58
g. 64
h. 68

19. Find the missing factor.

a. $8 \times n = 72$
b. $7 \times b = 119$
c. $5 \times t = 245$
d. $r \times 9 = 81$
e. $p \times 7 = 91$
f. $k \times 8 = 128$
g. $114 = 6 \times s$
h. $110 = 5 \times a$
i. $132 = c \times 4$

20. List all the factors of 12.

21. Mrs. Clark ordered 119 packages of cardboard. If there were 8 pieces in each package, how many pieces of cardboard did she get?

Feedback

Find each sum.

1. $\begin{array}{r} 26 \\ +59 \\ \hline \end{array}$

2. $\begin{array}{r} 64 \\ +38 \\ \hline \end{array}$

3. $\begin{array}{r} 47 \\ +26 \\ \hline \end{array}$

4. $\begin{array}{r} 66 \\ +26 \\ \hline \end{array}$

5. $\begin{array}{r} 49 \\ +28 \\ \hline \end{array}$

6. $\begin{array}{r} 9 \\ 8 \\ 7 \\ +6 \\ \hline \end{array}$

7. $\begin{array}{r} 7 \\ 6 \\ 5 \\ +8 \\ \hline \end{array}$

8. $\begin{array}{r} 17 \\ 23 \\ 15 \\ +16 \\ \hline \end{array}$

9. $\begin{array}{r} 23 \\ 17 \\ 26 \\ +18 \\ \hline \end{array}$

10. $\begin{array}{r} 41 \\ 19 \\ 23 \\ +11 \\ \hline \end{array}$

Choose $=$ or $\neq$.

11. $2 + 3 \bigcirc 3 + 2$
12. $7 - 4 \bigcirc 4 - 7$
13. $4 \times 6 \bigcirc 6 \times 4$
14. $14 \div 2 \bigcirc 2 \div 14$
15. $7 + 8 \bigcirc 8 + 7$
16. $6 - 1 \bigcirc 1 - 6$
17. $1 + 3 \bigcirc 3 + 1$
18. $8 - 4 \bigcirc 4 - 8$
19. $7 \times 2 \bigcirc 2 \times 7$
20. $3 \div 2 \bigcirc 2 \div 3$
21. $3 \times 4 \bigcirc 4 \times 3$
22. $7 \div 1 \bigcirc 1 \div 7$

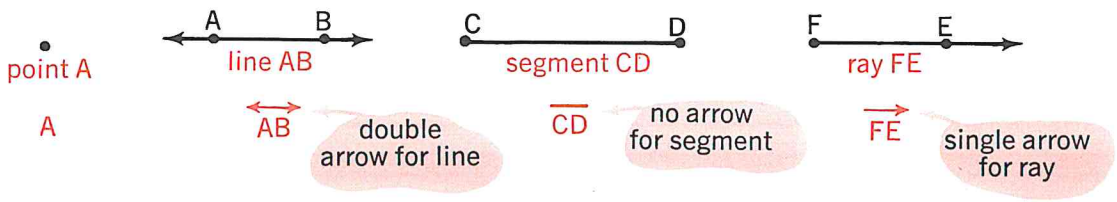
23. When it does not matter which of two numbers comes first, the operation is commutative.
    - **a.** Is addition commutative?
    - **b.** Is subtraction commutative?
    - **c.** Is multiplication commutative?
    - **d.** Is division commutative?

Complete each problem.

24. $\{1, 2, 3\} \cap \{2, 4, 6\} = \blacksquare$
25. $\{\text{Sam, Alice, Jill}\} - \{\text{Sam}\} = \blacksquare$
26. $\{1, 4, 6\} \cup \{0, 1, 2, 3, 4\} = \blacksquare$

Part 4 Geometry

Points, Lines, Segments, and Rays



A point gives a location.

A line is a set of points. A line has no endpoints. We can write $\overleftrightarrow{AB}$ for line AB.

A line segment is a subset of the set of points of a line. A line segment has two endpoints. We can write $\overline{CD}$ for line segment CD.

A ray is a subset of the set of points of a line. A ray has only one endpoint. We can write $\overrightarrow{FE}$ for ray FE. We name the endpoint first.

Pair each picture with its name.

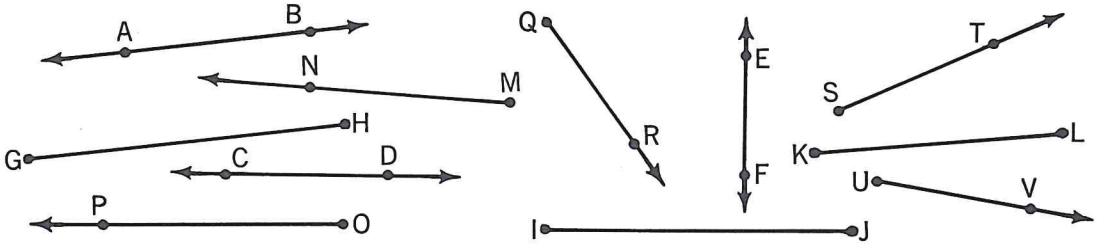
1.

2.

3.

A. $\overleftrightarrow{MN}$
B. $\overrightarrow{MN}$
C. $\overline{MN}$

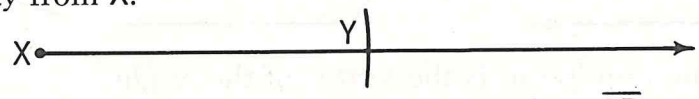
4. Below are pictures of lines, segments, and rays.
   a. Name the lines.
   b. Name the segments.
   c. Name the rays.



Copying a Line Segment

1. Use a compass and a ruler to copy $\overline{AB}$.

   a. Draw a ray from X.



   b. With X as a center and radius equal in length to $\overline{AB}$, construct an arc that intersects the ray.
   c. Let Y be the point of intersection.
   d. Is $\overline{XY}$ equal in length to $\overline{AB}$?
   e. When two different segments have the same length, they are **congruent.** Is $\overline{AB}$ congruent to $\overline{XY}$?

2. Construct a segment congruent to each of these segments.

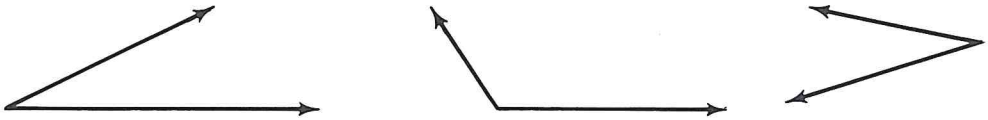
3. Use a ruler and compass to construct a segment twice as long as $\overline{CD}$.
   a. Draw a ray from X.
   b. With X as a center and radius equal in length to $\overline{CD}$, construct an arc that intersects the ray.
   c. With this point of intersection as a center and a radius equal in length to $\overline{CD}$, construct another arc that intersects the ray.
   d. Let Z be this point of intersection.
   e. Is $\overline{XZ}$ equal in length to twice $\overline{CD}$?

4. Construct a segment three times as long as $\overline{CD}$.

5. Segment RS is one inch long. Construct a six-inch segment.

Angles

An angle is the union of two rays that have a common endpoint.



The common endpoint is the **vertex** of the angle.

1. We name an angle by naming a point on one ray, then naming the vertex, and then naming a point on the second ray.

a. angle ABC or angle **CBA**

b. ∠ABC or ∠ **CBA**

c. The vertex is **B.**

d. angle DEF or angle ▢

e. ∠DEF or ∠ ▢

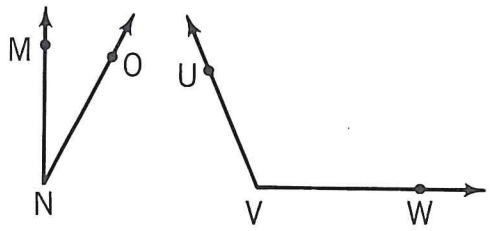
f. The vertex is ▢.

g. angle ▢ or angle ▢

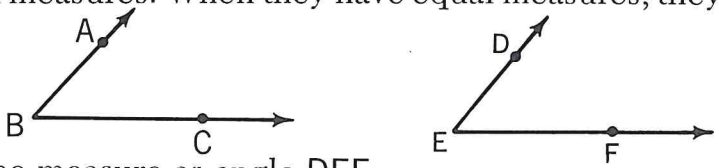
h. ∠XYZ or ∠ ▢

i. The vertex is ▢.

2. Angles may have unequal measures. Does angle MNO or angle UVW have the greater measure?



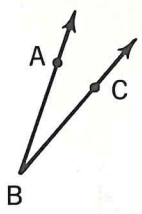
Angles may have equal measures. When they have equal measures, they are congruent.

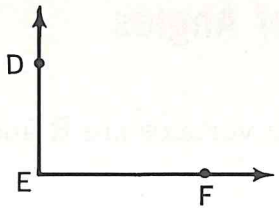


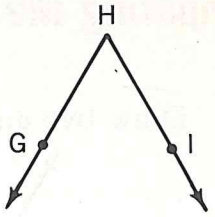
Angle ABC has the same measure as angle DEF.

Angle ABC is **congruent** to angle DEF or

∠ABC ≅ ∠DEF.





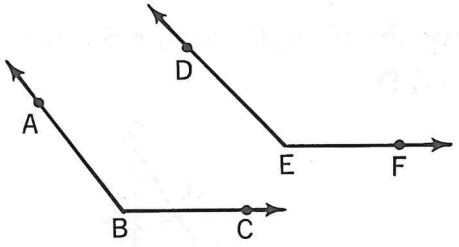


1. a. Which of the angles has the greatest measure?

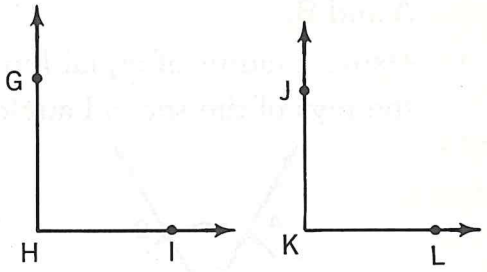
b. Which has the least measure?

2. Name the pairs of congruent angles.

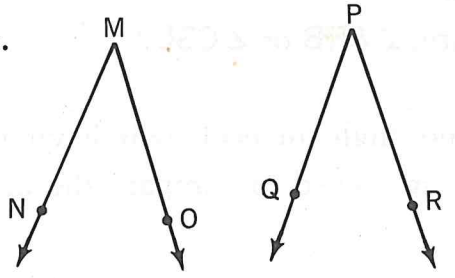
a.



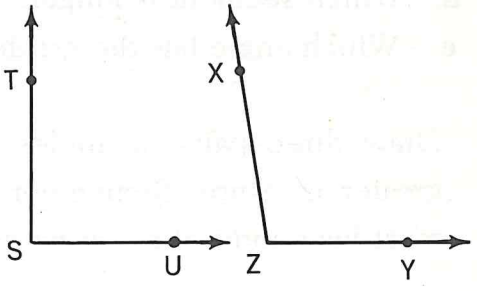
b.



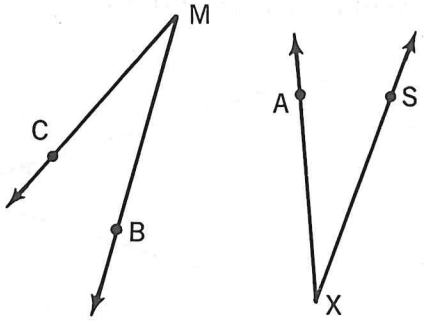
c.



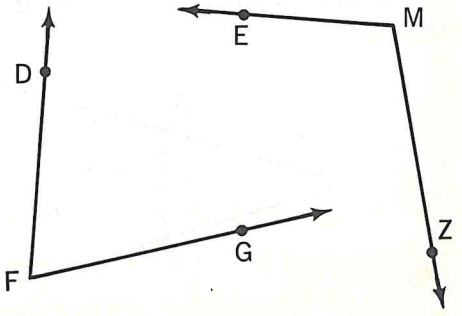
d.



e.

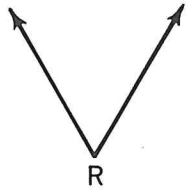


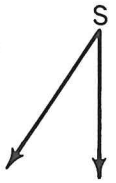
f.



Comparing Measures of Angles

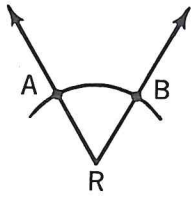
1. **a.** Draw two angles whose vertices are R and S.

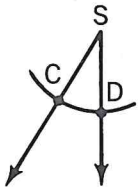




b. Draw an arc with center R intersecting the two rays of the angle at A and B.

c. Using a radius of equal length, draw an arc with center S intersecting the rays of the second angle at C and D.



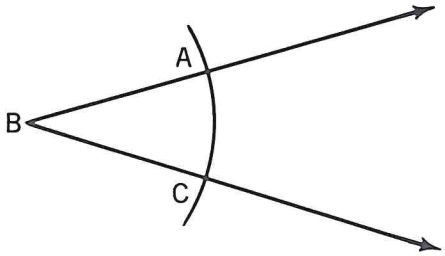


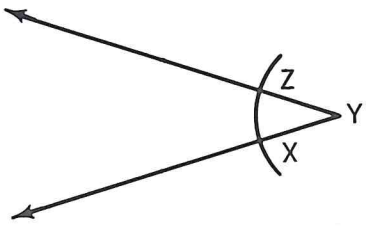
d. Which segment is longer, $\overline{AB}$ or $\overline{CD}$?

e. Which angle has the greater measure, $\angle ARB$ or $\angle CSD$?

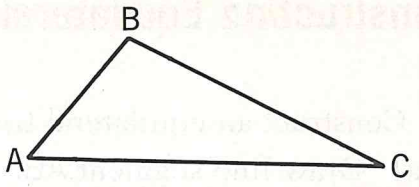
2. Draw three pairs of angles. Name the angle in each pair having the greater measure. Remember that the arcs used to compare the angles must have radii of equal length.

3. Max compares $\angle ABC$ and $\angle XYZ$. His work is shown below. He concludes that $\angle ABC$ is larger than $\angle XYZ$. What mistake did he make?



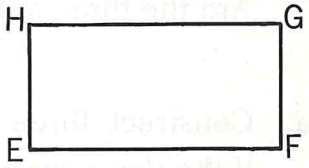


A triangle has three **interior angles.** The interior angles of triangle ABC are ∠CAB, ∠ABC, and ∠BCA.

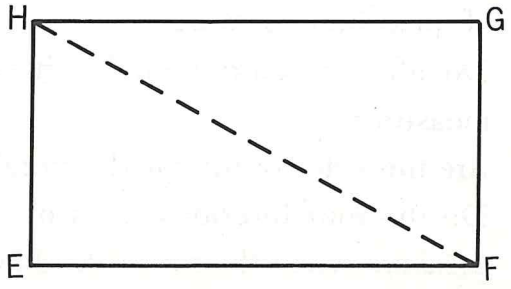


1. **a.** Draw a triangle that looks like triangle ABC.
   b. Construct an arc with A as the center that intersects sides AB and AC. Using a radius of the same length, construct arcs with points B and C as centers.
   c. Use the arcs to decide which of the three interior angles has the greatest measure.
   d. Which angle has the least measure?
   e. Is ∠ABC ≅ ∠BCA?

A rectangle has four interior angles. The interior angles of rectangle EFGH are ∠HEF, ∠EFG, ∠FGH, and ∠GHE.

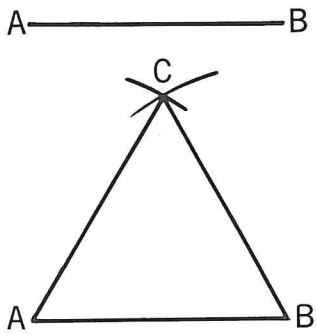


2. **a.** Use a sheet of paper for your rectangle. Label the four corners "E", "F", "G", and "H". Construct an arc with E as the center that cuts edges EF and EH. Using the same radius, construct arcs with points F, G, and H as centers.
   b. Use the arcs to decide which of the angles have the same measure.
   c. Is ∠HEF ≅ ∠EFG?
   d. Is ∠EFG ≅ ∠FGH?
   e. Is ∠HEF ≅ ∠FGH?
   f. Draw $\overline{HF}$ as shown in the figure.
   g. Is ∠EFH ≅ ∠GFH?
   h. Is ∠EFH ≅ ∠GHF?
   i. Is ∠EHF ≅ ∠GFH?



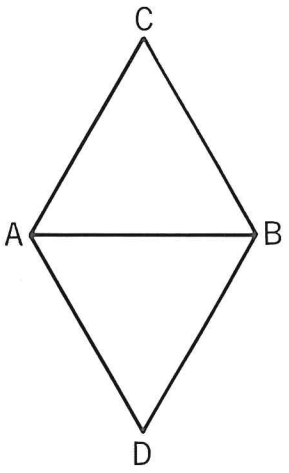
Constructing Equilateral Triangles

1. Construct an equilateral triangle with given base.
   - **a.** Draw line segment AB.
   - **b.** Construct an arc with center A and radius AB.
   - **c.** Construct an arc with center B and radius AB.
   - **d.** Make the two arcs intersect at C.
   - **e.** Draw $\overline{AC}$ and $\overline{BC}$.
   - **f.** Is $\overline{AC} \cong \overline{BC}$?
   - **g.** Is $\overline{AC} \cong \overline{AB}$?
   - **h.** Is $\overline{BC} \cong \overline{AB}$?
   - **i.** Are the three sides of the triangle ABC equal in length?



2. **a.** Construct three equilateral triangles and test each triangle to see if the three angles in each triangle have the same measure.
   - **b.** Do the three angles of each equilateral triangle have the same measure?

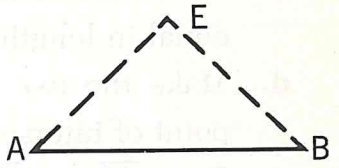
3. **a.** Draw a line segment AB.
   - **b.** Construct two equilateral triangles ACB and ADB on base AB.
   - **c.** Erase segment AB.
   - **d.** Draw arcs to compare the interior angles of quadrilateral ACBD.
   - **e.** Do all the interior angles have the same measure?
   - **f.** Are the sides of the quadrilateral congruent?
   - **g.** Do the four interior angles of a quadrilateral always have the same measure when the four sides have the same length?



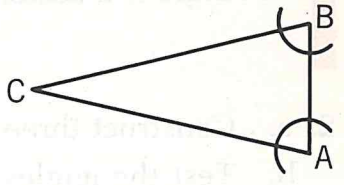
Constructing Isosceles Triangles

1. Construct an isosceles triangle, given the base and one of the two congruent sides.

   a. With $\overline{AB}$ as a base, construct an arc with center A and radius congruent to $\overline{CD}$.
   b. Construct an arc with center B and radius congruent to $\overline{CD}$.
   c. Make the two arcs intersect. Label the point of intersection "E".
   d. Draw $\overline{AE}$ and $\overline{BE}$.
   e. Is $\overline{AE} \cong \overline{BE}$?



2. Construct three more isosceles triangles and compare their angles. Follow these directions.
   a. Draw arcs with centers A and B and radii of equal length to compare the **base angles** CAB and CBA.
   b. Does angle CAB have the same measure as angle CBA?

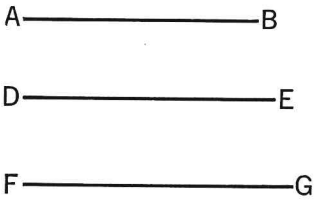


3. a. Construct an isosceles triangle whose base is 4 inches in length and whose sides are 3 inches in length.
   b. Compare the base angles by drawing arcs with radii of equal length.
   c. Do the two base angles of an isosceles triangle have equal measures?
   d. Are the two base angles of an isosceles triangle congruent?

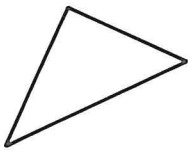
4. a. Construct an equilateral triangle DEF on base DE.
   b. Is $\overline{DF} \cong \overline{EF}$?
   c. Is triangle DEF an isosceles triangle?

Constructing Scalene Triangles

1. Construct a triangle when the three sides are given.
   a. Construct a line segment congruent to $\overline{AB}$.
   b. Construct an arc with center A and radius equal in length to $\overline{DE}$.
   c. Construct an arc with center B and radius equal in length to $\overline{FG}$.
   d. Make the two arcs intersect. Let C be the point of intersection.
   e. Draw $\overline{AC}$ and $\overline{BC}$.
   f. Are any of the sides congruent?



If no two sides of a triangle are congruent, the triangle is a **scalene** triangle.

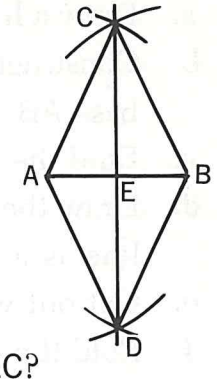


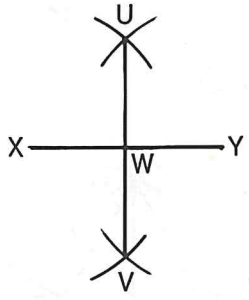
2. a. Construct three more scalene triangles.
   b. Test the angles of each triangle to see whether any of them are congruent.

3. Choose **0, 1, 2, 3,** or **4.**
   a. An equilateral triangle has ▪ sides equal in length.
   b. An equilateral triangle has ▪ interior angles that have the same measure.
   c. An isosceles triangle has at least ▪ sides equal in length.
   d. An isosceles triangle has at least ▪ interior angles that have the same measure.
   e. A scalene triangle has ▪ congruent sides.
   f. A scalene triangle has ▪ interior angles that have the same measure.

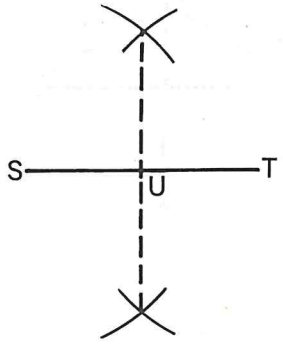
Bisecting Line Segments

1. **a.** Draw line segment AB.
   b. On base AB construct two isosceles triangles, ABC and ABD.
   c. Draw $\overline{CD}$. Segments AB and CD intersect. Label the point of intersection "E".
   d. Are $\overline{AE}$ and $\overline{EB}$ the same length?
   e. "Bisect" means "to cut into two equal parts." Does $\overline{CD}$ bisect $\overline{AB}$?
   f. Does angle AEC have the same measure as angle BEC?



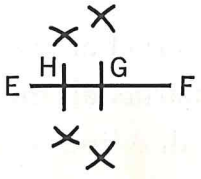


2. Bisect a given line segment.
   a. Draw a line segment XY.
   b. Construct arcs with centers X and Y and radius XY to intersect at U and V.
   c. Draw $\overline{UV}$. $\overline{UV}$ bisects $\overline{XY}$. W is the midpoint of $\overline{XY}$, because $\overline{XW}$ is equal in length to $\overline{WY}$.



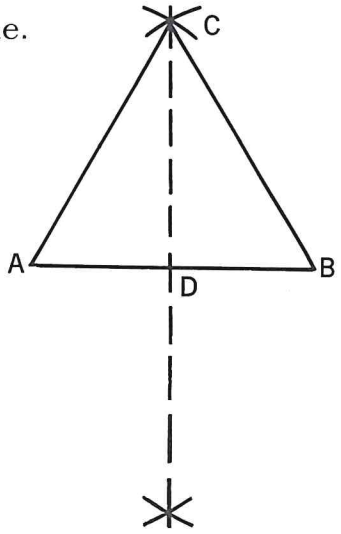
3. Find the midpoint of a given line segment.
   a. Draw segment ST.
   b. Bisect $\overline{ST}$.
   c. The point of intersection U is the midpoint of $\overline{ST}$.
   d. How many midpoints does a line segment have?

4. **a.** Draw segment EF.
   b. Find the midpoint G of $\overline{EF}$.
   c. Find the midpoint H of $\overline{EG}$.
   d. $\overline{EF}$ is how many times as long as $\overline{EH}$?

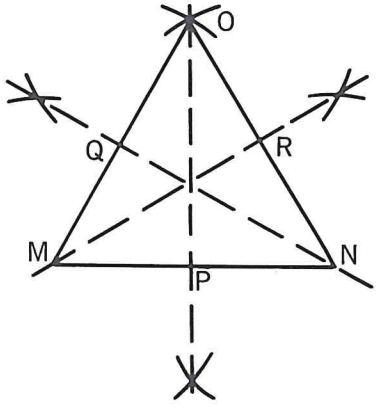


Lines of Symmetry in Triangles

1. Construct a line of symmetry of an equilateral triangle.
   - **a.** Draw a line segment AB of length about 4 in.
   - **b.** Construct an equilateral triangle ABC with base AB.
   - **c.** Find the midpoint D of $\overline{AB}$.
   - **d.** Draw the line passing through C and D. This line is a line of symmetry of triangle ABC.
   - **e.** Cut out your drawing of triangle ABC.
   - **f.** Fold the paper along the line CD.
   - **g.** Do the two parts fit each other exactly?

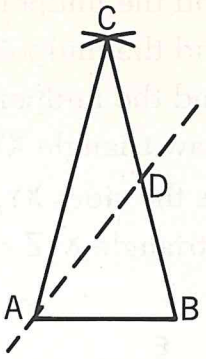


2. Construct all lines of symmetry of an equilateral triangle.
   - **a.** Draw a line segment MN about 3 in. long.
   - **b.** Construct an equilateral triangle MNO with base MN.
   - **c.** Find the midpoint P of $\overline{MN}$.
   - **d.** Draw the line through O and P. This is one line of symmetry.
   - **e.** Find the midpoint Q of $\overline{MO}$.
   - **f.** Draw the line through Q and N. This is a second line of symmetry.
   - **g.** Find the midpoint R of $\overline{ON}$.
   - **h.** Draw the line through M and R. This is a third line of symmetry.
   - **i.** How many lines of symmetry does an equilateral triangle have?

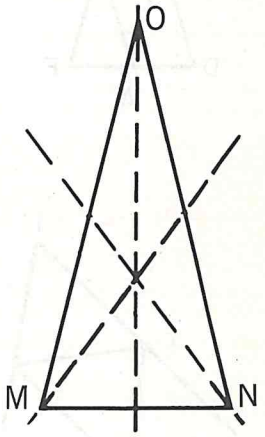


3. **a.** Construct an equilateral triangle with each side 2 inches long.
   - **b.** Construct all the lines of symmetry.
   - **c.** Do all 3 lines of symmetry intersect at one point?

1. **a.** Construct an isosceles triangle ABC with base AB 2 in. long and sides AC and BC each 4 in. long.
   b. Find the midpoint D of $\overline{BC}$.
   c. Draw the line through A and D.
   d. Cut out your drawing of triangle ABC.
   e. Fold the paper along the line AD.
   f. Is this line a line of symmetry of the triangle ABC?



2. **a.** Construct an isosceles triangle the same shape and size as the one in Exercise 1.
   b. Find the midpoints of all three sides of triangle MNO.
   c. Draw the three lines connecting midpoints and opposite vertices.
   d. Cut out your drawing of the triangle.
   e. Fold the paper in turn along each of the three lines.
   f. How many of the three lines are lines of symmetry of the triangle?
   g. How many lines of symmetry does triangle MNO have?

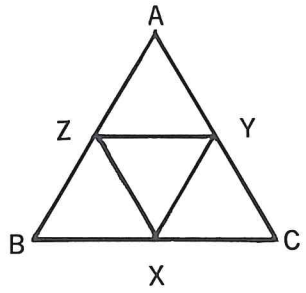


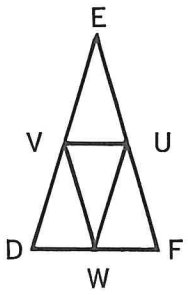
3. **a.** Draw a triangle that has no lines of symmetry.
   b. Cut out your drawing and show by folding that none of the lines connecting midpoints and vertices is a line of symmetry.

Complete each sentence.

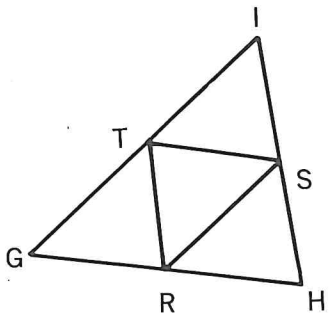
4. An equilateral triangle has ■ lines of symmetry.
5. An isosceles triangle that is not equilateral has ■ line of symmetry.
6. A scalene triangle has ■ lines of symmetry.

1. a. Construct an equilateral triangle ABC.
   b. Find the midpoint X of $\overline{BC}$.
   c. Find the midpoint Y of $\overline{AC}$.
   d. Find the midpoint Z of $\overline{AB}$.
   e. Draw triangle XYZ.
   f. Are the sides XY, YZ, and ZX equal in length?
   g. Is triangle XYZ equilateral?



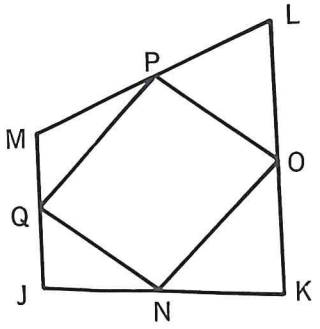


2. a. Construct an isosceles triangle with sides DE and EF twice the length of base DF.
   b. Find the midpoints of each side of triangle DEF and label them "W", "U", and "V".
   c. Connect the midpoints of these three sides.
   d. Is triangle UVW isosceles?



3. a. Draw a scalene triangle GHI.
   b. Find the midpoint S of $\overline{HI}$.
   c. Find the midpoint T of $\overline{IG}$.
   d. Find the midpoint R of $\overline{GH}$.
   e. Draw triangle RST.
   f. Color triangle GHI blue.
   g. Color triangle RST red.
   h. Are the red and blue triangles the same shape?

4. a. Draw a quadrilateral JKLM.
   b. Find the midpoints, N, O, P and Q, of the four sides.
   c. Draw quadrilateral NOPQ.
   d. Is $\overline{OP} \cong \overline{NQ}$?
   e. Is $\overline{PQ} \cong \overline{NO}$?



Feedback

Choose = or ≠.

1. $(6 + 8) + 1 \bullet 6 + (8 + 1)$
2. $(9 - 5) - 2 \bullet 9 - (5 - 2)$
3. $8 \times (3 \times 4) \bullet (8 \times 3) \times 4$
4. $(12 \div 4) \div 2 \bullet 12 \div (4 \div 2)$
5. $(3 + 1) + 7 \bullet 3 + (1 + 7)$
6. $8 - (2 - 1) \bullet (8 - 2) - 1$

7. When we have more than two numbers, and it does not matter on which pair of numbers we operate first, the operation is associative.
   - **a.** Is addition associative?
   - **b.** Is subtraction associative?
   - **c.** Is multiplication associative?
   - **d.** Is division associative?

Find each quotient.

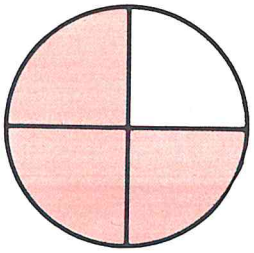
8. $8\overline{)648}$
9. $6\overline{)915}$
10. $9\overline{)1652}$
11. $7\overline{)8150}$
12. $3\overline{)5320}$
13. $6\overline{)4398}$
14. $5\overline{)7659}$
15. $8\overline{)2967}$

16. Bob sold 2 boxes of strawberries for 70¢. How much did he charge for each box?

17. Joe raises fresh corn. He sold 250 ears of corn on Monday, 314 ears of corn on Wednesday, and 289 ears of corn on Friday. How many ears of corn did he sell all together?

18. Bill raises chickens. He has a flock of 254 chickens. He sold 89 of them. How many does he have left?

Part 5 Fractions



The **region** enclosed by the circle is divided into 4 equal parts.
There are 3 parts shaded.
To indicate that we are talking about 3 of 4 equal parts, we write: $\frac{3}{4}$ ← numerator, ← denominator

The denominator tells how many equal parts there are.
The numerator tells how many parts we are talking about.

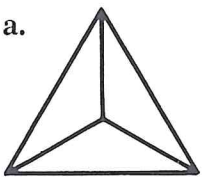
1. Complete each sentence.

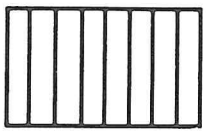
a. $\frac{1}{2}$ means there are ▢ equal parts.

b. $\frac{1}{5}$ means there are ▢ equal parts.

c. $\frac{3}{8}$ means there are ▢ equal parts.

2. How many equal parts are in each figure?

a. 

b. 

c.

d.

3. Complete each sentence.

a. $\frac{1}{3}$ means we are talking about ▢ part of 3 equal parts.

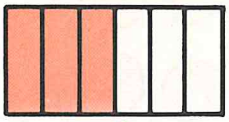
b. $\frac{2}{3}$ means we are talking about ▢ parts of 3 equal parts.

c. $\frac{7}{8}$ means we are talking about ▢ parts of 8 equal parts.

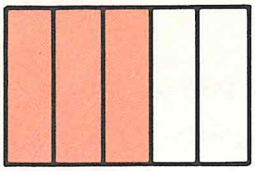
4. Write using numerals.

a. five-eighths b. one-sixth c. three-fourths d. two-thirds

What fraction tells what part of each whole figure is shaded?

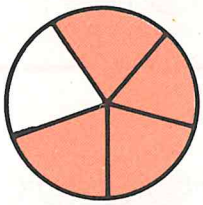
1. 

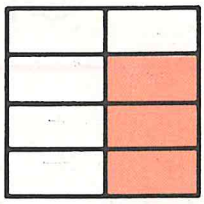
$\frac{3}{6}$

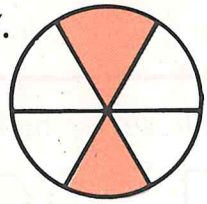
2. 

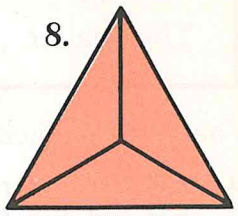
3.

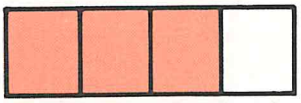
4.

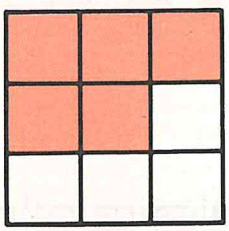
5. 

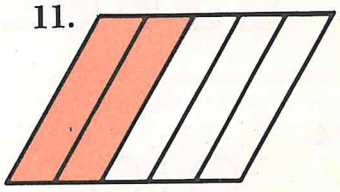
6. 

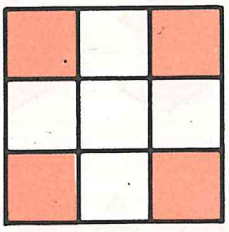
7. 

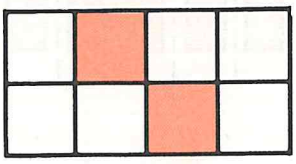
8. 

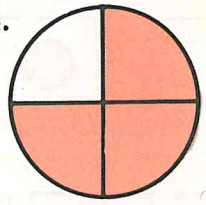
9. 

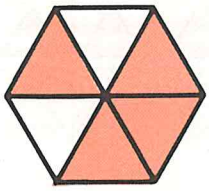
10. 

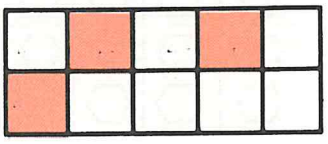
11. 

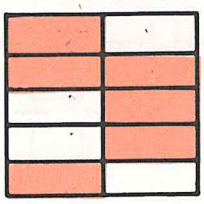
12. 

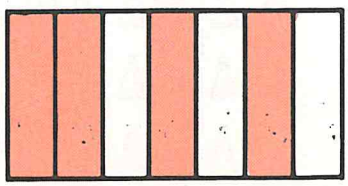
13. 

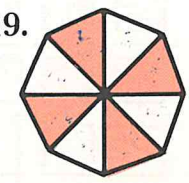
14. 

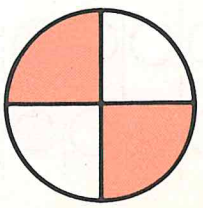
15. 

16. 

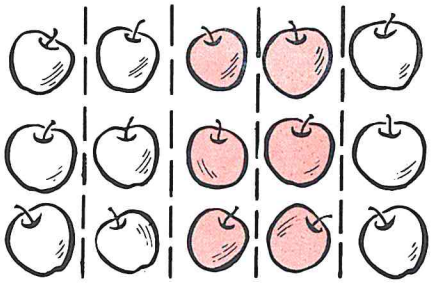
17. 

18. 

19. 

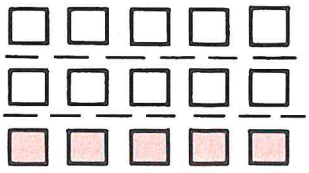
20. 

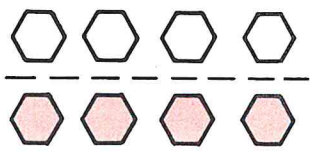
Each small set is a part of the large set.
We may show the parts by dotted lines.

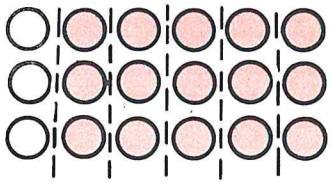


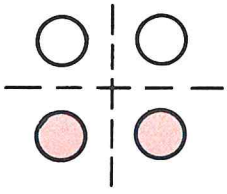
A set of things may be divided into parts.
This set of apples is divided into 5 parts.
The apples in 2 of the parts are shaded.
The shaded part is $\frac{2}{5}$ of the whole set.

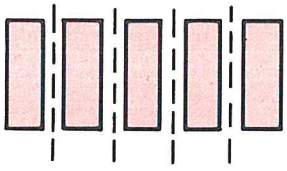
What part of each whole set is shaded?

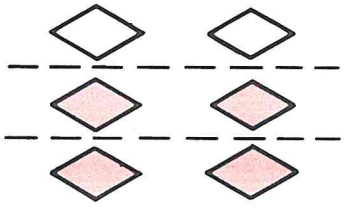
1. 

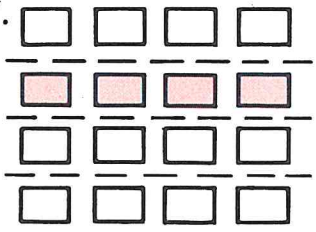
2. 

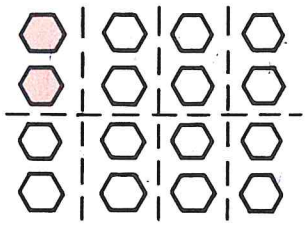
3. 

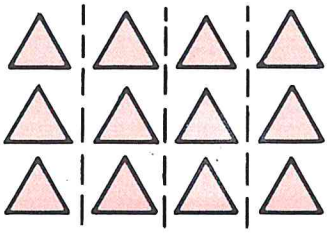
4. 

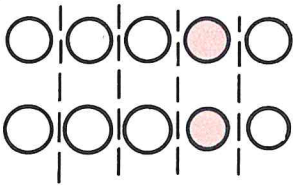
5. 

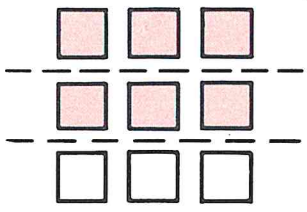
6. 

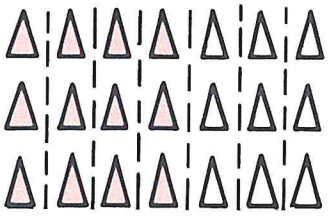
7. 

8. 

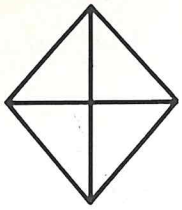
9. 

10. 

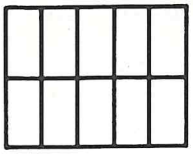
11. 

12. 

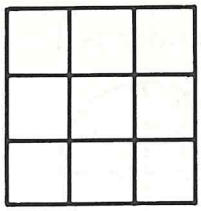
1. Copy each figure and shade it according to the fraction name written below it.

a. 

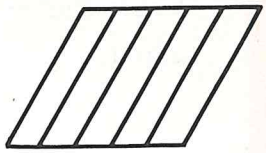
$\frac{1}{4}$

b. 

$\frac{7}{10}$

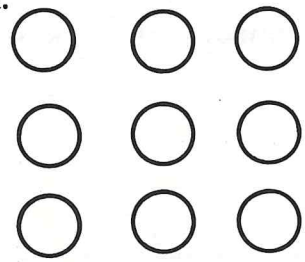
c. 

$\frac{2}{9}$

d. 

$\frac{5}{5}$

2. Copy each set of objects and divide it into small parts. Color the parts according to the fraction name written below it.

a. 

$\frac{2}{3}$

b.

$\frac{3}{4}$

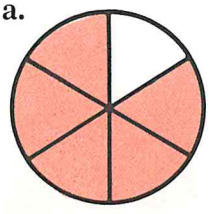
c.

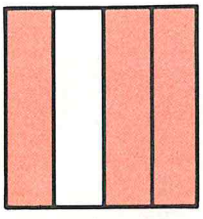
$\frac{9}{12}$

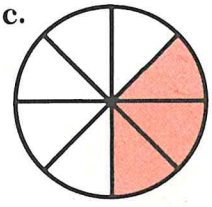
3. Give the denominator of each fraction in Exercise 1.

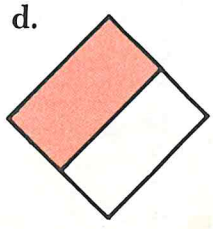
4. Give the numerator of each fraction in Exercise 2.

5. Write the fraction name that tells what part of each whole figure is shaded.

a. 

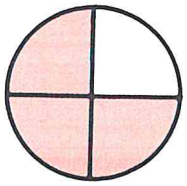
b. 

c. 

d. 

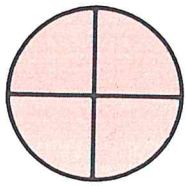
Fractions and Inequalities

When the numerator is less than the denominator, the fraction is less than 1.



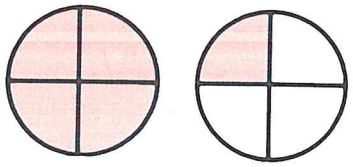
$\frac{3}{4} < 1$

When the numerator and the denominator are the same, the fraction is equal to 1.



$\frac{4}{4} = 1$

When the numerator is greater than the denominator, the fraction is greater than 1.



$\frac{5}{4} > 1$

1. a. List all the fractions below that are less than 1. Such fractions are often called **proper fractions.**

   b. List all the fractions below that are equal to or greater than 1. Such fractions are often called **improper fractions.**

   c. List all the fractions below that are equal to 1.

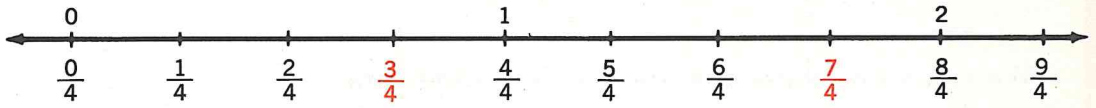
$\frac{4}{10}$ $\frac{6}{2}$ $\frac{5}{3}$ $\frac{4}{6}$ $\frac{8}{8}$ $\frac{12}{7}$ $\frac{18}{20}$ $\frac{3}{3}$ $\frac{12}{13}$ $\frac{13}{12}$ $\frac{11}{11}$

2. Choose $<$ or $=$ or $>$.

a. $\frac{5}{2}$ ● 1	d. $\frac{14}{15}$ ● 1	g. $\frac{9}{9}$ ● 1	j. $\frac{7}{8}$ ● 1
b. $\frac{13}{6}$ ● 1	e. $\frac{15}{15}$ ● 1	h. $\frac{3}{6}$ ● 1	k. $\frac{8}{8}$ ● 1
c. $\frac{7}{18}$ ● 1	f. $\frac{16}{15}$ ● 1	i. $\frac{50}{100}$ ● 1	l. $\frac{8}{7}$ ● 1

We can show whole numbers on a number line.

We can show fractions on a number line.



If two fractions have the same denominator, the one with the lesser numerator is the lesser number.

$$\frac{3}{4} < \frac{7}{4}$$

1. Count by halves from $\frac{0}{2}$ to $\frac{8}{2}$.
2. Count by sixths from $\frac{0}{6}$ to $\frac{15}{6}$.

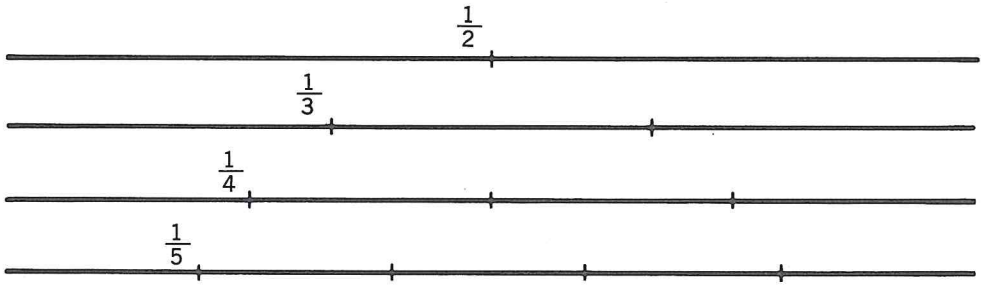
Choose < or >.

3. $\frac{3}{4} \bigcirc \frac{1}{4}$
4. $\frac{5}{5} \bigcirc \frac{7}{5}$
5. $\frac{10}{12} \bigcirc \frac{4}{12}$
6. $\frac{7}{7} \bigcirc \frac{4}{7}$
7. $\frac{3}{8} \bigcirc \frac{6}{8}$
8. $\frac{11}{9} \bigcirc \frac{2}{9}$
9. $\frac{1}{6} \bigcirc \frac{6}{6}$
10. $\frac{0}{2} \bigcirc \frac{1}{2}$
11. $\frac{2}{3} \bigcirc \frac{1}{3}$

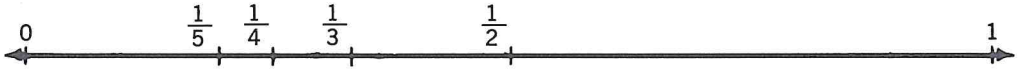
Write the fraction names in order starting with the name for the least fraction.

12. $\frac{3}{6}$ $\frac{1}{6}$ $\frac{4}{6}$ $\frac{10}{6}$ $\frac{0}{6}$ $\frac{8}{6}$ $\frac{6}{6}$ $\frac{15}{6}$ $\frac{2}{6}$
13. $\frac{1}{5}$ $\frac{8}{5}$ $\frac{2}{5}$ $\frac{5}{5}$ $\frac{6}{5}$ $\frac{4}{5}$ $\frac{0}{5}$ $\frac{3}{5}$ $\frac{7}{5}$
14. $\frac{3}{8}$ $\frac{4}{8}$ $\frac{9}{8}$ $\frac{0}{8}$ $\frac{7}{8}$ $\frac{2}{8}$ $\frac{6}{8}$ $\frac{8}{8}$ $\frac{1}{8}$
15. $\frac{4}{12}$ $\frac{10}{12}$ $\frac{11}{12}$ $\frac{2}{12}$ $\frac{16}{12}$ $\frac{13}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{0}{12}$

We can compare parts of a line segment.



We can show these fractions on a number line.



The fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ all have the same numerator.
If two fractions have the same numerator, the one with the greater denominator is the lesser fraction.

$$\frac{1}{5} < \frac{1}{2}$$

Choose < or >.

1. $\frac{1}{2}$ ● $\frac{1}{3}$
2. $\frac{1}{5}$ ● $\frac{1}{2}$
3. $\frac{1}{5}$ ● $\frac{1}{3}$
4. $\frac{1}{5}$ ● $\frac{1}{4}$
5. $\frac{2}{2}$ ● $\frac{2}{3}$
6. $\frac{2}{5}$ ● $\frac{2}{2}$
7. $\frac{4}{5}$ ● $\frac{4}{3}$
8. $\frac{9}{4}$ ● $\frac{9}{2}$

Write **greater** or **lesser.**

9. When the denominators are the same, the fraction with the greater numerator is the ■.
10. When the numerators are the same, the fraction with the greater denominator is the ■.

Choose < or >.

11. $\frac{1}{4}$ ● $\frac{1}{3}$
12. $\frac{2}{5}$ ● $\frac{4}{5}$
13. $\frac{9}{9}$ ● $\frac{3}{9}$
14. $\frac{1}{4}$ ● $\frac{1}{2}$
15. $\frac{3}{4}$ ● $\frac{3}{7}$
16. $\frac{2}{6}$ ● $\frac{6}{6}$
17. $\frac{25}{5}$ ● $\frac{25}{100}$
18. $\frac{25}{100}$ ● $\frac{75}{100}$

Addition of Whole Numbers and Fractions

We can add two whole numbers.

$0 + 3 = 3$

We can add a whole number and a fraction.

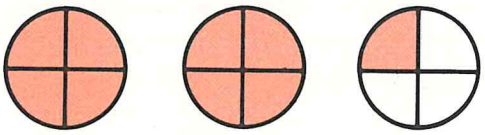
$1 + \frac{1}{2} = 1\frac{1}{2}$ $\qquad$ $3 + \frac{2}{3} = 3\frac{2}{3}$

Numbers like $1\frac{1}{2}$ and $3\frac{2}{3}$ are often called **mixed numbers.**

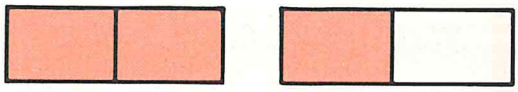
Find each sum.

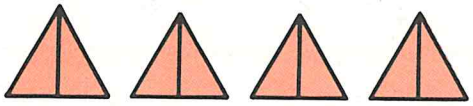
1. $4 + \frac{1}{5} =$ ■
2. $3 + \frac{2}{7} =$ ■
3. $4 + \frac{3}{8} =$ ■
4. $6 + \frac{1}{5} =$ ■
5. $2 + \frac{3}{5} =$ ■
6. $5 + \frac{5}{6} =$ ■
7. $7 + \frac{2}{3} =$ ■
8. $8 + \frac{1}{4} =$ ■
9. $6 + \frac{1}{2} =$ ■
10. $9 + \frac{1}{6} =$ ■
11. $8 + \frac{2}{5} =$ ■
12. $2 + \frac{3}{4} =$ ■

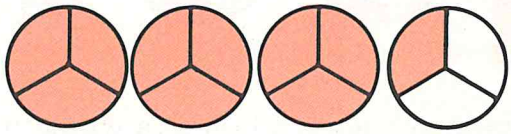
Name the shaded parts in two ways.

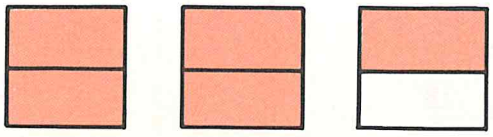
13. 

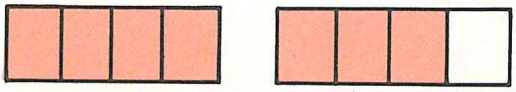
$\frac{9}{4}$ or $2\frac{1}{4}$

14. 

15. 

16. 

17. 

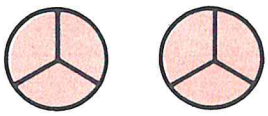
18. 

Some fractions equal whole numbers.

We have 3 whole pies.
We cut them to serve twelve people.
We have 12 fourths.

$\frac{12}{4} = 3$

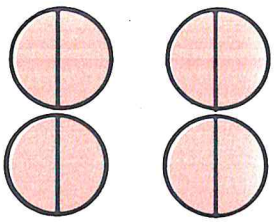
Give the correct equation for each of these groups of figures.

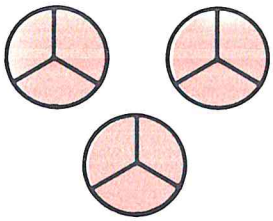
1. 

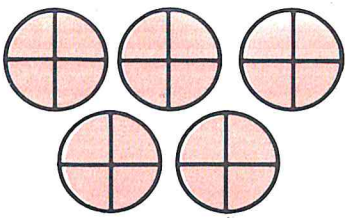
$\frac{6}{3} = 2$

2.

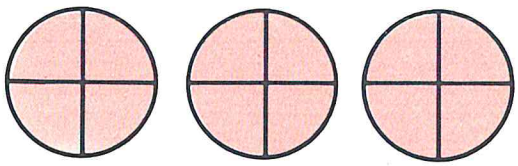
3.

4. 

5. 

6. 

We divide to find the whole number.



We have 12 parts.
4 parts make one whole.

$\frac{12}{4} = 12 \div 4 = 3$

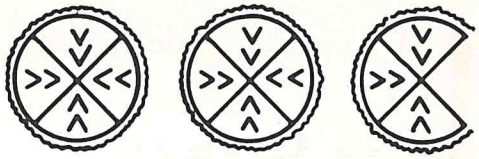
Express each fraction as a whole number.

7. $\frac{9}{9}$
8. $\frac{16}{4}$
9. $\frac{6}{2}$
10. $\frac{10}{2}$
11. $\frac{12}{3}$
12. $\frac{17}{17}$
13. $\frac{14}{2}$
14. $\frac{28}{7}$
15. $\frac{18}{3}$
16. $\frac{72}{9}$
17. $\frac{12}{12}$
18. $\frac{0}{7}$

Some fractions equal mixed numbers.

We have 2 whole pies and 3 fourths.
One piece of pie is gone.
We have 11 fourths left.

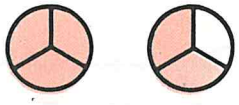
$\frac{11}{4} = 2\frac{3}{4}$

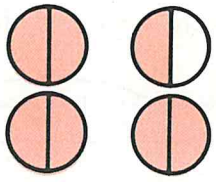


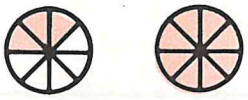
Give the correct equation for each of these groups of figures.

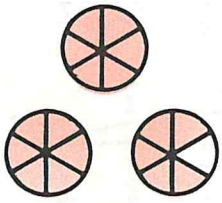
1.

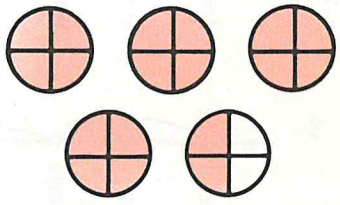
$\frac{7}{6} = 1\frac{1}{6}$

2. 

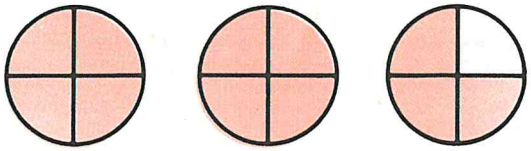
3. 

4. 

5. 

6. 

We divide the total number of parts by the number of parts it takes to make one whole.



We have 11 parts.
4 parts make one whole.
We divide 11 by 4.

$\frac{11}{4} = 11 \div 4 = 2\frac{3}{4}$

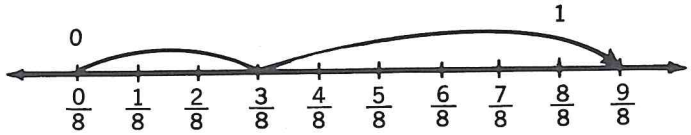
Divide to find another name.

7. $\frac{8}{3} = 2\frac{2}{3}$
8. $\frac{9}{2} =$ ■
9. $\frac{11}{5} =$ ■
10. $\frac{8}{1} =$ ■
11. $\frac{9}{6} =$ ■
12. $\frac{20}{3} =$ ■
13. $\frac{14}{3} =$ ■
14. $\frac{51}{7} =$ ■
15. $\frac{17}{4} =$ ■
16. $\frac{43}{9} =$ ■
17. $\frac{19}{6} =$ ■
18. $\frac{27}{8} =$ ■

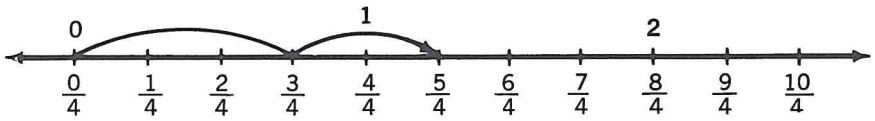
Addition of Fractions

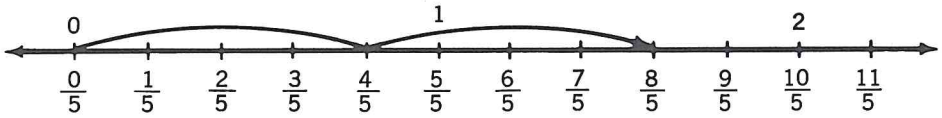
We can show addition of fractions on a number line.

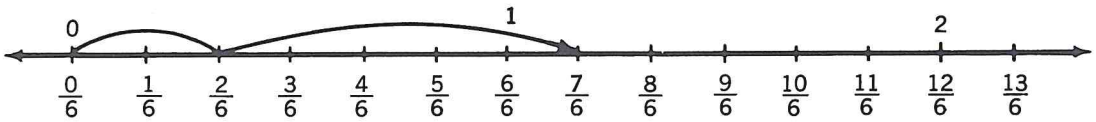
$\frac{3}{8} + \frac{6}{8} = \frac{9}{8} = 1\frac{1}{8}$



Write an equation for each number line.

1.


2.


3.


Find each sum.

4. $\frac{1}{5} + \frac{3}{5} =$ ■
5. $\frac{3}{8} + \frac{5}{8} =$ ■
6. $\frac{3}{4} + \frac{2}{4} =$ ■
7. $\frac{2}{6} + \frac{3}{6} =$ ■
8. $\frac{7}{8} + \frac{2}{8} =$ ■
9. $\frac{3}{4} + \frac{1}{4} =$ ■
10. $\frac{2}{3} + \frac{2}{3} =$ ■
11. $\frac{5}{6} + \frac{2}{6} =$ ■
12. $\frac{3}{8} + \frac{4}{8} =$ ■
13. $\frac{3}{5} + \frac{3}{5} =$ ■
14. $\frac{4}{7} + \frac{4}{7} =$ ■
15. $\frac{4}{10} + \frac{9}{10} =$ ■
16. $\frac{4}{9} + \frac{5}{9} =$ ■
17. $\frac{3}{7} + \frac{6}{7} =$ ■
18. $\frac{4}{6} + \frac{3}{6} =$ ■
19. $\frac{3}{8} + \frac{4}{8} =$ ■
20. $\frac{2}{3} + \frac{5}{3} =$ ■
21. $\frac{5}{9} + \frac{7}{9} =$ ■

22. Fractions whose denominators are the same are sometimes called like fractions. To add like fractions, add the ■.

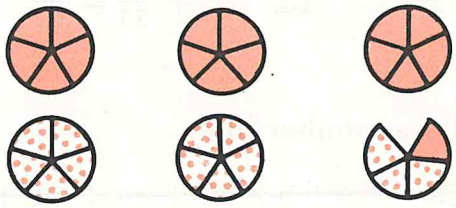
Find each sum.

$\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$
$= 1$

$\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$
$= 1\frac{1}{3}$

1. $\frac{1}{3} + \frac{4}{3} =$ ■
2. $\frac{2}{5} + \frac{3}{5} =$ ■
3. $\frac{4}{7} + \frac{1}{7} =$ ■
4. $\frac{1}{6} + \frac{4}{6} =$ ■
5. $\frac{3}{5} + \frac{4}{5} =$ ■
6. $\frac{3}{8} + \frac{6}{8} =$ ■
7. $\frac{9}{7} + \frac{4}{7} =$ ■
8. $\frac{8}{9} + \frac{7}{9} =$ ■
9. $\frac{2}{6} + \frac{3}{6} =$ ■
10. $\frac{7}{10} + \frac{6}{10} =$ ■
11. $\frac{5}{12} + \frac{3}{12} =$ ■
12. $\frac{11}{12} + \frac{2}{12} =$ ■
13. $\frac{6}{8} + \frac{1}{8} =$ ■
14. $\frac{3}{4} + \frac{1}{4} =$ ■
15. $\frac{8}{9} + \frac{8}{9} =$ ■

We can add mixed numbers



$$3\frac{1}{5} + 2\frac{3}{5} = 5\frac{4}{5}$$

To add mixed numbers, first add the fractions, and then add the whole numbers.

First step:
$$\begin{array}{r} 3\frac{1}{5} \\ +\ 2\frac{3}{5} \\ \hline \frac{4}{5} \end{array}$$

Second step:
$$\begin{array}{r} 3\frac{1}{5} \\ +\ 2\frac{3}{5} \\ \hline 5\frac{4}{5} \end{array}$$

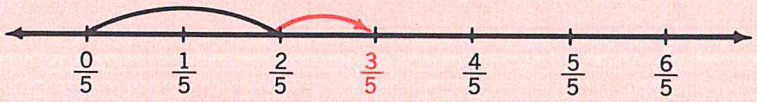
Find each sum.

16. $2\frac{1}{4} + 3\frac{1}{4}$
17. $5\frac{2}{4} + 2\frac{1}{4}$
18. $5\frac{5}{12} + 2\frac{2}{12}$
19. $7\frac{3}{24} + 4\frac{4}{24}$
20. $1\frac{1}{3} + 2\frac{1}{3}$
21. $4\frac{4}{7} + 1\frac{1}{7}$
22. $2\frac{2}{8} + 3\frac{5}{8}$
23. $7\frac{4}{9} + 1\frac{3}{9}$

Subtraction of Fractions

Copy and complete each problem. Use a number line if you need help.

$\frac{2}{5} + \square = \frac{3}{5}$

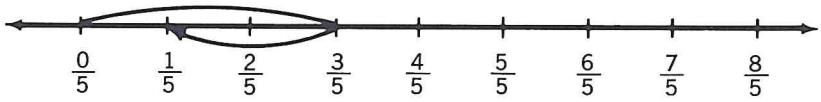


$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

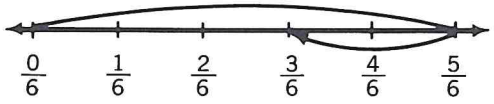
1. $\frac{1}{3} + \square = \frac{2}{3}$
2. $\frac{2}{5} + \square = \frac{3}{5}$
3. $\frac{1}{4} + \square = \frac{3}{4}$
4. $\frac{3}{7} + \square = \frac{6}{7}$
5. $\frac{5}{8} + \square = \frac{7}{8}$
6. $\square + \frac{4}{3} = \frac{5}{3}$
7. $\square + \frac{7}{14} = \frac{9}{14}$
8. $\square + \frac{5}{7} = \frac{10}{7}$
9. $\square + \frac{3}{18} = \frac{12}{18}$
10. $\square + \frac{2}{22} = \frac{12}{22}$
11. $\frac{10}{8} + \square = \frac{13}{8}$
12. $\frac{7}{16} + \square = \frac{11}{16}$
13. $\square + \frac{5}{12} = \frac{10}{12}$
14. $\frac{3}{15} + \square = \frac{11}{15}$
15. $\square + \frac{7}{23} = \frac{15}{23}$

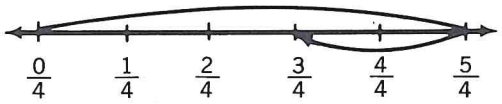
We may show subtraction of fractions on a number line.

$\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$



Write an equation for each number line.

16. 

17. 

Subtract the fractions.

18. $\frac{3}{6} - \frac{2}{6} = \square$
19. $\frac{5}{7} - \frac{3}{7} = \square$
20. $\frac{6}{8} - \frac{3}{8} = \square$
21. $\frac{7}{5} - \frac{3}{5} = \square$
22. $\frac{9}{10} - \frac{4}{10} = \square$
23. $\frac{8}{16} - \frac{3}{16} = \square$
24. $\frac{4}{4} - \frac{1}{4} = \square$
25. $\frac{4}{12} - \frac{3}{12} = \square$
26. $\frac{11}{12} - \frac{7}{12} = \square$

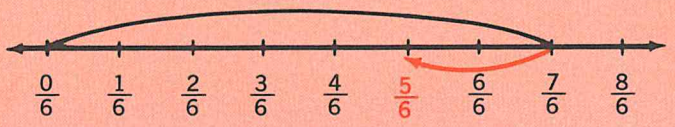
27. To subtract like fractions, subtract the $\square$.

Find each sum or difference.

1. $\frac{2}{6} + \frac{5}{6} = \square$
2. $\frac{5}{6} - \frac{2}{6} = \square$
3. $\frac{3}{9} + \frac{4}{9} = \square$
4. $\frac{4}{5} + \frac{4}{5} = \square$
5. $\frac{9}{12} + \frac{2}{12} = \square$
6. $\frac{11}{13} - \frac{7}{13} = \square$
7. $\frac{1}{3} + \frac{2}{3} = \square$
8. $\frac{9}{12} - \frac{4}{12} = \square$
9. $\frac{5}{10} - \frac{4}{10} = \square$

Copy and complete each problem. Use a number line if you need help.

$\frac{7}{6} - \square = \frac{5}{6}$



$\frac{7}{6} - \frac{2}{6} = \frac{5}{6}$

10. $\frac{7}{6} - \square = \frac{5}{6}$
11. $\frac{2}{3} - \square = \frac{1}{3}$
12. $\frac{3}{4} - \square = \frac{1}{4}$
13. $\frac{4}{5} - \square = \frac{2}{5}$
14. $\frac{7}{10} - \square = \frac{4}{10}$
15. $\frac{11}{12} - \square = \frac{8}{12}$
16. $\frac{6}{17} - \square = \frac{2}{17}$
17. $\frac{3}{13} - \square = \frac{1}{13}$
18. $\frac{9}{11} - \square = \frac{6}{11}$
19. $\frac{7}{8} - \square = \frac{4}{8}$
20. $\square - \frac{1}{4} = \frac{2}{4}$
21. $\square - \frac{2}{5} = \frac{1}{5}$
22. $\square - \frac{4}{10} = \frac{1}{10}$
23. $\square - \frac{2}{12} = \frac{3}{12}$
24. $\square - \frac{2}{9} = \frac{4}{9}$
25. $\square - \frac{3}{6} = \frac{2}{6}$
26. $\square - \frac{3}{17} = \frac{1}{17}$
27. $\square - \frac{2}{11} = \frac{3}{11}$

28. Joyce's house is $\frac{2}{8}$ of a mile from school. One day she went to her friend's house after school. Her friend lives $\frac{3}{8}$ of a mile past the school. How far is it from Joyce's house to her friend's house if she passes the school to get to her friend's house?

Add or subtract.

1. $\frac{5}{3} + \frac{2}{3} =$ ■
2. $\frac{3}{7} + \frac{8}{7} =$ ■
3. $\frac{19}{12} - \frac{11}{12} =$ ■
4. $\frac{20}{20} - \frac{7}{20} =$ ■
5. $\frac{15}{10} - \frac{8}{10} =$ ■
6. $\frac{16}{17} + \frac{16}{17} =$ ■
7. $\frac{11}{12} - \frac{8}{12} =$ ■
8. $\frac{75}{100} + \frac{10}{100} =$ ■
9. $\frac{9}{4} - \frac{3}{4} =$ ■
10. $\frac{12}{7} + \frac{5}{7} =$ ■
11. $\frac{8}{24} + \frac{9}{24} =$ ■
12. $\frac{14}{5} + \frac{26}{5} =$ ■
13. $\frac{9}{10} - \frac{7}{10} =$ ■
14. $\frac{29}{23} - \frac{17}{23} =$ ■
15. $\frac{3}{9} + \frac{5}{9} =$ ■
16. $\frac{8}{16} - \frac{1}{16} =$ ■
17. $\frac{8}{12} + \frac{6}{12} =$ ■
18. $\frac{9}{21} - \frac{7}{21} =$ ■
19. $\frac{9}{13} + \frac{16}{13} =$ ■
20. $\frac{30}{100} - \frac{25}{100} =$ ■
21. $\frac{4}{6} - \frac{1}{6} =$ ■
22. $\frac{7}{15} + \frac{13}{15} =$ ■
23. $\frac{23}{24} - \frac{18}{24} =$ ■
24. $\frac{18}{27} + \frac{4}{27} =$ ■

25. Mr. Gomez has only one-sixth of a pound of mixed nails. He needs five-sixths of a pound. What part of a pound must he buy to have enough nails?

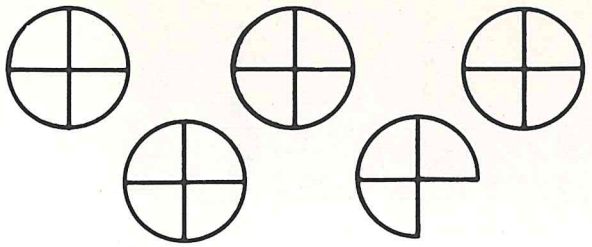
26. Sylvia has two-thirds of a yard of dress material. She is given another two-thirds of a yard of the same material. How much dress material does she now have?

27. Phyllis has one-half of a tube of paint more than Truman. Truman has one-half of a tube of paint. How much paint does Phyllis have?

28. Mike's science project is about the growth rate of plants. One day his corn plant measured $\frac{7}{16}$ inch tall. Two days later it measured $\frac{18}{16}$ inches. How much did it grow in the two days?

29. Jim's corn plant grew $\frac{13}{16}$ inch in two days. Which of the two corn plants grew more in the two days?

We can subtract mixed numbers.



$$\begin{array}{r} 4\frac{3}{4} \\ -1\frac{1}{4} \\ \hline 3\frac{2}{4} \end{array}$$

To subtract mixed numbers, first subtract the fractions, and then subtract the whole numbers.

First step:
$$\begin{array}{r} 4\frac{3}{4} \\ -1\frac{1}{4} \\ \hline \frac{2}{4} \end{array}$$

Second step:
$$\begin{array}{r} 4\frac{3}{4} \\ -1\frac{1}{4} \\ \hline 3\frac{2}{4} \end{array}$$

Find each difference.

1. $4\frac{5}{7} - 2\frac{3}{7}$
2. $9\frac{4}{17} - 1\frac{2}{17}$
3. $8\frac{6}{9} - 3\frac{2}{9}$
4. $7\frac{2}{3} - 3\frac{1}{3}$
5. $6\frac{7}{8} - 2\frac{3}{8}$
6. $9\frac{3}{4} - 2\frac{2}{4}$
7. $7\frac{4}{5} - 4\frac{1}{5}$
8. $8\frac{6}{7} - 3\frac{2}{7}$

Find each sum or difference.

9. $4\frac{4}{7} - 3\frac{1}{7}$
10. $8\frac{4}{8} - 5\frac{3}{8}$
11. $8\frac{3}{5} - 3\frac{1}{5}$
12. $9\frac{1}{4} + 10\frac{2}{4}$
13. $4\frac{5}{12} - 2\frac{2}{12}$
14. $2\frac{2}{8} + 3\frac{5}{8}$
15. $7\frac{4}{9} - 1\frac{3}{9}$
16. $4\frac{3}{6} - 2\frac{2}{6}$
17. $2\frac{1}{3} + 3\frac{1}{3}$
18. $7\frac{2}{9} + 2\frac{4}{9}$
19. $8\frac{9}{11} - 2\frac{2}{11}$
20. $6\frac{7}{12} + 1\frac{1}{12}$

We can add or subtract mixed numbers, whole numbers, and fractions.

$$\begin{array}{r} 5 \\ +1\frac{1}{4} \\ \hline 6\frac{1}{4} \end{array} \qquad \begin{array}{r} 5\frac{4}{7} \\ -\frac{2}{7} \\ \hline 5\frac{2}{7} \end{array} \qquad \begin{array}{r} \frac{1}{8} \\ +3\frac{3}{8} \\ \hline 3\frac{4}{8} \end{array} \qquad \begin{array}{r} 9\frac{5}{6} \\ -\frac{1}{6} \\ \hline 9\frac{4}{6} \end{array}$$

Add or subtract.

1. $3\frac{1}{4} + 2\frac{2}{4}$
2. $9\frac{1}{12} + \frac{7}{12}$
3. $8\frac{4}{11} - \frac{3}{11}$
4. $\frac{1}{16} + 3\frac{4}{16}$
5. $4\frac{3}{9} - \frac{2}{9}$
6. $3\frac{3}{4} + 4$
7. $5\frac{1}{10} + 7\frac{7}{10}$
8. $4\frac{1}{4} + 5\frac{1}{4}$
9. $\frac{1}{6} + 7\frac{4}{6}$
10. $7\frac{5}{8} - 2\frac{2}{8}$
11. $9\frac{3}{5} - \frac{1}{5}$
12. $8\frac{3}{7} - 5\frac{1}{7}$
13. $5\frac{2}{5} - 2\frac{2}{5}$
14. $\frac{2}{9} + 1\frac{2}{9}$
15. $\frac{5}{12} + 2\frac{5}{12}$
16. $4\frac{9}{11} - \frac{7}{11}$

17. Yesterday Janice colored the pictures on $3\frac{1}{3}$ pages of her coloring book. Today she colored $4\frac{1}{3}$ pages. How many pages did she color during the two days?

18. Bob drew a line segment $7\frac{9}{16}$ inches long. He erased $3\frac{1}{16}$ inches of it. How long was the remaining line segment?

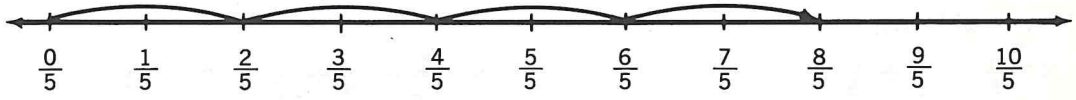
19. Sandy wants to make curtains for her windows. It will take $4\frac{3}{4}$ yd. to make the curtains. She has $\frac{1}{4}$ yd. How much more material does she need?

Multiplication of Fractions

We can show multiplication of whole numbers on a number line.

$$4 \times 2 = 2 + 2 + 2 + 2$$

We can show multiplication of a fraction by a whole number on a number line.



$$4 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$$
$$= \frac{8}{5}$$
$$= 1\frac{3}{5}$$

Copy and complete each problem.

1. $2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \square$
2. $3 \times \frac{5}{6} = \frac{\square}{6} + \frac{\square}{6} + \frac{\square}{6} = \square$
3. $4 \times \frac{2}{7} = \frac{\square}{7} + \frac{\square}{7} + \frac{\square}{7} + \frac{\square}{7} = \square$
4. $4 \times \frac{3}{8} = \frac{3}{\square} + \frac{3}{\square} + \frac{3}{\square} + \frac{3}{\square} = \square$

Write each problem as an addition problem and find the sum.

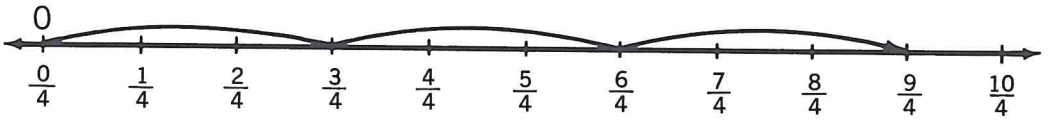
5. $4 \times \frac{3}{5} = \square$
6. $3 \times \frac{2}{3} = \square$
7. $6 \times \frac{1}{5} = \square$
8. $8 \times \frac{2}{3} = \square$
9. $3 \times \frac{3}{7} = \square$
10. $4 \times \frac{2}{5} = \square$
11. $2 \times \frac{4}{5} = \square$
12. $5 \times \frac{2}{3} = \square$
13. $7 \times \frac{1}{6} = \square$

Write each problem as a multiplication problem and find the product.

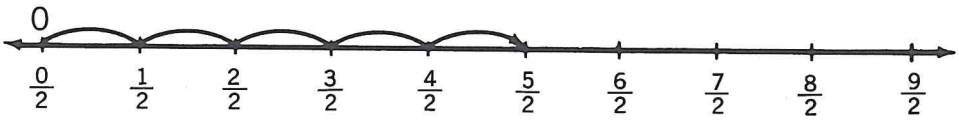
14. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \square$
15. $\frac{5}{6} + \frac{5}{6} = \square$
16. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \square$
17. $\frac{6}{7} + \frac{6}{7} + \frac{6}{7} = \square$
18. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \square$
19. $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = \square$

Give both an addition equation and a multiplication equation for each number line.

1.



2.



Find each sum.

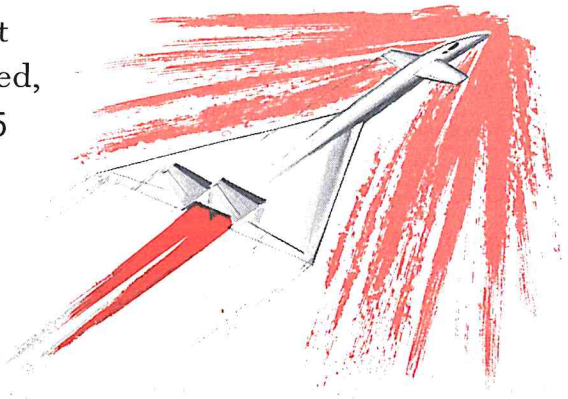
3. $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{9}{5} = 1\frac{4}{5}$
4. $\frac{4}{7} + \frac{4}{7} + \frac{4}{7} =$ ■
5. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} =$ ■
6. $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} + \frac{2}{7} =$ ■
7. $\frac{3}{6} + \frac{3}{6} + \frac{3}{6} + \frac{3}{6} =$ ■
8. $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} =$ ■
9. $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} =$ ■
10. $\frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8} =$ ■
11. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} =$ ■
12. $\frac{3}{7} + \frac{3}{7} + \frac{3}{7} =$ ■

Find each product.

13. $7 \times \frac{2}{5} = \frac{14}{5} = 2\frac{4}{5}$
14. $9 \times \frac{3}{5} =$ ■
15. $7 \times \frac{1}{4} =$ ■
16. $8 \times \frac{2}{3} =$ ■
17. $3 \times \frac{4}{5} =$ ■
18. $6 \times \frac{7}{8} =$ ■
19. $5 \times \frac{5}{6} =$ ■
20. $2 \times \frac{3}{5} =$ ■
21. $8 \times \frac{9}{10} =$ ■
22. $3 \times \frac{3}{7} =$ ■
23. $4 \times \frac{4}{5} =$ ■
24. $7 \times \frac{7}{9} =$ ■

25. To multiply a fraction by a whole number, multiply the ■ of the fraction by the whole number.

26. A supersonic plane can travel about $\frac{5}{9}$ of a mile in 1 second. At this speed, how many miles does it travel in 5 seconds?



Division of Fractions

We can divide a whole number by any whole number except zero. We cannot divide by zero.

$$8 \div 2 = 4$$

We can divide a fraction by any whole number except zero.

$$\frac{8}{5} \div 2 = \frac{4}{5}$$

Complete each division problem.

1. $\frac{25}{4} \div 5 = \frac{5}{4}$
2. $\frac{18}{3} \div 6 =$ ■
3. $\frac{81}{5} \div 9 =$ ■
4. $\frac{49}{8} \div 7 =$ ■
5. $\frac{36}{10} \div 4 =$ ■
6. $\frac{42}{9} \div 7 =$ ■
7. $\frac{36}{11} \div 6 =$ ■
8. $\frac{64}{9} \div 8 =$ ■
9. $\frac{35}{8} \div 5 =$ ■
10. $\frac{48}{17} \div 6 =$ ■
11. $\frac{24}{15} \div 3 =$ ■
12. $\frac{40}{7} \div 8 =$ ■
13. $\frac{24}{5} \div 4 =$ ■
14. $\frac{63}{10} \div 9 =$ ■
15. $\frac{56}{9} \div 7 =$ ■

Complete each problem.

16. $2 + \frac{2}{5} =$ ■
17. $3 \times \frac{2}{3} =$ ■
18. $\frac{8}{5} \div 2 =$ ■
19. $\frac{9}{5} - \frac{6}{5} =$ ■
20. $\frac{7}{4} + \frac{2}{4} =$ ■
21. $6 \times \frac{2}{7} =$ ■
22. $4 + \frac{3}{7} =$ ■
23. $\frac{2}{5} + \frac{3}{5} =$ ■
24. $\frac{9}{7} \div 3 =$ ■
25. $\frac{3}{7} + \frac{4}{7} =$ ■
26. $5 \times \frac{2}{9} =$ ■
27. $\frac{8}{3} - \frac{7}{3} =$ ■
28. $\frac{28}{10} \div 4 =$ ■
29. $3 \times \frac{3}{7} =$ ■
30. $8 \times \frac{5}{9} =$ ■

31. Mr. Hart had a strip of metal $\frac{6}{4}$ of a yard in length. He cut off $\frac{3}{4}$ of a yard. How long was the strip of metal that he had left?

32. Toni had $\frac{4}{8}$ of a yard of ribbon. She used $\frac{1}{8}$ of a yard to make a bow. How much ribbon did she have left?

Feedback

Choose = or > or <.

1. $1 \bigcirc \frac{2}{3}$
2. $\frac{1}{3} \bigcirc \frac{1}{2}$
3. $1 \bigcirc \frac{7}{6}$
4. $\frac{2}{6} \bigcirc \frac{1}{3}$
5. $\frac{1}{4} \bigcirc \frac{1}{5}$
6. $\frac{1}{2} \bigcirc \frac{2}{4}$
7. $\frac{5}{4} \bigcirc 1$
8. $\frac{2}{3} \bigcirc \frac{4}{6}$
9. $\frac{2}{4} \bigcirc \frac{1}{4}$
10. $1 \bigcirc \frac{3}{3}$
11. $\frac{5}{3} \bigcirc \frac{4}{4}$
12. $\frac{4}{5} \bigcirc \frac{4}{3}$

Find each sum or difference.

13. $\frac{1}{3} + \frac{1}{3} = \square$
14. $\frac{3}{4} + \frac{2}{4} = \square$
15. $\frac{2}{6} + \frac{3}{6} = \square$
16. $\frac{1}{7} + \frac{3}{7} = \square$
17. $\frac{3}{10} + \frac{4}{10} = \square$
18. $\frac{3}{2} - \frac{1}{2} = \square$
19. $\frac{5}{6} - \frac{3}{6} = \square$
20. $\frac{4}{5} - \frac{2}{5} = \square$
21. $\frac{6}{6} - \frac{1}{6} = \square$

Complete each problem.

22. $\frac{1}{4} + \square = \frac{3}{4}$
23. $3 \times \frac{2}{3} = \square$
24. $\square + \frac{2}{6} = \frac{3}{6}$
25. $1 + \frac{1}{6} = \square$
26. $2 \times \frac{2}{5} = \square$
27. $\frac{1}{6} + \square = \frac{5}{6}$
28. $\frac{8}{9} \div 2 = \square$
29. $\frac{4}{12} - \square = \frac{1}{12}$
30. $3 \times \frac{2}{7} = \square$

31. Helen made 4 bows. If she used $\frac{1}{3}$ yard of ribbon for each bow, how many yards of ribbon did she use?

32. Bob ate $\frac{1}{6}$ of a pie and Bill ate $\frac{2}{6}$ of the pie. How much of the pie did they eat together?

33. Trisha has five-twelfths of a box of jelly beans, Debbie has seven-twelfths of a box, and John has one-twelfth.
    a. How many twelfths of a box do they have all together?
    b. Is this greater than, equal to, or less than a whole box?

Feedback

Find each quotient.

1. $7\overline{)56}$
2. $6\overline{)489}$
3. $3\overline{)98}$
4. $9\overline{)372}$
5. $7\overline{)7205}$
6. $5\overline{)1462}$
7. $8\overline{)9146}$
8. $4\overline{)3408}$

Find each product.

9. $\begin{array}{r} 369 \\ \times\ 8 \\ \hline \end{array}$
10. $\begin{array}{r} 834 \\ \times\ 7 \\ \hline \end{array}$
11. $\begin{array}{r} 953 \\ \times\ 60 \\ \hline \end{array}$
12. $\begin{array}{r} 469 \\ \times\ 70 \\ \hline \end{array}$
13. $\begin{array}{r} 456 \\ \times\ 18 \\ \hline \end{array}$
14. $\begin{array}{r} 859 \\ \times\ 67 \\ \hline \end{array}$
15. $\begin{array}{r} 368 \\ \times\ 38 \\ \hline \end{array}$
16. $\begin{array}{r} 298 \\ \times\ 49 \\ \hline \end{array}$

17. Give one of these laws as a reason for the truth of each equation.

commutative law for addition — associative law for addition
commutative law for multiplication — associative law for multiplication

a. $7 \times 8 = 8 \times 7$
b. $8 \times (1 \times 2) = (8 \times 1) \times 2$
c. $4 + 6 = 6 + 4$
d. $9 \times 3 = 3 \times 9$
e. $13 + 7 = 7 + 13$
f. $4 + (6 + 7) = (4 + 6) + 7$
g. $3 + (8 + 1) = 3 + (1 + 8)$
h. $(3 \times 2) \times 9 = 3 \times (2 \times 9)$

18. Mrs. Jones buys a coat that costs twenty-four dollars and ninety-eight cents and a sweater that costs five dollars. How much does she spend?

19. The school librarian has six hundred twenty-five dollars and fifty cents to buy new books. She spends four hundred twenty-three dollars and forty cents. How much does she have left?

Part 6 More Division

Complete each problem.

1. a. $9 \times 10 =$ ■
   b. $9 \times 100 =$ ■
   c. $9 \times 1000 =$ ■

2. a. $10 \times 8 = 8 \times 10 =$ ■
   b. $100 \times 8 = 8 \times 100 =$ ■
   c. $1000 \times 8 = 8 \times 1000 =$ ■

3. a. $5 \times 10 =$ ■
   b. $5 \times 100 =$ ■
   c. $5 \times 1000 =$ ■

4. a. $10 \times 7 = 7 \times 10 =$ ■
   b. $100 \times 7 = 7 \times 100 =$ ■
   c. $1000 \times 7 = 7 \times 1000 =$ ■

5. a. $18 \times 10 =$ ■
   b. $36 \times 100 =$ ■
   c. $42 \times 10 =$ ■
   d. $51 \times 1000 =$ ■
   e. $26 \times 100 =$ ■
   f. $45 \times 1000 =$ ■

6. a. $10 \times 72 =$ ■
   b. $1000 \times 81 =$ ■
   c. $100 \times 19 =$ ■
   d. $1000 \times 37 =$ ■
   e. $10 \times 25 =$ ■
   f. $100 \times 48 =$ ■

7. a. $37 \times 100 =$ ■
   b. $42 \times 1000 =$ ■
   c. $61 \times 10 =$ ■
   d. $39 \times 1000 =$ ■
   e. $84 \times 10 =$ ■
   f. $57 \times 100 =$ ■

8. a. $100 \times 40 =$ ■
   b. $1000 \times 53 =$ ■
   c. $10 \times 17 =$ ■
   d. $100 \times 63 =$ ■
   e. $10 \times 28 =$ ■
   f. $1000 \times 94 =$ ■

9. Mr. Taylor bought 100 boxes of canned soup. If each box contained 24 cans of soup, how many cans of soup did he buy?

10. Millie knew 14 children in her school. Her older brother Bill knew 10 times as many children. How many children did Bill know?

Multiplying Multiples of Ten

Copy and complete each problem and give a reason for each step.

$7 \times 20 = 7 \times (2 \times 10)$	because $20 = 2 \times 10$
$= (7 \times 2) \times 10$	because of the associative law of multiplication
$= 14 \times 10$	because $7 \times 2 = 14$
$= 140$	because $14 \times 10 = 140$

1. $4 \times 30 = 4 \times (3 \times 10)$
   $= (4 \times \_\_) \times 10$
   $= \_\_ \times 10$
   $= \blacksquare$

2. $5 \times 70 = 5 \times (\_\_ \times 10)$
   $= (5 \times \_\_) \times 10$
   $= \_\_ \times 10$
   $= \blacksquare$

3. $7 \times 40 = 7 \times (4 \times \_\_)$
   $= (7 \times \_\_) \times \_\_$
   $= \_\_ \times \_\_$
   $= \blacksquare$

4. $9 \times 60 = 9 \times (\_\_ \times \_\_)$
   $= (9 \times \_\_) \times \_\_$
   $= \_\_ \times \_\_$
   $= \blacksquare$

Complete each problem.

5. a. 6×4 tens $=$ 24 tens $= 240$
   b. 8×9 tens $= \blacksquare$
   c. 7×7 tens $= \blacksquare$
   d. 5×8 tens $= \blacksquare$
   e. 6×6 tens $= \blacksquare$
   f. 9×7 tens $= \blacksquare$
   g. 8×8 tens $= \blacksquare$
   h. 4×4 tens $= \blacksquare$

6. a. 4 tens $\times 6 =$ 240
   b. 7 tens $\times 8 = \blacksquare$
   c. 4 tens $\times 8 = \blacksquare$
   d. 5 tens $\times 5 = \blacksquare$
   e. 3 tens $\times 9 = \blacksquare$
   f. 6 tens $\times 8 = \blacksquare$
   g. 9 tens $\times 5 = \blacksquare$
   h. 3 tens $\times 6 = \blacksquare$

We can divide 480 by 6.

$$\begin{array}{r} 80 \\ 6\overline{)480} \\ \underline{480} \\ 0 \end{array}$$ because 6×8 tens $= 480$

We now wish to divide 480 by 60.

$$60\overline{)480}$$

We think of 480 as 48 tens and 60 as 6 tens.

$$\begin{array}{r} 8 \\ 60\overline{)480} \\ \underline{480} \end{array}$$ because 6 tens $\times 8 = 48$ tens $= 480$

Find each quotient and give a reason for each step.

1. a. $80\overline{)320}$ b. $30\overline{)240}$ c. $70\overline{)490}$ d. $60\overline{)420}$
2. a. $50\overline{)250}$ b. $80\overline{)560}$ c. $20\overline{)160}$ d. $30\overline{)270}$
3. a. $40\overline{)280}$ b. $40\overline{)240}$ c. $90\overline{)360}$ d. $50\overline{)450}$

Copy and complete each problem.

$$\begin{array}{r} 5 \\ 50\overline{)262} \\ \underline{250} \\ 12 \end{array}$$ because 5 tens $\times 5 = 250$

4. a. $30\overline{)242}$ b. $70\overline{)506}$ c. $40\overline{)174}$ d. $80\overline{)583}$
5. a. $60\overline{)375}$ b. $20\overline{)140}$ c. $60\overline{)526}$ d. $50\overline{)465}$
6. a. $80\overline{)715}$ b. $90\overline{)415}$ c. $30\overline{)161}$ d. $70\overline{)382}$

Multiplying Multiples of Ten by Multiples of Ten

Copy and complete each problem.

$$\begin{aligned} 60 \times 20 &= (6 \times 10) \times (2 \times 10) \\ &= (6 \times 2) \times (10 \times 10) \\ &= 12 \times 100 \\ &= 1200 \end{aligned}$$

1. $80 \times 70 = (8 \times 10) \times (\_ \times 10)$
$= (8 \times \_) \times (10 \times 10)$
$= \_ \times 100$
$= \blacksquare$

2. $30 \times 40 = (\_ \times 10) \times (4 \times \_)$
$= (\_ \times \_) \times (\_ \times \_)$
$= \_ \times 100$
$= \blacksquare$

3. $30 \times 50 = (\_ \times 10) \times (5 \times 10)$
$= (\_ \times \_) \times (\_ \times \_)$
$= \_ \times \_$
$= \blacksquare$

4. $40 \times 60 = (\_ \times \_) \times (\_ \times \_)$
$= (\_ \times \_) \times (\_ \times \_)$
$= \_ \times \_$
$= \blacksquare$

$10 \times 10 = 100$
1 ten × 1 ten = 1 hundred

$10 \times 1 = 10$
1 ten × 1 = 1 ten

5. a. 6 tens × 3 tens = **18 hundreds**
   b. 5 tens × 7 tens = ■
   c. 6 tens × 4 tens = ■
   d. 2 tens × 9 tens = ■
   e. 6 tens × 8 tens = ■
   f. 9 tens × 3 tens = ■
   g. 4 tens × 5 tens = ■

6. a. 6 tens × 3 tens = **1800**
   b. 8 tens × 5 tens = ■
   c. 7 tens × 2 tens = ■
   d. 3 tens × 8 tens = ■
   e. 9 tens × 6 tens = ■
   f. 4 tens × 5 tens = ■
   g. 5 tens × 3 tens = ■

Dividing by Multiples of Ten

We can divide 480 by 60.

$$\begin{array}{r} 8 \\ 60\overline{)480} \\ \underline{480} \\ \end{array}$$

because 6 tens × 8 = 48 tens = 480

Now we wish to divide problems like this:

$$60\overline{)4800}$$

We think of 4800 as 480 tens and of 60 as 6 tens.

$$\begin{array}{r} 80 \\ 60\overline{)4800} \\ \underline{4800} \\ \end{array}$$

80 = 8 tens

because 6 tens × 8 tens = 4800

Find each quotient.

1. a. $30\overline{)210}$
   b. $30\overline{)2100}$

2. a. $70\overline{)280}$
   b. $70\overline{)2800}$

3. a. $50\overline{)400}$
   b. $50\overline{)4000}$

4. a. $80\overline{)560}$
   b. $80\overline{)5600}$

5. a. $90\overline{)810}$
   b. $90\overline{)8100}$

6. a. $40\overline{)200}$
   b. $40\overline{)2000}$

7. a. $20\overline{)120}$
   b. $20\overline{)1200}$

8. a. $50\overline{)450}$
   b. $50\overline{)4500}$

9. a. $70\overline{)420}$
   b. $70\overline{)4200}$

10. a. $40\overline{)240}$
    b. $40\overline{)2400}$

11. a. $60\overline{)300}$
    b. $60\overline{)3000}$

12. a. $30\overline{)180}$
    b. $30\overline{)1800}$

We estimate before we divide.

Step 1

$$\begin{array}{r} 4 \\ 60\overline{)2934} \\ 2400 \\ \hline 534 \end{array}$$

$60 \times 40 < 2900$

because 6 tens × 4 tens = 2400

Step 2

$$\begin{array}{r} 48 \\ 60\overline{)2934} \\ 2400 \\ \hline 534 \\ 480 \\ \hline 54 \end{array}$$

because 6 tens × 8 = 48 tens = 480

Copy and complete each problem.

1. $$\begin{array}{r} \blacksquare 5 \\ 90\overline{)5874} \\ 5400 \\ \hline 474 \\ 450 \\ \hline 24 \end{array}$$

because 9 tens × ■ tens = 54 hundreds = 5400

because 9 tens × ■ = 450

2. $$\begin{array}{r} \blacksquare 1 \\ 80\overline{)6528} \\ 6400 \\ \hline 128 \\ \blacksquare \\ \hline 48 \end{array}$$

because 8 tens × ■ tens = 64 hundreds = 6400

because 8 tens × 1 = ■

3. $$\begin{array}{r} \blacksquare 7 \\ 30\overline{)1436} \\ 1200 \\ \hline 236 \\ 210 \\ \hline 26 \end{array}$$

because 3 tens × ■ tens = 12 hundreds = 1200

because 3 tens × ■ = 210

Copy and complete each problem.

1. $90\overline{)8526}$

$$\begin{array}{r} 9\blacksquare \\ 8100 \\ \hline 426 \\ 360 \\ \hline \blacksquare \end{array}$$

2. $70\overline{)586}$

$$\begin{array}{r} 8 \\ \blacksquare \\ \hline \blacksquare \end{array}$$

3. $50\overline{)4961}$

$$\begin{array}{r} 9\blacksquare \\ \blacksquare \\ \hline 461 \\ \blacksquare \\ \hline 11 \end{array}$$

4. $80\overline{)6095}$

$$\begin{array}{r} 7\blacksquare \\ 5600 \\ \hline 495 \\ \blacksquare \\ \hline \blacksquare \end{array}$$

5. $20\overline{)1479}$

$$\begin{array}{r} \blacksquare 3 \\ 1400 \\ \hline 79 \\ \blacksquare \\ \hline 19 \end{array}$$

6. $70\overline{)2913}$

$$\begin{array}{r} \blacksquare 1 \\ 2800 \\ \hline 113 \\ 70 \\ \hline \blacksquare \end{array}$$

7. $90\overline{)2497}$

8. $20\overline{)498}$

9. $50\overline{)6324}$

10. $40\overline{)3686}$

11. $10\overline{)198}$

12. $60\overline{)3729}$

13. $60\overline{)799}$

14. $40\overline{)3690}$

15. $20\overline{)842}$

16. A clock showed that an experiment lasted 3894 minutes. How many hours was this?

17. If 1520 people viewed an art exhibit in 20 hours and the same number of people viewed the exhibit during each hour, how many people viewed the exhibit each hour?

Multiplying Tens and Hundreds

$1 \times 1 \text{ ten} = 1 \text{ ten}$
$1 \times 10 = 10$

$1 \text{ ten} \times 1 \text{ ten} = 1 \text{ hundred}$
$10 \times 10 = 100$

$1 \times 1 \text{ hundred} = 1 \text{ hundred}$
$1 \times 100 = 100$

$1 \text{ ten} \times 1 \text{ hundred} = 1 \text{ thousand}$
$10 \times 100 = 1000$

Complete each problem.

1. a. 5 tens × 3 tens = **1500**
   b. 7 tens × 5 tens = ■
   c. 6 tens × 3 tens = ■
   d. 8 tens × 9 tens = ■
   e. 4 tens × 5 tens = ■

2. a. 50 × 50 = **2500**
   b. 60 × 40 = ■
   c. 30 × 20 = ■
   d. 80 × 50 = ■
   e. 90 × 50 = ■

3. a. 30 × **40** = 1200
   b. 20 × ■ = 1400
   c. 60 × ■ = 5400
   d. 10 × ■ = 600
   e. 40 × ■ = 2000

4. a. 4 tens × 3 hundreds = **12000**
   b. 9 tens × 2 hundreds = ■
   c. 6 tens × 4 hundreds = ■
   d. 8 tens × 8 hundreds = ■
   e. 3 tens × 7 hundreds = ■

5. a. 80 × 700 = **56000**
   b. 30 × 600 = ■
   c. 70 × 700 = ■
   d. 40 × 400 = ■
   e. 60 × 200 = ■

6. a. 50 × **300** = 15000
   b. 70 × ■ = 4900
   c. 30 × ■ = 21000
   d. 50 × ■ = 30000
   e. 40 × ■ = 1200

We estimate before we divide.

Step 1

$$\begin{array}{r} 9 \\ 80\overline{)74282} \\ 72000 \\ \hline 2282 \end{array}$$

$80 \times 900 < 74000$

because 8 tens × 9 hundreds = 72000

Step 2

$$\begin{array}{r} 92 \\ 80\overline{)74282} \\ 72000 \\ \hline 2282 \\ 1600 \\ \hline 682 \end{array}$$

$80 \times 20 < 2200$

because 8 tens × 2 tens = 1600

Step 3

$$\begin{array}{r} 928 \\ 80\overline{)74282} \\ 72000 \\ \hline 2282 \\ 1600 \\ \hline 682 \\ 640 \\ \hline 42 \end{array}$$

$80 \times 8 < 680$

because 8 tens × 8 = 640

Copy and complete each problem.

1. $70\overline{)54238}$

$$\begin{array}{r} 7\square4 \\ 70\overline{)54238} \\ 49000 \\ \hline 5238 \\ \square \\ \hline 338 \\ 280 \\ \hline \square \end{array}$$

2. $40\overline{)5862}$

$$\begin{array}{r} \square4\square \\ 40\overline{)5862} \\ 4000 \\ \hline 1862 \\ \square \\ \hline 262 \\ 240 \\ \hline \square \end{array}$$

3. $80\overline{)74682}$

$$\begin{array}{r} 9\square3 \\ 80\overline{)74682} \\ 72000 \\ \hline \square \\ 2400 \\ \hline \square \\ 240 \\ \hline \square \end{array}$$

4. $30\overline{)24681}$

5. $50\overline{)24685}$

6. $20\overline{)4688}$

Solve each problem.

1. $30\overline{)9251}$	2. $40\overline{)8888}$	3. $80\overline{)35022}$
4. $20\overline{)3502}$	5. $80\overline{)52834}$	6. $70\overline{)2913}$
7. $80\overline{)6095}$	8. $70\overline{)59836}$	9. $40\overline{)6837}$
10. $90\overline{)8723}$	11. $30\overline{)4562}$	12. $90\overline{)76923}$
13. $60\overline{)9863}$	14. $70\overline{)36284}$	15. $40\overline{)89430}$
16. $50\overline{)46582}$	17. $70\overline{)65760}$	18. $30\overline{)6872}$
19. $80\overline{)72614}$	20. $20\overline{)4956}$	21. $90\overline{)56846}$

Find each answer. Show all of your work.

22. Mr. Carlson works at a mint operating the machine that puts pennies into rolls. Each roll has 50 pennies in it. At one machine there are 63,245 pennies. How many rolls of pennies is this?

23. Another machine puts 20 quarters in a roll. At this machine there are 85,263 quarters. How many rolls of quarters is this?

24. Mr. Lynch wants to buy 6324 stamps of a certain kind. If these stamps come in sheets of 60 stamps, how many sheets must he buy?

25. Miss Park has a photograph album with 1294 photographs. If each page has 20 photographs, how many pages did she fill?

26. The auditorium has 16,800 seats. If each row has 80 seats, how many rows are there?

Estimating

To divide by a two-digit number not ending in zero, we use a two-digit number ending in zero as an estimate. We can estimate a number to the nearest multiple of ten by looking at the digit in the ones' place. If this last digit is 1, 2, 3, or 4 we think of it as a 0. If this last digit is 5, 6, 7, 8, or 9 we think of it as a 0 and add 1 ten to the tens' place.

We think: 42 is 40 to the nearest multiple of ten.
26 is 30 to the nearest multiple of ten.

Tell what number we should use as an estimate.

$28\overline{)695}$ Use 30 as an estimate.

1. $23\overline{)795}$	4. $66\overline{)989}$	7. $35\overline{)598}$	10. $33\overline{)574}$
2. $21\overline{)636}$	5. $25\overline{)638}$	8. $47\overline{)863}$	11. $32\overline{)597}$
3. $42\overline{)886}$	6. $37\overline{)864}$	9. $28\overline{)632}$	12. $23\overline{)978}$

13. If these numbers are divisors, tell what numbers we use as the estimates.

a. 45	d. 55	g. 72	j. 92	m. 51
b. 69	e. 84	h. 16	k. 43	n. 82
c. 21	f. 36	i. 88	l. 64	o. 76

Find each product.

14. a. $20 \times 9 =$ ■	15. a. $40 \times 20 =$ ■	16. a. $30 \times 100 =$ ■
b. $23 \times 9 =$ ■	b. $38 \times 20 =$ ■	b. $32 \times 100 =$ ■
17. a. $20 \times 30 =$ ■	18. a. $50 \times 8 =$ ■	19. a. $60 \times 100 =$ ■
b. $24 \times 30 =$ ■	b. $47 \times 8 =$ ■	b. $64 \times 100 =$ ■

Using Estimates in Division

When we divide 695 by 28, we use 30 as an estimate.

$$\begin{array}{r} 2 \\ 28\overline{)695} \\ \underline{560} \\ 135 \end{array}$$ because {the estimate of 28 is 30; 3 tens × 2 tens < 695 < 3 tens × 3 tens}

$$\begin{array}{r} 24 \\ 28\overline{)695} \\ \underline{560} \\ 135 \\ \underline{112} \\ 23 \end{array}$$ because 3 tens × 4 < 135 < 3 tens × 5; which is less than 28

Copy and complete each problem.

1. $\begin{array}{r} 2\blacksquare \\ 36\overline{)819} \\ \underline{720} \\ 99 \\ \underline{72} \\ \blacksquare \end{array}$ because {the estimate of 36 is ___; ■ × 2 tens < 819 < ■ × 3 tens}; because ■ × 2 < 99 < ■ × 3; which is less than ■

2. $\begin{array}{r} 31 \\ 24\overline{)753} \\ \underline{72} \\ 33 \\ \underline{24} \\ \blacksquare \end{array}$ because {the estimate of 24 is ___; ■ × 3 tens < ■ < ■ × 4 tens}; because ■ × 1 < ■ < ■ × 2; which is less than ■

3. $\begin{array}{r} 2\blacksquare \\ 42\overline{)968} \\ \underline{840} \\ 128 \\ \underline{126} \\ \blacksquare \end{array}$ because {the estimate of 42 is ___; ■ × 2 tens < ■ < ■ × 3 tens}; because ■ × 3 < 128 < ■ × 4; which is less than ■

Sometimes we must show 0 tens in our quotient.

```
  204
33)6734
 6600
  134
  132
    2
```

We must show 0 tens.

because {the estimate of 33 is 30
{30 × 2 hundreds < 6734 < 30 × 3 hundreds

because 30 × 4 < 134 < 30 × 5

Sometimes we must show 0 ones in our quotient.

```
  140
49)6908
 4900
 2008
 1960
   48
```

We must show 0 ones.

because {the estimate of 49 is 50
{50 × 1 hundred < 6908 < 50 × 2 hundreds

because 50 × 4 tens < 2008 < 50 × 5 tens

Copy and complete each problem. Supply the missing reasons.

1.
```
  30
21)646
 630
```
because ___

2.
```
  15■
58)9202
 5800
 3402
 2900
  502
  ■
   38
```
because ___

because ___

because ___

3.
```
  2■3
42)8526
 8400
  126
  ■
```
because ___

because ___

4.
```
  1■2
79)8121
 7900
  221
  ■
  ■
```
because ___

because ___

Find each quotient.

5. 32)192
6. 53)4770
7. 41)8200
8. 29)6023
9. 38)823
10. 88)9199
11. 75)1845
12. 34)2380
13. 28)9021
14. 92)6440
15. 61)3668
16. 67)4000

Sometimes the method of estimating the divisor does not work and we must change our quotient.
For example, let us divide 32 by 13.

$$\begin{array}{r} 3 \\ 13\overline{)32} \\ \underline{39} \end{array}$$

because {the estimate of 13 is 10
$10 \times 3 < 32 < 4 \times 10$
but $13 \times 3 > 32$

Since 3 is too great because $13 \times 3 = 39$ and $39 > 32$, we must use a lesser whole number. We try 2.

$$\begin{array}{r} 2 \\ 13\overline{)32} \\ \underline{26} \end{array}$$

The product must not be greater than the dividend, and the remainder must always be less than the divisor. We sometimes need to change our quotient because the remainder is too great.
For example, let us divide 67 by 16.

$$\begin{array}{r} 3 \\ 16\overline{)67} \\ \underline{48} \\ 19 \end{array}$$

because {the estimate of 16 is 20
$20 \times 3 < 68 < 20 \times 4$
but $19 > 16$

Here the remainder was too great, so we try 4.

$$\begin{array}{r} 4 \\ 16\overline{)67} \\ \underline{64} \\ 3 \end{array}$$

Find each quotient.

1. $12\overline{)35}$
2. $44\overline{)90}$
3. $11\overline{)34}$
4. $14\overline{)70}$
5. $16\overline{)100}$
6. $19\overline{)82}$
7. $21\overline{)83}$
8. $17\overline{)136}$

Copy and complete each problem.

1. $66\overline{)989}$

$$\begin{array}{r} 14 \\ 66\overline{)989} \\ \square \\ \hline 329 \\ \square \\ \hline \square \end{array}$$

2. $25\overline{)538}$

$$\begin{array}{r} 2\square \\ 25\overline{)538} \\ 500 \\ \hline 38 \\ \square \\ \hline \square \end{array}$$

3. $37\overline{)854}$

$$\begin{array}{r} \square 3 \\ 37\overline{)854} \\ 740 \\ \hline 114 \\ \square \\ \hline \square \end{array}$$

4. $62\overline{)843}$

$$\begin{array}{r} 1\square \\ 62\overline{)843} \\ \square \\ \hline 223 \\ \square \\ \hline \square \end{array}$$

5. $31\overline{)741}$

$$\begin{array}{r} \square 3 \\ 31\overline{)741} \\ \square \\ \hline 121 \\ \square \\ \hline \square \end{array}$$

6. $85\overline{)6290}$

$$\begin{array}{r} \square 4 \\ 85\overline{)6290} \\ 5950 \\ \hline 340 \\ \square \\ \hline \square \end{array}$$

7. $52\overline{)8679}$

$$\begin{array}{r} 1\square 6 \\ 52\overline{)8679} \\ \square \\ \hline 3479 \\ 3120 \\ \hline \square \\ 312 \\ \hline \square \end{array}$$

8. $48\overline{)23758}$

$$\begin{array}{r} 49\square \\ 48\overline{)23758} \\ \square \\ \hline 4558 \\ \square \\ \hline 238 \\ \square \\ \hline \square \end{array}$$

9. $63\overline{)8892}$

$$\begin{array}{r} 1\square 1 \\ 63\overline{)8892} \\ \square \\ \hline \square \\ 2520 \\ \hline \square \\ 63 \\ \hline \square \end{array}$$

10. $31\overline{)8062}$

$$\begin{array}{r} \square 6\square \\ 31\overline{)8062} \\ \square \\ \hline 1862 \\ \square \\ \hline \square \end{array}$$

11. $16\overline{)4924}$

$$\begin{array}{r} 3\square 7 \\ 16\overline{)4924} \\ \square \\ \hline \square \\ 112 \\ \hline \square \end{array}$$

12. $88\overline{)45832}$

$$\begin{array}{r} \square 2\square \\ 88\overline{)45832} \\ 44000 \\ \hline \square \\ \square \\ \hline \square \end{array}$$

13. Mr. Walsh washes 216 windows of a building with 27 windows on each floor. How many stories high is the building?

14. A printing press printed a total of 56,145 pages for a high school yearbook. If each book contained 57 pages, how many books were printed?

Checking Division

After we divide, we may multiply to check our work.

$$\begin{array}{r} 3 \\ 15\overline{)45} \\ \underline{45} \\ \end{array}$$

Check: $15 \times 3 = 45$

If there is a remainder when we divide, we multiply the divisor and the quotient and then add the remainder.

$$\begin{array}{r} 3 \\ 15\overline{)47} \\ \underline{45} \\ 2 \end{array}$$

Check:
$$\begin{array}{r} 15 \\ \times\ 3 \\ \hline 45 \\ +\ 2 \\ \hline 47 \end{array}$$

Find each quotient. Check each problem by multiplication.

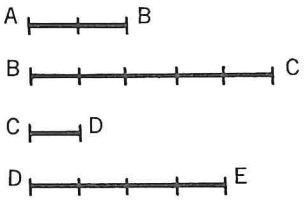
1. $38\overline{)4630}$
2. $43\overline{)29582}$
3. $67\overline{)475}$
4. $52\overline{)76321}$
5. $27\overline{)699}$
6. $63\overline{)58462}$
7. $32\overline{)647}$
8. $66\overline{)23598}$
9. $40\overline{)28376}$
10. $24\overline{)4286}$
11. $86\overline{)572}$
12. $42\overline{)98634}$

13. Mr. Ellis sold the same number of pairs of socks each day for 45 days. He sold a total of 7830 pairs of socks in that time. How many pairs of socks did he sell each day?

14. Mrs. Christopher ordered 8352 buttons. She bought three dozen of each style of button. How many different styles of buttons did she buy?

15. Mrs. Lawrence sold 672 hats in 56 days. If she sold the same number of hats each day, how many hats did she sell each day?

Finding Averages



length of $\overline{AB} = 2$ cm.
length of $\overline{BC} = 5$ cm.
length of $\overline{CD} = 1$ cm.
length of $\overline{DE} = 4$ cm.

The length of all the line segments is 12 cm.

There are 4 line segments. $2 + 5 + 1 + 4 = 12$

$$12 \div 4 = 3$$

The average length of the segments is 3 cm.

1. Mike weighs 80 pounds, Terry weighs 72 pounds, and Billy weighs 76 pounds. What is their average weight?
2. Find the average of 3, 4, 6, 8, and 9.
3. Find the average of 80, 72, 90, and 84.

Sometimes the sum is given and we need only divide to find the average. The length of 4 line segments is 12 cm. What is the average length of the segments?

$$12 \div 4 = 3$$

The average length of the segments is 3 cm.

4. Mr. Tate drove 1829 miles in 7 days. What was the average number of miles he drove each day?
5. In 18 days before Halloween, Mrs. Marconi sold 3226 costumes. What was the average number of costumes she sold each day?
6. Mrs. Rommanoff sold 75,643 Christmas cards in the 37 days before Christmas. What was the average number of cards she sold each day?

Find each quotient. Then check each problem by multiplication.

1. $26\overline{)4862}$	4. $38\overline{)31592}$	7. $18\overline{)4384}$	10. $81\overline{)9863}$
2. $47\overline{)987}$	5. $69\overline{)4949}$	8. $21\overline{)7396}$	11. $24\overline{)1648}$
3. $90\overline{)43851}$	6. $85\overline{)6275}$	9. $80\overline{)9888}$	12. $39\overline{)2467}$

13. Mr. Kulinski drove 3146 miles on his vacation trip. If he was gone 18 days, what was the average number of miles he drove each day?

14. A sawmill received 24 logs weighing a total of 87,243 pounds. What was the average number of pounds each log weighed?

15. Mr. Samuel's bakery sold 62,495 cookies in 87 days. What was the average number of cookies sold each day?

16. Mr. Delaney sold 5496 shirts in 68 days. What was the average number of shirts he sold each day?

17. In West City there are 245 children in the fourth grade, 304 in the fifth grade, and 281 in the sixth grade. How many children all together are there in all three grades in West City?

18. There are 850 children in two elementary schools. If 394 children are boys, how many of the children are girls?

19. Nancy collected leaves. She collected 18 leaves on Monday, 15 leaves on Tuesday, 14 leaves on Wednesday, 11 leaves on Thursday, and 13 leaves on Friday. What was the average number of leaves collected each day?

When we divide money, we must show dollars and cents in the answer, but we estimate in the same way.

$$\begin{array}{r} \$0.97 \\ 5\overline{)\$4.85} \\ 4\ 50 \\ \hline 35 \\ 35 \\ \hline 0 \end{array}$$

We show 0 dollars.

$$\begin{array}{r} \$\ 1.89 \\ 25\overline{)\$47.25} \\ 25\ 00 \\ \hline 22\ 25 \\ 20\ 00 \\ \hline 2\ 25 \\ 2\ 25 \\ \hline 0 \end{array}$$

The decimal point in the answer should be placed right above the decimal point in the dividend when we divide by a whole number.

Complete each problem. Be sure to put the dollar sign and the decimal point in the correct place in each answer.

1. $33\overline{)\$158.73}$
2. $26\overline{)\$161.20}$
3. $48\overline{)\$25.44}$
4. $52\overline{)\$32.76}$
5. $46\overline{)\$15.64}$
6. $75\overline{)\$225.00}$
7. $35\overline{)\$104.30}$
8. $94\overline{)\$62.98}$
9. $36\overline{)\$87.48}$
10. $53\overline{)\$49.82}$
11. $29\overline{)\$184.15}$
12. $72\overline{)\$428.40}$

13. Seventy-five adults went to a fair. The cashier collected $111.75 from them. What was the price of admission for adults?

14. Eighty-four children went to the fair. The cashier collected $29.40 from them. What was the price of admission for children?

15. Sixty-eight teenagers went to this fair. The cashier collected $51.00 from them. What was the price of admission for teenagers?

Complete each problem.

1. $91\overline{)4277}$
2. $78\overline{)1872}$
3. $45\overline{)1530}$
4. $64\overline{)3072}$
5. $68\overline{)2652}$
6. $89\overline{)4895}$
7. $13\overline{)4387}$
8. $6\overline{)2488}$
9. $92\overline{)\$229.08}$
10. $42\overline{)\$24.36}$
11. $65\overline{)\$477.75}$
12. $83\overline{)\$121.18}$

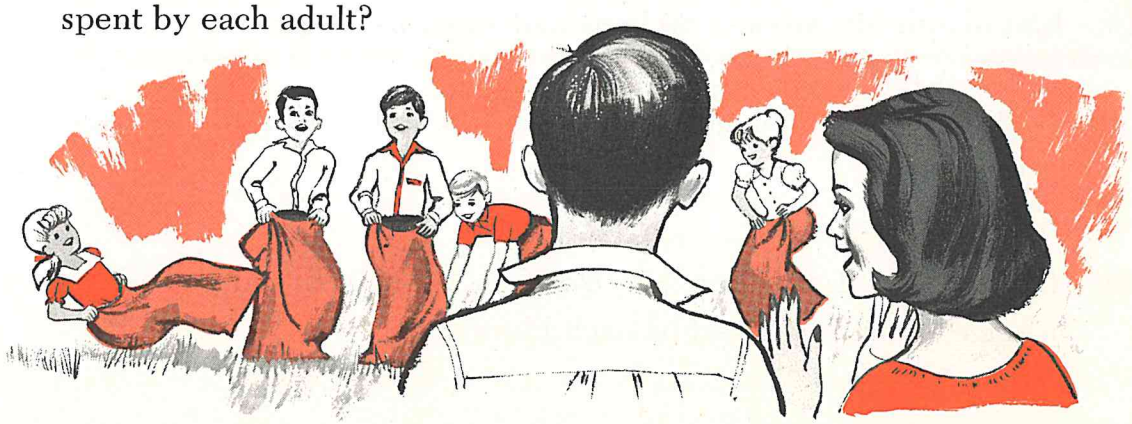
13. A person breathes about 840 times an hour. About how many times a minute does he breathe?

14. If it takes Mrs. Donohue 60 minutes to read 32,486 words, how many words does she read in one minute?

15. There is a total of 15,072 people living in 24 large apartment buildings. What is the average number of people in each building?

16. Seventy-nine children went to a picnic and bought something to eat and drink. They spent a total of $11.85. What was the average amount spent by each child?

17. Fifty-six adults went with the children to the picnic and bought food and prizes. They spent a total of $17.92. What was the average amount spent by each adult?



Feedback

Find each quotient.

1.	$5\overline{)7385}$	**4.**	$81\overline{)7581}$	**7.**	$24\overline{)10249}$	**10.**	$91\overline{)24640}$
2.	$45\overline{)5476}$	**5.**	$95\overline{)3250}$	**8.**	$17\overline{)80942}$	**11.**	$76\overline{)53537}$
3.	$3\overline{)1726}$	**6.**	$30\overline{)6031}$	**9.**	$63\overline{)64598}$	**12.**	$84\overline{)79840}$

Find each product.

13.	845 × 80	**14.**	927 × 38	**15.**	637 × 27	**16.**	9428 × 9

Solve each problem.

17. Joe raises fresh corn to sell to local markets. He has 450 ears of corn, and wishes to put one dozen ears in each bag.

a. How many bags will he need?

b. How many ears of corn will be left over?

18. The milkman sells 3 quarts of milk each day to each family on his route. In 5 days he sold 615 quarts of milk. How many families are on his route?

19. Last month Mr. Morse's 89 hens laid 156 dozen eggs.

a. How many eggs is this?

b. What is the average number of eggs laid last month by one of his hens?

20. The city voted to spend $37,925.76 on its 6 libraries. What was the average amount received by each library?

Feedback

Find each sum.

1. $\begin{array}{r} 160 \\ 171 \\ 282 \\ \underline{103} \end{array}$

2. $\begin{array}{r} 111 \\ 142 \\ 130 \\ \underline{173} \end{array}$

3. $\begin{array}{r} 178 \\ 160 \\ 111 \\ \underline{210} \end{array}$

4. $\begin{array}{r} 276 \\ 134 \\ 293 \\ \underline{182} \end{array}$

5. $\begin{array}{r} 167 \\ 171 \\ 282 \\ \underline{103} \end{array}$

Complete each problem.

6. $\frac{3}{6} + \frac{2}{6} = \blacksquare$
7. $\frac{3}{8} + \blacksquare = \frac{6}{8}$
8. $\frac{4}{7} \div 2 = \blacksquare$
9. $3 \times \frac{2}{5} = \blacksquare$
10. $4 + \frac{2}{3} = \blacksquare$
11. $1 - \frac{2}{3} = \blacksquare$
12. $\frac{7}{8} - \blacksquare = \frac{4}{8}$
13. $2 \times \frac{3}{7} = \blacksquare$
14. $\frac{3}{4} + \blacksquare = 1$

Complete each problem.

15. $\{1, 4, 8\} \cup \{2, 4, 6\} = \blacksquare$
16. $\{2, 3\} \subset \{2, 4, \blacksquare, 5\}$
17. $4 \in \{\blacksquare, 10\}$
18. $\{1, 5, 10\} \cap \{0, 6, 8\} = \blacksquare$
19. $\{\text{Alice, Sue}\} - \{\text{Kay}\} = \blacksquare$
20. $\{n: 4 < n < 7\} = \blacksquare$

21. Mr. Roberts raises puppies to sell. During the past month, he sold 23 puppies at $56.75 each. How much money did he receive from the sale of the puppies?

22. Four children made a stamp exhibit for school. Jack brought two hundred seventy stamps, Joe brought one hundred eighty-five, Ed brought two hundred ten, and Amy brought two hundred twelve. How many stamps were in the exhibit?

Part 7 Measuring

Linear Measure

Length is measured in units of inches, feet, yards, rods, and miles.

12 inches (in.) = 1 foot (ft.)
36 inches = 1 yard (yd.)
3 feet = 1 yard
$16\frac{1}{2}$ feet = 1 rod (rd.)
$5\frac{1}{2}$ yards = 1 rod
5280 feet = 1 mile (mi.)
1760 yards = 1 mile
320 rods = 1 mile

What number makes each sentence true?

1. 2 rd. = ■ yd.
2. 46 ft. = 15 yd. ■ ft.
3. 5 ft. 3 in. = ■ in.
4. 2 yd. 1 ft. 4 in. = ■ in.
5. 3 mi. = ■ rd.
6. 6 rd. = ■ yd.
7. 5 mi. = ■ yd.
8. 8 rd. = ■ yd.
9. ■ yd. = 6 mi.
10. ■ rd. = 5280 ft.
11. ■ ft. = 2 mi.
12. ■ rd. = 33 ft.
13. ■ ft. = 4 rd.

14. ■ ft. = 1760 yd.
15. ■ ft. = 144 in.
16. ■ in. = 4 yd.
17. ■ yd. = 4 rd.
18. ■ mi. = 640 rd.

Solve each problem.

1. At a track meet Dan ran the 50-yard dash. How many feet did he run?

2. At the same track meet Bob jumped 2 yards 4 inches and Phil jumped 72 inches. Who jumped farther?

3. In the high-jump event John jumped a height of 5 feet 6 inches and George jumped a height of 68 inches. Who jumped higher?

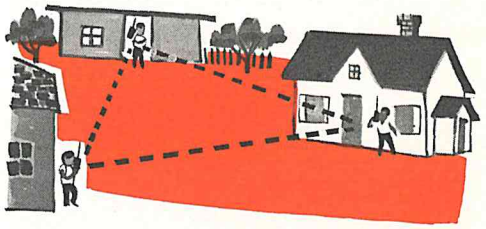
4. Scott is 4 feet 6 inches tall, and Greg is 53 inches tall. Who is taller?

5. Leslie walks 4 miles to the library. How many rods is this? How many yards? How many feet?

6. Mr. Gordon cut a 10-foot board into four equal parts for book shelves. How many inches long was each shelf?

7. Lisa had a paper streamer 12 yards long. She cut it into pieces 18 feet long. How many pieces did she have?

8. Jim's house is 32 yards from Gary's house and 93 feet from Ernie's house. Whose house is closer to Jim's?



9. Mrs. Lewis had 3 yards of string. She used all but 4 inches of it. How many inches did she use?

Time

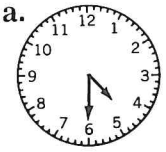
Time is measured in units of seconds, minutes, hours, and days.

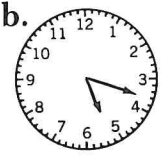
60 seconds (sec.) = 1 minute (min.)

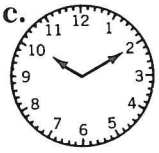
60 minutes = 1 hour (hr.)

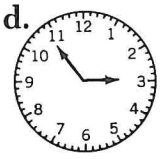
24 hours = 1 day (da.)

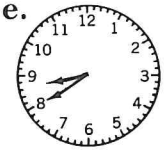
1. Give the time for each of the clocks below.

a. 

b. 

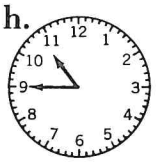
c. 

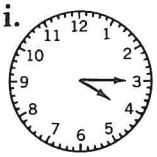
d. 

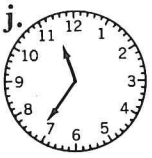
e. 

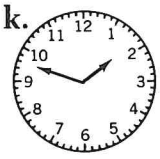
f.

g.

h. 

i. 

j. 

k. 

l.

2. The Johnson family left for the airport at 9:30 A.M. They arrived at 10:13 A.M. How long did it take them to drive to the airport?

3. Ann helps her mother bake and sew from 2:15 P.M. to 3:40 P.M. How long does she work?

4. Sue takes a train to visit her cousin. The train leaves at 11:05 A.M. and arrives 3 hours and 25 minutes later. What time does Sue arrive?

5. Mr. Brent drives his car at an average of 40 miles an hour. How far does he go in 5 hours?

Calendar

Long periods of time are measured in units of weeks, months, years, decades, and centuries.

7 days (da.) = 1 week (wk.)
52 weeks = 1 year (yr.)
365 days = 1 year
12 months (mo.) = 1 year
10 years = 1 decade
100 years = 1 century

February						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28						

March						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

April						
S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

1. Eleanor starts her 3-week vacation on March 30. On what date does her vacation end?

2. Betty is seven on February 3 and Lorie is five on February 28. What is the difference in age of the two girls?

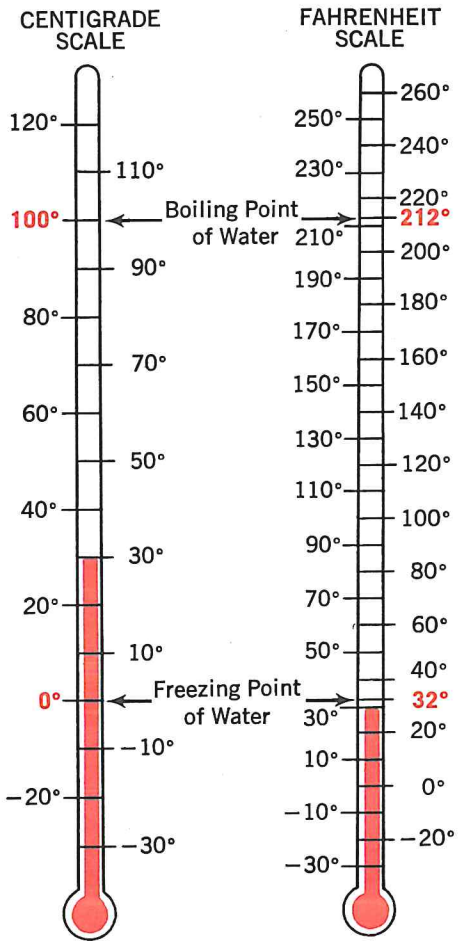
3. How many months are there between October 1 and January 1?

4. The Cook family has lived in the same city for 7 years. How many months is this?

5. Complete each problem.

   a. 180 min. = ■ hr.
   b. 7 min. = ■ sec.
   c. 72 hr. = ■ days
   d. 7 yr. = ■ mo.
   e. 1 wk. = ■ hr.
   f. 3 decades = ■ yr.
   g. 50 yr. = ■ decades
   h. 20 decades = ■ yr.
   i. 3 centuries = ■ yr.
   j. 4 centuries = ■ decades

Temperature



We use different scales to measure temperature.
The thermometer on the right has a Fahrenheit scale. The thermometer on the left has a centigrade scale.
We write "C" for centigrade.
We write "F" for Fahrenheit.
Below zero temperatures are written with a minus sign. For example, "−10°C" means 10 degrees below zero on the centigrade scale.

Answer each question.

1. Is it warmer when the temperature is 10°C or when it is −10°C?
2. Is it colder when the temperature is 30°F or when it is 30°C?
3. At what centigrade temperature does water freeze?
4. At what centigrade temperature does water boil?
5. Is it warmer when the temperature is 65°F or when it is 65°C?
6. Is it colder when the temperature is 0°C or when it is 0°F?
7. Is it hotter when the temperature is 100°C or when it is 100°F?
8. How many degrees are there between −100°C and 100°C?
9. How many degrees are there between 5°C and 121°C?
10. If a thermometer in the freezer reads 30, is it a centigrade or a Fahrenheit thermometer?

Liquid Measure

Liquids are measured in units of teaspoons, tablespoons, fluid ounces, cups, pints, quarts, and gallons.

3 teaspoons (t.) = 1 tablespoon (tbsp.)
2 tablespoons = 1 fluid ounce (fl. oz.)
8 fluid ounces = 1 cup (c.)
16 fluid ounces = 1 pint (pt.)
2 cups = 1 pint
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

The word "cup" means a measuring cup which is used for cooking. A teacup usually holds less liquid than a measuring cup.

Complete each problem.

1. 1 gal. = 8 pt.
2. 24 pt. = ■ qt.
3. 24 fl. oz. = ■ c.
4. 60 pt. = ■ gal.
5. 2 gal. = ■ qt.
6. 9 gal. = ■ pt.
7. 1 gal. = ■ fl. oz.
8. 5 c. = ■ fl. oz.
9. 3 gal. = ■ c.
10. 3 pt. = ■ fl. oz.
11. 54 gal. = ■ qt.
12. 12 gal. = ■ pt.
13. 5 pt. = ■ c.
14. 8 fl. oz. = ■ c.
15. 1 qt. = ■ c.
16. 3 tbsp. = 9 t.
17. 16 qt. = 32 ■
18. 9 t. = 3 ■
19. 6 pt. = 96 ■
20. 8 qt. = 2 ■
21. 32 fl. oz. = 2 ■
22. 6 tbsp. = 3 ■
23. 8 pt. = 4 ■
24. 12 t. = 2 ■
25. 3 c. = 24 ■
26. 64 fl. oz. = 2 ■
27. 32 c. = 2 ■
28. 1 qt. = 32 ■
29. 8 pt. = 1 ■
30. 3 fl. oz. = 6 ■

Weight Measure

Weight is measured in units of ounces, pounds, hundredweights, and tons.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Complete each sentence.

1. 1 lb. 13 oz. = ▢ oz.
2. 96 oz. = ▢ lb.
3. 128 oz. = ▢ lb.
4. 5 lb. = ▢ oz.
5. 2 T. 5 cwt. = ▢ lb.
6. 3 tons = ▢ cwt.
7. 170 oz. = ▢ lb. and ▢ oz.
8. 7 cwt. = ▢ lb.
9. 3710 lb. = ▢ T. and ▢ lb.
10. 5 tons = ▢ lb.

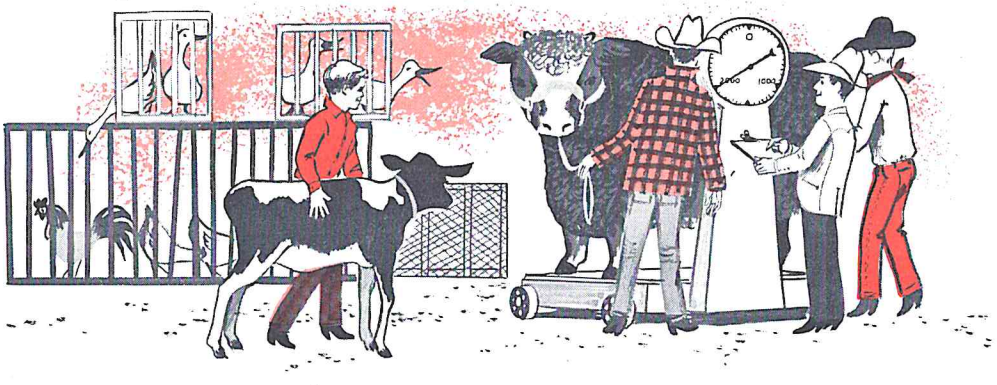
Solve the following problems.

11. Mr. Sims bought a bull weighing 2000 pounds. He paid $100.00 per cwt. How much did the bull cost?

12. He also bought a calf weighing 450 lb. He paid $20.00 per cwt. How much did he pay?

13. Mr. Scott sold his chickens by the crate. Each crate contained 24 chickens and each chicken weighed 4 lb. Each empty crate weighed 4 lb. How much did each full crate weigh?

14. His truck could carry $1\frac{1}{2}$ tons of weight. How many crates of chickens could he put on his truck?



Money

The coins we use are pennies, nickels, dimes, quarters, and half dollars.

1 penny = 1 cent (1¢ or \$.01)
1 nickel = 5 cents (5¢ or \$.05)
1 dime = 10 cents (10¢ or \$.10)
1 quarter = 25 cents (25¢ or \$.25)
1 half dollar = 50 cents (50¢ or \$.50)
1 dollar = 100 cents (\$1.00)

1. David buys a shirt for \$1.98 and a belt for \$1.50. How much change does he receive from \$5.00?

2. Elaine spends \$.35 for ribbons, \$.25 for thread, \$.20 for pins and \$.49 for needles. She gives the clerk \$2.00. How much change does she receive?

3. Scott buys a model car for \$.50, 4 books at \$.25 each, and a game for \$.29. How much change does he receive from \$5.00?

4. Lisa buys for her mother a plant that costs \$7.98. She gives the clerk \$10.00. How much change does she receive?

5. Which has the greater value: 4 dimes, 3 nickels, and 9 pennies or 5 dimes, 2 nickels, and 3 pennies?

6. Which has the greater value: 2 quarters, 4 dimes, 5 nickels, and 7 pennies or 1 quarter, 7 dimes, and 8 nickels?

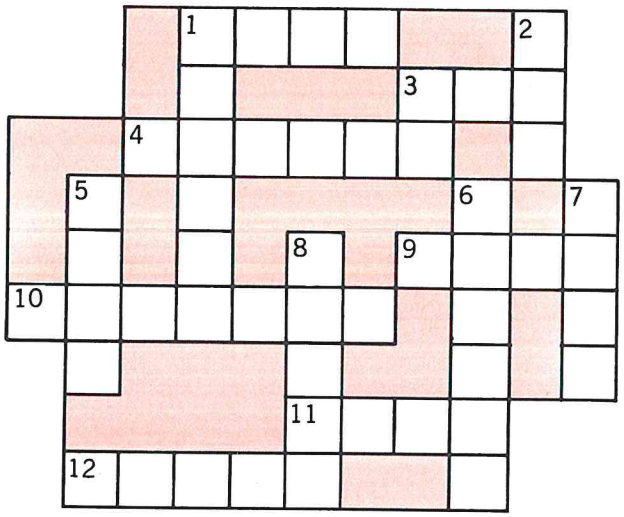
7. John has 5 quarters. How many dimes must he have to make a total of \$1.95?

8. Dave has 5 dimes. Jan has twice as many quarters as Dave has dimes. How much money does she have? How much money does Dave have? How much more money has Jan than Dave?

1. Greg has 6 quarters. How many dimes must he have to make a total of $2.50?

2. Bob has 7 dimes. Ken has twice as many quarters as Bob has dimes.
   a. How much money does Ken have?
   b. How much more money does Ken have than Bob?

3. Copy the crossword puzzle and solve it.



Across

1. Coin worth $\frac{1}{10}$ of a dollar.
3. 20 nickels = ___ dollar.
4. $1.10 equals ___ dimes.
9. $2.50 is worth the same as ___ half dollars.
10. Coin worth $\frac{1}{4}$ of a dollar.
11. $.45 is worth the same as ___ nickels.
12. A half dollar is worth ___ cents.

Down

1. 4 quarters = 1 ___.
2. One dollar = ___ dimes.
3. Word meaning "upon."
5. 1 dollar = ___ quarters.
6. Coin worth $\frac{1}{20}$ of a dollar.
7. Value of coin worth $\frac{1}{100}$ of a dollar.
8. Coin worth $\frac{1}{10}$ of a dime.

Time Zones

It takes 24 hours for the Earth to rotate about its axis. Because the Earth rotates, we see the sun rise in the East and set in the West. Places east of us have sunrise earlier than we do. Places west of us have sunrise later than we do.

When the sun is approximately overhead, we say that it is noon. The Earth is divided into 24 time zones so that clocks everywhere will say noon when the sun is approximately overhead.

When it is noon (12 M.) in one time zone, it is one hour earlier (11 A.M.) in the first zone west of it, and one hour later (1 P.M.) in the first zone east of it.

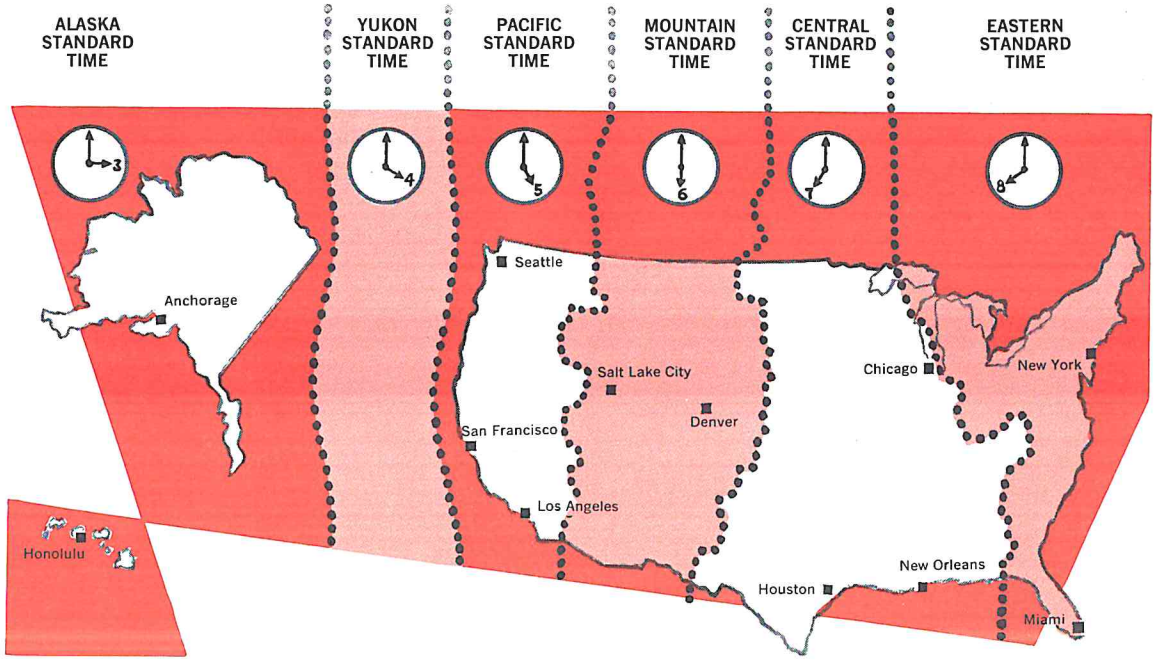
Look at the globe to answer these questions.

1. Do I set my watch forward or back if I go
   - **a.** from Hawaii to Texas?
   - **b.** from Maine to Arizona?
   - **c.** from Alabama to Montana?
   - **d.** from Utah to Alaska?
   - **e.** from Europe to America?
   - **f.** from Asia to Europe?
   - **g.** from Africa to Asia?
   - **h.** from Australia to Europe?

2. Use an encyclopedia to find the answers.
   - **a.** What is Greenwich Mean Time?
   - **b.** Do the time zones run from the North Pole to the South Pole?
   - **c.** Find at least one important city in each of the U.S. time zones.

3. If the sun is just rising in New York, will it be light or dark in Chicago?

4. If the sun is just setting in Manila, will it be light or dark in Berlin?



When it is 3 o'clock in Alaska and Hawaii, it is 5 o'clock on the West Coast, 6 o'clock in the Rockies, 7 o'clock in the Mid-West, and 8 o'clock on the East Coast.

Alaska and the East Coast have a time difference of 5 hours. Add 5 hours to the Alaskan time to find the East Coast time. Subtract 5 hours from the East Coast time to find the Alaskan time.

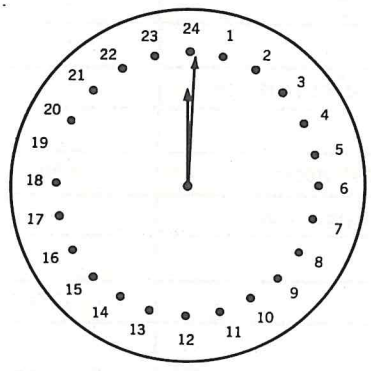
1. If it is 1 A.M. in Honolulu, what time is it in Los Angeles? in Denver? in New Orleans? in Miami?
2. If it is 8 P.M. in New York, what time is it in Miami? in San Francisco? in Chicago? in Salt Lake City?
3. Jean phones from Los Angeles to her friend in New York. Jean's watch says 11 A.M. What time is it in New York?
4. If you fly from Chicago to New Orleans, do you need to reset your watch?
5. If you fly from San Francisco to Miami do you need to set your watch back or forward? How many hours do you change it?

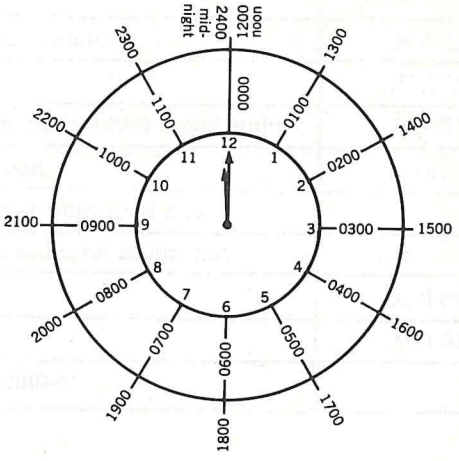
The 24-hour Clock

We refer to the hours from midnight to noon as the A.M. hours and the hours from noon to midnight as the P.M. hours. They are written as:

7 A.M. 12 M. 7 P.M. 1 A.M. 12 P.M.

The military services and the airlines work 24 hours a day, and find these A.M. and P.M. hours confusing. They use a 24-hour clock, measuring the time from midnight to midnight.





On the left is a 24-hour clock. The diagram on the right shows how the 24-hour clock corresponds to the 12-hour clock.

Times referring to a 24-hour clock are given by a 4-digit numeral. These numerals are not read in the ordinary way.

"0005" is read "zero zero zero five"; "0300" is read "zero three hundred"; "0817" is read "zero eight seventeen"; "1100" is read "eleven hundred"; "1241" is read "twelve forty-one"; "2207" is read "twenty-two zero seven".

Read these times aloud.

1. 0007
2. 0800
3. 0924
4. 0613
5. 1300
6. 1408
7. 1521
8. 1933
9. 2145
10. 2359
11. 0042
12. 1340

On the 24-hour clock the new day begins at midnight; 0001 is 1 minute past midnight.
The A.M. hours are 0001 through 1200; 1200 is noon.
The P.M. hours are 1201 through 2400; 2400 is midnight.

1. Copy and complete the table.

Time on a 12-hour clock	In words	Time on a 24-hour clock
12:02 A.M.	two minutes past midnight	0002
3:20 A.M.		0320
9:37 A.M.	nine hours thirty-seven minutes past midnight	
12 noon	noon	
	one hour nine minutes past noon	1309
	two hours forty-five minutes past noon	
8:20 P.M.		
11:46 P.M.		2346
	midnight	

2. Jay's plane leaves at 0940. What time is this on a 12-hour clock?
3. Nancy's plane arrives in New York at 1738. What time is this on a 12-hour clock?
4. Soldiers must get up at 0530. What time is this on a 12-hour clock?
5. They have marching drill at 1300. What time is this on a 12-hour clock?
6. They must be in bed by 2200. What time is this on a 12-hour clock?

Change from 24-hour time to 12-hour time.

7. 0735
8. 1430
9. 1950
10. 1615
11. 1215
12. 2207
13. 1532
14. 0220

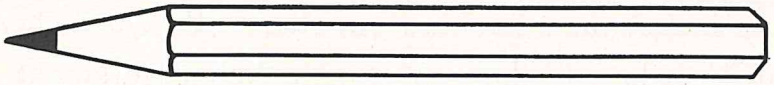
Change from 12-hour time to 24-hour time.

15. 12:15 A.M.
16. 5:30 A.M.
17. 4:15 A.M.
18. 10:30 A.M.
19. 1:02 P.M.
20. 4:30 P.M.
21. 3:10 P.M.
22. 11:20 P.M.

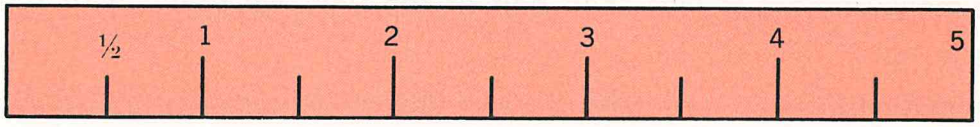
Measurement is Approximate

We measure many things. We measure length, capacity, time, and weight. We use many different instruments to measure each of these properties.

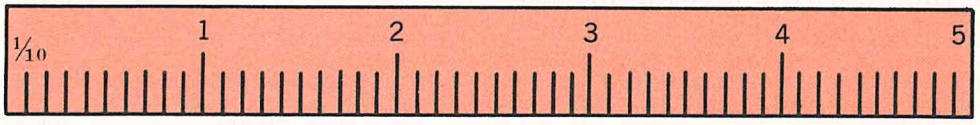
The measurement of a quantity is only an approximate measurement. The smaller the unit used to measure, the greater is the precision of the measurement.



If we use a ruler marked into units of $\frac{1}{2}$ inch, we find that the picture of the pencil is almost 4 inches long.



If we use a ruler marked into units of $\frac{1}{10}$ inch, we find that the picture of the pencil is slightly more than $3\frac{9}{10}$ inches long.



If we had a ruler marked into even smaller units, we might be able to make an even closer measurement of the picture of the pencil.

It often is not necessary for us to know measurement to a great degree of precision. However, when we give measurements we may need to indicate how precise they are by telling the smallest unit we used to measure.

Give the length of these line segments to the nearest $\frac{1}{2}$ inch; to the nearest $\frac{1}{4}$ inch; to the nearest $\frac{1}{8}$ inch.

1. A ____ B
2. C_____D
3. E_____F
4. G_____H
5. I___ J
6. K _____ L
7. M____N
8. P_____Q
9. R_____S

Rounding Measurements

To give a measurement to the nearest stated unit, we often have to round the measurement.

Round the measurement of the black strip to the nearest inch.

The strip is longer than 4 inches but shorter than 5 inches. It is also longer than $4\frac{1}{2}$ inches. To the nearest inch, the measurement is 5 inches.

Rules for rounding measurements:

If a measurement is less than half of the unit, we round it to the next smaller unit.

If a measurement is one half or more of the unit, we round it to the next larger unit.

Round the following measurements.

1. To the nearest pound:
   - a. $81\frac{3}{4}$ lb.
   - b. $14\frac{1}{4}$ lb.
   - c. $20\frac{2}{3}$ lb.
   - d. $44\frac{1}{3}$ lb.
   - e. $39\frac{5}{8}$ lb.
   - f. $11\frac{3}{4}$ lb.

2. To the nearest quart:
   - a. 4 qt. 3 pt.
   - b. 7 qt. 1 cup
   - c. 6 qt. 2 pt.
   - d. 1 qt. 2 cups
   - e. 3 qt. 5 cups
   - f. 2 qt. 1 pt. 1 cup

3. To the nearest hour:
   - a. 3:07
   - b. 12:50
   - c. 4:35
   - d. 2:15
   - e. 5:45
   - f. 8:20

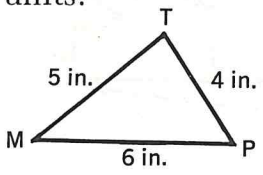
4. To the nearest half-hour:
   - a. 5:22
   - b. 7:40
   - c. 10:37
   - d. 11:59
   - e. 9:42
   - f. 1:31

5. To the nearest dollar:
   - a. \$1.14
   - b. \$3.05
   - c. \$5.94
   - d. \$10.59
   - e. \$51.33
   - f. \$101.78

6. To the nearest half dollar:
   - a. \$4.98
   - b. \$3.37
   - c. \$8.75
   - d. \$6.29
   - e. \$1.18
   - f. \$5.85

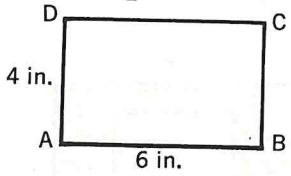
Perimeters and Areas

The distance around a figure is its perimeter. Perimeter is measured in linear units.



Perimeter triangle MPT = (6 + 4 + 5) in.
= 15 in.

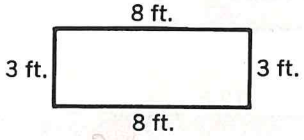
The area of a figure is the measure of the surface it encloses. Area is measured in square units.



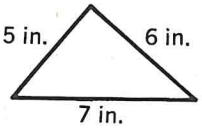
Area rectangle ABCD = (4 × 6) sq. in.
= 24 sq. in.

Find the perimeters of these figures.

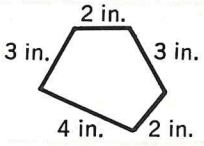
1.



2.

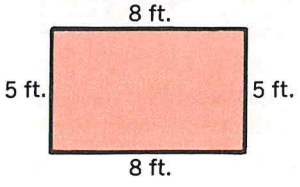


3.

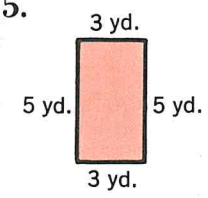


Find the areas of these figures.

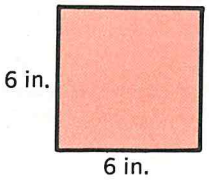
4.



5.



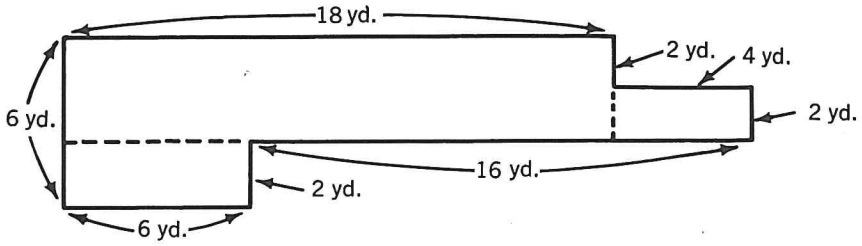
6.



Complete each sentence.

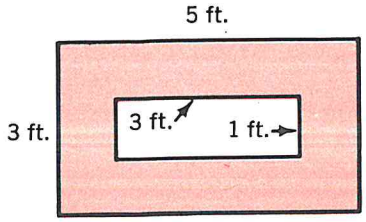
7. The lengths of the sides of a triangle are a, b, and c.
   The perimeter P = ___ + ___ + ___.
8. The length of a rectangle is l and the width is w. The area A = ___ × ___.
9. The length of a rectangle is l and the width is w.
   The perimeter P = ___ + ___ + ___ + ___ = 2___ + 2___.

1. What is the perimeter of the field shown below? What is its area? (Hint: First find the area of the 3 rectangles.)

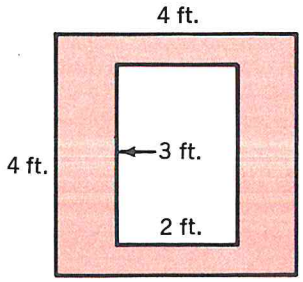


Find the area of the shaded part of each figure. (Hint: Subtract the area of the small rectangle from the area of the large rectangle.)

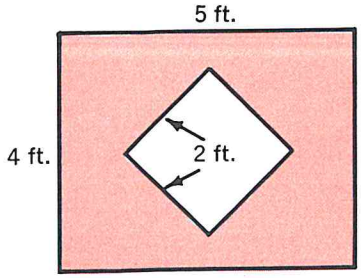
2.



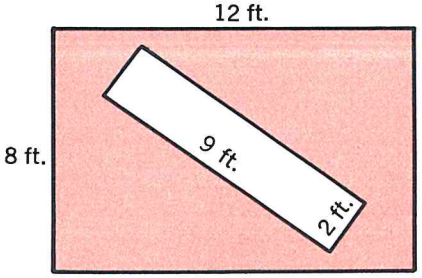
3.



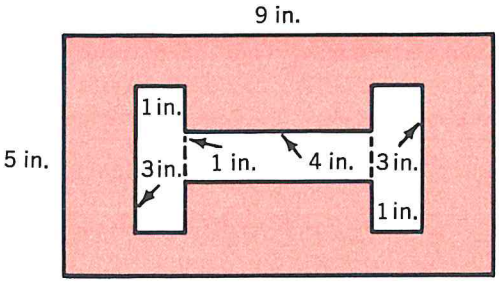
4.



5.



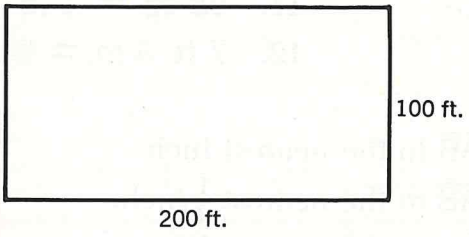
6. Find the area of the shaded part of the figure. The H-shaped figure is made up of 3 rectangles of the dimensions given.



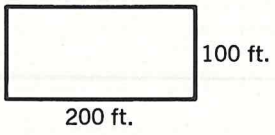
Drawing to a Scale

Often we need to picture something smaller than its actual size. We use a scale drawing so that the picture is the same shape as the thing being pictured.

A lot is 100 ft. by 200 ft. We cannot draw a picture that is 100 ft. by 200 ft. In this case we can let 1 in. represent 100 ft. Our drawing then looks like this:



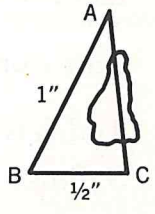
We could have let $\frac{1}{2}$ in. represent 100 ft. Then our drawing would look like this:



1. Mr. Jacobs bought a lot that was 150 ft. by 200 ft. Let 1 in. represent 200 ft. Make a drawing of the lot.

2. Bert measured the gym floor. It was 60 ft. by 90 ft. Let 1 in. represent 30 ft. Make a drawing of the gym floor.

3. Jim drew the drawing at the right for a treasure hunt. He let 1 in. represent 50 ft. How many feet is it from A to B to C?



4. Carol drew a map. She let 1 in. represent 50 mi. If one state was drawn 7 in. in length, how many miles long was this state?

Feedback

Complete each problem.

1. 2 yd. 3 in. = ■ in.
2. 24 ft. = ■ yd.
3. 6 yr. = ■ mo.
4. 22 pt. = ■ qt.
5. 3 gal. = ■ qt.
6. 6 c. = ■ pt.
7. 6 lb. = ■ oz.
8. 2 T. = ■ lb.
9. 3 days = ■ hr.
10. 16 pt. = ■ gal.
11. 98 oz. = ■ lb. 2 oz.
12. 7 ft. 4 in. = ■ in.

13. a. Measure $\overline{AB}$ to the nearest inch.
    b. Measure $\overline{AB}$ to the nearest $\frac{1}{2}$ inch.
    c. Measure $\overline{AB}$ to the nearest $\frac{1}{4}$ inch.
    d. Measure $\overline{AB}$ to the nearest $\frac{1}{8}$ inch.

A B

14. Is it warmer when the temperature is 40°C or 40°F?

15. Is it colder when the temperature is 18°C or −18°C?

16. A jet flies 520 miles an hour. How many miles does it fly in a minute?

17. Bill is 4 ft. 10 in. tall and Kathy is 3 ft. 11 in. tall. Bill is how much taller than Kathy?

18. Ann works from 3:45 P.M. to 4:30 P.M. How long does Ann work?

19. Ted has 2 quarters, 3 dimes, 4 nickels, 6 pennies. Tim has 1 quarter, 5 dimes, 6 nickels, 2 pennies. Who has more money?

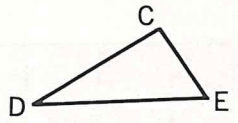
20. Mr. Thomas cut a 6-foot board into 4 pieces that were the same length. How long was each piece?

Feedback

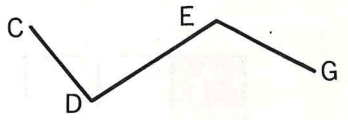
Pair each set description with a figure.

1. $\overline{CD} \cup \overline{DE} \cup \overline{EG}$
2. $\overline{CD} \cap \overline{EG} = \{P\}$
3. $\overline{CD} \cup \overline{DE} \cup \overline{EC}$

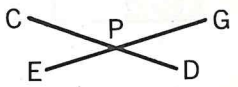
a.



b.



c.



Complete each problem.

4. $\{n: n < 3 \text{ and } n \text{ is a whole number}\} =$ ■
5. $\{n: n < 9 \text{ and } n \text{ is an odd positive number}\} =$ ■
6. $\{n: 4 < n < 6 \text{ and } n \text{ is a whole number}\} =$ ■
7. $\{n: n < 25 \text{ and } n \text{ is a prime number}\} =$ ■
8. $\{n: 3 < n < 13 \text{ and } n \text{ is an even number}\} =$ ■

Find each product.

9. $\begin{array}{r} 846 \\ \times\ 7 \\ \hline \end{array}$
10. $\begin{array}{r} 9084 \\ \times\ 8 \\ \hline \end{array}$
11. $\begin{array}{r} 36 \\ \times 85 \\ \hline \end{array}$
12. $\begin{array}{r} 946 \\ \times\ 67 \\ \hline \end{array}$

Find each difference.

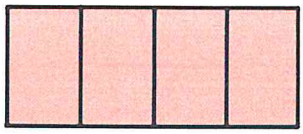
13. $\begin{array}{r} 946 \\ -197 \\ \hline \end{array}$
14. $\begin{array}{r} 900 \\ -258 \\ \hline \end{array}$
15. $\begin{array}{r} 6432 \\ -1947 \\ \hline \end{array}$
16. $\begin{array}{r} 8010 \\ -2874 \\ \hline \end{array}$

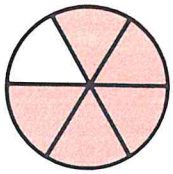
Find each quotient.

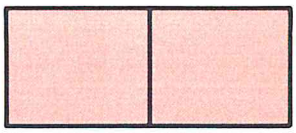
17. $8\overline{)1468}$
18. $21\overline{)8514}$
19. $19\overline{)5301}$
20. $36\overline{)7490}$

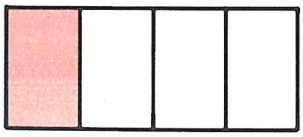
21. Mr. Blanchard's orchard contains 704 trees. There is an equal number of apricot trees, peach trees, pear trees, and fig trees. How many of each kind of tree does he have?

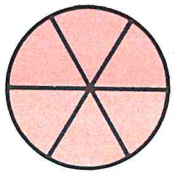
Part 8 More Fractions

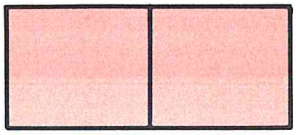
a. 

b. 

c. 







Find the diagram that shows:

1. $\frac{11}{6} = 1\frac{5}{6}$ b.
2. $\frac{4}{2} = 2$
3. $\frac{5}{4} = 1\frac{1}{4}$
4. $2 = \frac{4}{2}$
5. $1\frac{5}{6} = \frac{11}{6}$
6. $1\frac{1}{4} = \frac{5}{4}$

Find a whole number that is equal to each fraction.

7. $\frac{18}{6}$
8. $\frac{14}{7}$
9. $\frac{25}{5}$
10. $\frac{21}{3}$
11. $\frac{36}{4}$
12. $\frac{20}{5}$
13. $\frac{12}{6}$
14. $\frac{18}{2}$

Find a mixed number that is equal to each fraction.

15. $\frac{9}{4}$
16. $\frac{11}{3}$
17. $\frac{8}{5}$
18. $\frac{7}{4}$
19. $\frac{13}{2}$
20. $\frac{25}{6}$
21. $\frac{15}{4}$
22. $\frac{19}{6}$
23. $\frac{11}{6}$
24. $\frac{27}{4}$
25. $\frac{31}{5}$
26. $\frac{45}{7}$

Find a whole number or a mixed number that is equal to each fraction.

27. $\frac{3}{3}$
28. $\frac{4}{4}$
29. $\frac{23}{6}$
30. $\frac{15}{5}$
31. $\frac{13}{4}$
32. $\frac{8}{8}$
33. $\frac{27}{5}$
34. $\frac{9}{9}$
35. $\frac{24}{12}$
36. $\frac{10}{10}$
37. $\frac{18}{18}$
38. $\frac{11}{4}$

We sometimes need to express a whole number or a mixed number as a fraction.

$$1 = \frac{4}{4}$$

$$2 = 1 + 1 \qquad 2\frac{1}{4} = 1 + 1 + \frac{1}{4}$$
$$= \frac{4}{4} + \frac{4}{4} \qquad = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= \frac{8}{4} \qquad = \frac{9}{4}$$

Copy and complete each problem.

1. $2 = 1 + 1$
$= \frac{\blacksquare}{3} + \frac{\blacksquare}{3}$
$= \blacksquare$

2. $1\frac{1}{5} = 1 + \frac{1}{5}$
$= \frac{\blacksquare}{5} + \frac{1}{5}$
$= \blacksquare$

3. $2\frac{1}{8} = 1 + 1 + \frac{1}{8}$
$= \frac{\blacksquare}{8} + \frac{\blacksquare}{8} + \frac{1}{8}$
$= \blacksquare$

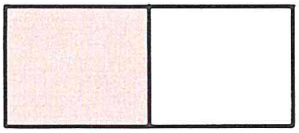
4. $1\frac{2}{3} = 1 + \frac{2}{3}$
$= \frac{\blacksquare}{3} + \frac{2}{3}$
$= \blacksquare$

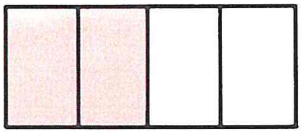
5. $2 = 1 + 1$
$= \frac{\blacksquare}{6} + \frac{\blacksquare}{6}$
$= \blacksquare$

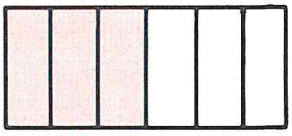
6. $3\frac{1}{4} = 1 + 1 + 1 + \frac{1}{4}$
$= \frac{\blacksquare}{4} + \frac{\blacksquare}{4} + \frac{\blacksquare}{4} + \frac{1}{4}$
$= \blacksquare$

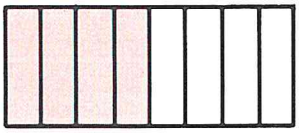
Express each whole or mixed number as a fraction.

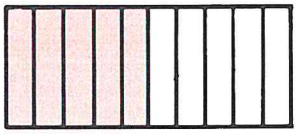
7. $3\frac{1}{5} = \frac{\blacksquare}{5}$
8. $4\frac{1}{3} = \frac{\blacksquare}{3}$
9. $2 = \frac{\blacksquare}{8}$
10. $2\frac{1}{3} = \frac{\blacksquare}{3}$
11. $4\frac{1}{2} = \frac{\blacksquare}{2}$
12. $3\frac{1}{3} = \frac{\blacksquare}{3}$
13. $2\frac{1}{2} = \frac{\blacksquare}{2}$
14. $2 = \frac{\blacksquare}{6}$
15. $3\frac{3}{4} = \frac{\blacksquare}{4}$
16. $2\frac{2}{5} = \frac{\blacksquare}{5}$
17. $4\frac{1}{4} = \frac{\blacksquare}{4}$
18. $2\frac{2}{9} = \frac{\blacksquare}{9}$
19. $3\frac{1}{2} = \frac{\blacksquare}{2}$
20. $4\frac{3}{4} = \frac{\blacksquare}{4}$
21. $2\frac{3}{7} = \frac{\blacksquare}{7}$

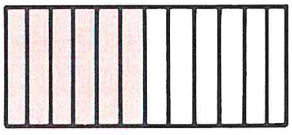
a. 

b. 

c. 

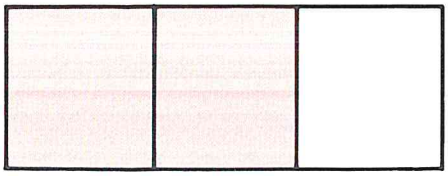
d. 

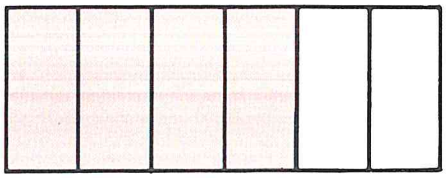
e. 

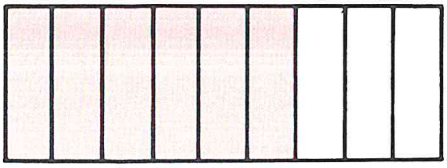
f. 

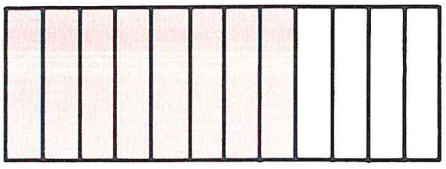
Find the diagrams above that show:

1. $\frac{1}{2} = \frac{2}{4}$ a. b.
2. $\frac{3}{6} = \frac{1}{2}$
3. $\frac{2}{4} = \frac{6}{12}$
4. $\frac{4}{8} = \frac{1}{2}$
5. $\frac{3}{6} = \frac{6}{12}$
6. $\frac{5}{10} = \frac{1}{2}$
7. $\frac{6}{12} = \frac{1}{2}$
8. $\frac{2}{4} = \frac{3}{6}$
9. $\frac{5}{10} = \frac{2}{4}$

g. 

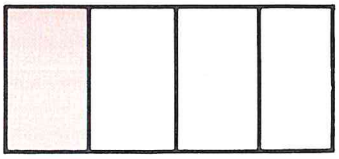
h. 

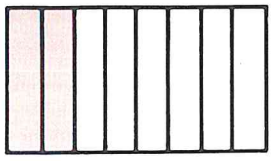
i. 

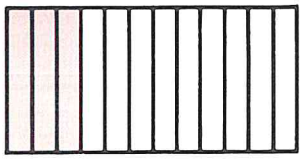
j. 

Choose = or ≠. Look at the diagrams above.

10. $\frac{2}{3}$ ● $\frac{4}{6}$
11. $\frac{4}{6}$ ● $\frac{8}{12}$
12. $\frac{6}{9}$ ● $\frac{2}{3}$
13. $\frac{4}{6}$ ● $\frac{6}{9}$
14. $\frac{8}{12}$ ● $\frac{2}{3}$
15. $\frac{6}{9}$ ● $\frac{8}{12}$

k. 

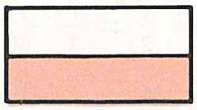
l. 

m. 

16. $\frac{2}{8}$ ● $\frac{1}{4}$
17. $\frac{1}{4}$ ● $\frac{3}{12}$
18. $\frac{3}{12}$ ● $\frac{2}{8}$

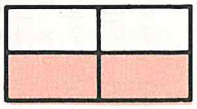
Renaming Fractions

Complete each problem.

1. a.  $\frac{1 \times 1}{2 \times 1} = \square$

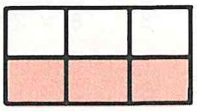
   b. 1 of ■ parts is colored.

   c. Is $\frac{1}{2}$ of the interior colored?

2. a.  $\frac{1 \times 2}{2 \times 2} = \square$

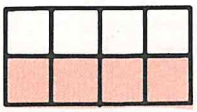
   b. 2 of ■ parts are colored.

   c. Is $\frac{1}{2}$ of the interior colored?

3. a.  $\frac{1 \times 3}{2 \times 3} = \square$

   b. 3 of ■ parts are colored.

   c. Is $\frac{1}{2}$ of the interior colored?

4. a.  $\frac{1 \times 4}{2 \times 4} = \square$

   b. 4 of ■ parts are colored.

   c. Is $\frac{1}{2}$ of the interior colored?

We know that $n \times 1 = n$.

$3 \times 1 = 3$ $\qquad$ $\frac{1}{2} \times 1 = \frac{1}{2}$

We know that 1 has many names.

$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}$

We can use these facts to rename any fraction.

To find other names for $\frac{1}{2}$ we may write:

$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$ $\qquad$ $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ $\qquad$ $\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

Choose = or ≠.

5. a. $\frac{1}{2} \bigcirc \frac{2}{4}$

   b. $\frac{1}{2} \bigcirc \frac{4}{8}$

   c. $\frac{1}{2} \bigcirc \frac{3}{6}$

6. a. $\frac{3}{3} \bigcirc 1$

   b. $\frac{2}{2} \bigcirc 1$

   c. $\frac{4}{4} \bigcirc 1$

7. a. $\frac{1 \times 4}{2 \times 4} \bigcirc \frac{1 \times 2}{2 \times 2}$

   b. $\frac{1 \times 3}{2 \times 3} \bigcirc \frac{1 \times 2}{2 \times 4}$

   c. $\frac{1}{2} \bigcirc \frac{1 \times 3}{2 \times 3}$

Find the new name for each fraction.

$$\frac{4}{7} = \frac{4 \times 3}{7 \times 3} = \frac{12}{21}$$

1. $\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \square$
2. $\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \square$
3. $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \square$
4. $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \square$
5. $\frac{1}{2} = \frac{1 \times 50}{2 \times 50} = \square$
6. $\frac{2}{7} = \frac{2 \times 3}{7 \times 3} = \square$
7. $\frac{4}{5} = \frac{4 \times 5}{5 \times 5} = \square$
8. $\frac{1}{2} = \frac{1 \times 8}{2 \times 8} = \square$
9. $\frac{3}{7} = \frac{3 \times 4}{7 \times 4} = \square$
10. $\frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \square$
11. $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \square$
12. $\frac{1}{6} = \frac{1 \times 7}{6 \times 7} = \square$
13. $\frac{1}{3} = \frac{1 \times 6}{3 \times 6} = \square$
14. $\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \square$
15. $\frac{5}{7} = \frac{5 \times 10}{7 \times 10} = \square$
16. $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \square$
17. $\frac{5}{8} = \frac{5 \times 20}{8 \times 20} = \square$
18. $\frac{7}{9} = \frac{7 \times 3}{9 \times 3} = \square$

Give another name for each fraction.

19. $\frac{2}{3}$
20. $\frac{3}{4}$
21. $\frac{2}{5}$
22. $\frac{4}{7}$
23. $\frac{7}{9}$
24. $\frac{1}{10}$
25. $\frac{7}{12}$
26. $\frac{9}{11}$

Give five other names for each fraction.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$$

27. $\frac{1}{3}$
28. $\frac{1}{4}$
29. $\frac{1}{5}$
30. $\frac{1}{9}$
31. $\frac{2}{9}$
32. $\frac{3}{8}$
33. $\frac{4}{9}$
34. $\frac{5}{9}$
35. $\frac{2}{3}$
36. $\frac{2}{7}$
37. $\frac{7}{8}$
38. $\frac{3}{5}$

39. Chuck, Sam, and Ralph each had 24 marbles. Chuck divided his marbles into two bags and put one bag into his pocket. Sam divided his marbles into 6 bags and put three of the bags into his pocket. Ralph divided his marbles into 8 bags and put 4 of the bags into his pocket. Did each boy have half of his marbles in his pocket?

We sometimes need to rename a fraction using a given denominator.

$\frac{3}{7} = \frac{n}{28}$

$\frac{3 \times 4}{7 \times 4} = \frac{n}{28}$ because $28 \div 7 = 4$ and $\frac{4}{4} = 1$

$\frac{12}{28} = \frac{n}{28}$ because $3 \times 4 = 12$ and $7 \times 4 = 28$

$n = 12$ because if the denominators of 2 equal fractions are equal, the numerators are equal.

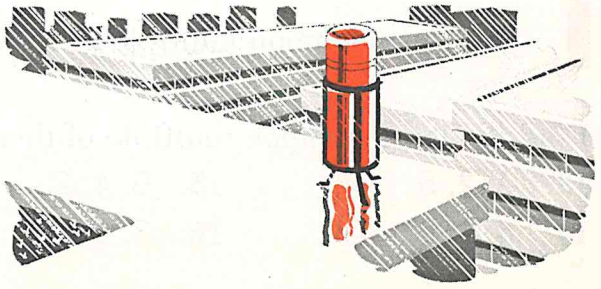
Find each missing numerator.

1. $\frac{2}{3} = \frac{4}{6}$
2. $\frac{4}{7} = \frac{\blacksquare}{14}$
3. $\frac{1}{3} = \frac{\blacksquare}{12}$
4. $\frac{1}{2} = \frac{\blacksquare}{36}$
5. $\frac{5}{8} = \frac{\blacksquare}{24}$
6. $\frac{2}{5} = \frac{\blacksquare}{30}$
7. $\frac{7}{11} = \frac{\blacksquare}{33}$
8. $\frac{1}{4} = \frac{\blacksquare}{16}$
9. $\frac{3}{4} = \frac{\blacksquare}{40}$
10. $\frac{1}{2} = \frac{\blacksquare}{28}$
11. $\frac{2}{3} = \frac{\blacksquare}{21}$
12. $\frac{3}{8} = \frac{\blacksquare}{32}$
13. $\frac{5}{6} = \frac{\blacksquare}{18}$
14. $\frac{0}{3} = \frac{\blacksquare}{15}$
15. $\frac{7}{9} = \frac{\blacksquare}{54}$
16. $\frac{1}{8} = \frac{\blacksquare}{72}$
17. $\frac{4}{11} = \frac{\blacksquare}{77}$
18. $\frac{2}{3} = \frac{\blacksquare}{39}$

Choose $<$ or $=$ or $>$. First rename one fraction.

19. $\frac{2}{3} \bullet \frac{5}{12}$
20. $\frac{1}{2} \bullet \frac{5}{14}$
21. $\frac{3}{4} \bullet \frac{23}{36}$
22. $\frac{4}{7} \bullet \frac{9}{14}$
23. $\frac{1}{4} \bullet \frac{5}{12}$
24. $\frac{3}{8} \bullet \frac{6}{16}$
25. $\frac{1}{5} \bullet \frac{5}{25}$
26. $\frac{3}{4} \bullet \frac{17}{28}$
27. $\frac{5}{5} \bullet \frac{20}{20}$

28. Wednesday it rained $\frac{7}{8}$ of an inch. The next day it rained $\frac{17}{24}$ of an inch. Which day did it rain more?



Finding the Least Common Multiple

The least positive number which is a multiple of two or more numbers is their **least common multiple.**

To find the least common multiple of 3 and 2, we first find the set A of positive multiples of 3. $A = \{3, 6, 9, 12, 15, 18, 21, \ldots\}$

Then we find the set B of positive multiples of 2.

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, \ldots\}$

Finally, we find the intersection of sets A and B. This is the set of positive multiples of both 3 and 2. $A \cap B = \{6, 12, 18, \ldots\}$

The least common multiple of 3 and 2 is 6.

Find the least common multiple of each pair of numbers.

1. 6, 3	**4.** 3, 6	**7.** 4, 6	**10.** 7, 6
2. 3, 4	**5.** 4, 5	**8.** 8, 6	**11.** 5, 6
3. 5, 2	**6.** 3, 7	**9.** 8, 4	**12.** 4, 12

To find the least common multiple of 6, 12, and 4, we first find the set A of positive multiples of 6. $A = \{6, 12, 18, 24, 30, \ldots\}$

We find the set B of positive multiples of 12.

$B = \{12, 24, \ldots\}$

We find the set C of positive multiples of 4.

$C = \{4, 8, 12, 16, 20, 24, 28, \ldots\}$

Finally, we find the intersection of sets A, B, and C.

$A \cap B \cap C = \{12, 24, \ldots\}$

The least common multiple of 6, 12, and 4 is 12.

Find the least common multiple of these numbers.

13. 3, 2, 5	**15.** 5, 4, 2	**17.** 3, 4, 5, 6
14. 4, 3, 2	**16.** 3, 12, 6	**18.** 2, 3, 8, 6

Finding the Least Common Denominator

Sometimes in adding or subtracting two fractions we must rename one or both fractions so that they have a common denominator.

The least common multiple of two or more denominators is the **least common denominator.**

We can find the least common denominator of $\frac{3}{4}$ and $\frac{2}{3}$.
We need to find the least common multiple of 4 and 3.

$$A = \{4, 8, 12, 16, 20, 24, 28, \ldots\}$$
$$B = \{3, 6, 9, 12, 15, 18, 21, 24, 27, \ldots\}$$
$$A \cap B = \{12, 24, \ldots\}$$

The least common denominator of $\frac{3}{4}$ and $\frac{2}{3}$ is 12.
We need to rename both $\frac{3}{4}$ and $\frac{2}{3}$ as fractions with the same denominator.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Find the least common denominator and rename the fractions.

1. $\frac{1}{2}$ and $\frac{1}{12}$
2. $\frac{5}{6}$ and $\frac{7}{12}$
3. $\frac{2}{5}$ and $\frac{1}{3}$
4. $\frac{1}{4}$ and $\frac{1}{5}$
5. $\frac{1}{3}$ and $\frac{3}{7}$
6. $\frac{3}{5}$ and $\frac{1}{2}$
7. $\frac{1}{6}$ and $\frac{2}{9}$
8. $\frac{2}{7}$ and $\frac{3}{4}$
9. $\frac{3}{8}$ and $\frac{5}{6}$
10. $\frac{1}{8}$ and $\frac{11}{12}$
11. $\frac{5}{9}$ and $\frac{1}{2}$
12. $\frac{3}{5}$ and $\frac{5}{6}$

Choose $<$ or $>$. First find like fractions.

13. $\frac{4}{5} \bullet \frac{5}{6}$
14. $\frac{2}{9} \bullet \frac{1}{6}$
15. $\frac{5}{7} \bullet \frac{2}{3}$
16. $\frac{3}{8} \bullet \frac{1}{5}$
17. $\frac{3}{4} \bullet \frac{5}{6}$
18. $\frac{7}{10} \bullet \frac{3}{4}$
19. $\frac{1}{2} \bullet \frac{4}{7}$
20. $\frac{4}{5} \bullet \frac{3}{4}$
21. $\frac{2}{5} \bullet \frac{3}{8}$
22. $\frac{2}{3} \bullet \frac{4}{5}$
23. $\frac{5}{11} \bullet \frac{1}{2}$
24. $\frac{3}{7} \bullet \frac{1}{4}$

To add or subtract fractions with common denominators, we add or subtract their numerators.

$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$ $\qquad$ $\frac{4}{8} - \frac{1}{8} = \frac{3}{8}$

To add or subtract fractions with denominators that are not the same, we first rename one or both fractions so that they have common denominators.

$\frac{1}{3} + \frac{2}{15} = \frac{5}{15} + \frac{2}{15}$ $\qquad$ because $\frac{1}{3} = \frac{5}{15}$

$= \frac{7}{15}$ $\qquad$ because $5 + 2 = 7$

Find each sum or difference.

1. $\frac{3}{8} + \frac{2}{8} =$ ▢
2. $\frac{2}{5} + \frac{2}{5} =$ ▢
3. $\frac{4}{7} + \frac{1}{7} =$ ▢
4. $\frac{3}{6} + \frac{2}{6} =$ ▢
5. $\frac{3}{4} - \frac{2}{4} =$ ▢
6. $\frac{7}{8} - \frac{4}{8} =$ ▢
7. $\frac{4}{5} - \frac{1}{5} =$ ▢
8. $\frac{5}{6} - \frac{4}{6} =$ ▢

Copy and complete each problem.

9. $\frac{3}{4} + \frac{1}{12} = \frac{\square}{12} + \frac{1}{12} =$ ▢
10. $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{\square}{8} =$ ▢
11. $\frac{3}{10} + \frac{1}{5} = \frac{3}{10} + \frac{\square}{10} =$ ▢
12. $\frac{2}{3} + \frac{1}{6} = \frac{\square}{6} + \frac{1}{6} =$ ▢
13. $\frac{3}{7} + \frac{3}{14} = \frac{\square}{14} + \frac{3}{14} =$ ▢
14. $\frac{3}{4} + \frac{1}{8} = \frac{\square}{8} + \frac{1}{8} =$ ▢
15. $\frac{3}{5} + \frac{2}{15} = \frac{\square}{15} + \frac{2}{15} =$ ▢
16. $\frac{1}{3} + \frac{5}{9} = \frac{\square}{9} + \frac{5}{9} =$ ▢
17. $\frac{2}{3} - \frac{1}{12} = \frac{\square}{12} - \frac{1}{12} =$ ▢
18. $\frac{3}{7} - \frac{1}{14} = \frac{\square}{14} - \frac{1}{14} =$ ▢
19. $\frac{5}{12} - \frac{1}{6} = \frac{5}{12} - \frac{\square}{12} =$ ▢
20. $\frac{2}{5} - \frac{1}{10} = \frac{\square}{10} - \frac{1}{10} =$ ▢
21. $\frac{3}{4} - \frac{5}{12} = \frac{\square}{12} - \frac{5}{12} =$ ▢
22. $\frac{2}{3} - \frac{1}{6} = \frac{\square}{6} - \frac{1}{6} =$ ▢
23. $\frac{1}{2} - \frac{4}{10} = \frac{\square}{10} - \frac{4}{10} =$ ▢
24. $\frac{11}{12} - \frac{1}{6} = \frac{11}{12} - \frac{\square}{12} =$ ▢

Find each sum or difference.

1. $\frac{2}{3} + \frac{1}{5} = \frac{\square}{15} + \frac{3}{15} = \square$
2. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{\square}{6} = \square$
3. $\frac{3}{8} - \frac{1}{6} = \frac{9}{24} - \frac{\square}{24} = \square$
4. $\frac{2}{5} + \frac{1}{2} = \frac{\square}{10} + \frac{5}{10} = \square$
5. $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{\square}{12} = \square$
6. $\frac{4}{5} - \frac{3}{4} = \frac{16}{20} - \frac{\square}{20} = \square$
7. $\frac{1}{6} + \frac{2}{9} = \frac{3}{18} + \frac{\square}{18} = \square$
8. $\frac{2}{3} - \frac{3}{7} = \frac{\square}{21} - \frac{9}{21} = \square$
9. $\frac{1}{6} + \frac{1}{4} = \frac{2}{12} + \frac{\square}{12} = \square$
10. $\frac{6}{7} - \frac{3}{4} = \frac{\square}{28} - \frac{21}{28} = \square$
11. $\frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{\square}{18} = \square$
12. $\frac{1}{2} + \frac{1}{7} = \frac{7}{14} + \frac{\square}{14} = \square$
13. $\frac{2}{5} - \frac{1}{3} = \frac{\square}{15} - \frac{5}{15} = \square$
14. $\frac{2}{3} - \frac{1}{2} = \frac{\square}{6} - \frac{3}{6} = \square$
15. $\frac{4}{11} + \frac{1}{2} = \frac{\square}{22} + \frac{11}{22} = \square$
16. $\frac{3}{4} + \frac{1}{5} = \frac{\square}{20} + \frac{4}{20} = \square$

Add or subtract. Rename a fraction if you need to.

$$\frac{17}{21} + \frac{1}{3} = \frac{17}{21} + \frac{7}{21}$$
$$= \frac{24}{21}$$

17. $\frac{5}{12} + \frac{1}{2} = \square$
18. $\frac{7}{9} - \frac{2}{3} = \square$
19. $\frac{4}{5} - \frac{3}{10} = \square$
20. $\frac{7}{7} - \frac{1}{14} = \square$
21. $\frac{1}{9} + \frac{1}{3} = \square$
22. $\frac{5}{8} + \frac{1}{4} = \square$
23. $\frac{3}{4} - \frac{5}{12} = \square$
24. $\frac{5}{18} + \frac{2}{3} = \square$
25. $\frac{11}{7} - \frac{9}{14} = \square$
26. $\frac{5}{12} - \frac{5}{24} = \square$
27. $\frac{11}{18} - \frac{5}{9} = \square$
28. $\frac{15}{7} + \frac{19}{21} = \square$
29. $\frac{1}{2} + \frac{7}{24} = \square$
30. $\frac{15}{22} - \frac{5}{11} = \square$
31. $\frac{3}{4} - \frac{1}{24} = \square$

32. Tom and Al are shoveling snow off a walk. Tom has shoveled $\frac{5}{18}$ of the walk. Al has shoveled $\frac{2}{9}$ of it.
   a. How much have they shoveled together?
   b. How much more do they have to shovel?

Add or subtract.

1. $\frac{13}{24} - \frac{1}{8} =$ ■
2. $\frac{7}{16} + \frac{1}{2} =$ ■
3. $\frac{2}{3} - \frac{5}{18} =$ ■
4. $\frac{3}{5} - \frac{2}{15} =$ ■
5. $\frac{4}{7} + \frac{5}{21} =$ ■
6. $\frac{11}{10} - \frac{1}{2} =$ ■
7. $\frac{17}{20} + \frac{4}{5} =$ ■
8. $\frac{1}{12} + \frac{1}{24} =$ ■
9. $\frac{9}{16} - \frac{1}{4} =$ ■
10. $\frac{1}{2} + \frac{7}{18} =$ ■
11. $\frac{8}{21} - \frac{1}{3} =$ ■
12. $\frac{3}{8} + \frac{5}{16} =$ ■
13. $\frac{9}{14} + \frac{1}{2} =$ ■
14. $\frac{14}{15} + \frac{2}{3} =$ ■
15. $\frac{5}{6} - \frac{13}{24} =$ ■
16. $\frac{17}{25} - \frac{2}{5} =$ ■
17. $\frac{51}{100} - \frac{1}{2} =$ ■
18. $\frac{23}{24} - \frac{3}{4} =$ ■

19. Ron had a strip of balsa wood that was $\frac{2}{3}$ yd. long. Jeff used $\frac{1}{6}$ yd. of the strip. How long was the remaining piece?

20. Jan bought two pieces of felt. One was $\frac{3}{8}$ yd. long and the other was $\frac{1}{4}$ yd. long. How many yards of felt did she buy all together?

21. Mary cut $\frac{1}{2}$ yd. of ribbon for a bow from a piece of ribbon $\frac{7}{8}$ yd. long. How much ribbon did she have left?

22. Gary practiced playing his horn for $\frac{1}{2}$ hour and worked on a model plane for $\frac{2}{3}$ hour. How long all together did he practice on his horn and work on his plane?

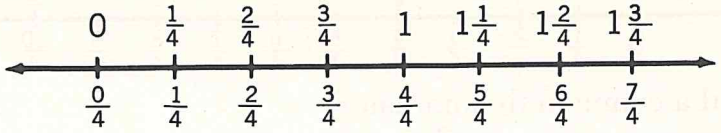
23. Helen used $\frac{2}{3}$ cup of sugar making cookies and $\frac{1}{6}$ cup of sugar making a pie. How much sugar did she use all together?

24. Stuart had $\frac{3}{4}$ gal. of paint. If he used $\frac{1}{8}$ gal. to paint a door, how much did he have left?

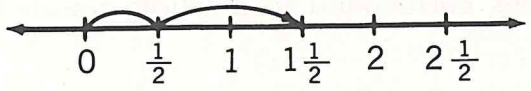
25. Pat is riding her bicycle to school. She knows that it is $\frac{9}{10}$ mi. from her house to the school. If the speedometer on her bike shows that she has already gone $\frac{3}{10}$ mi., how much farther does she have to go?

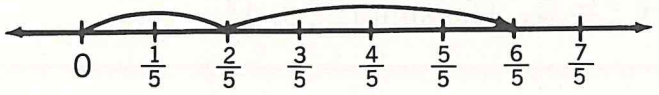
26. Catherine gave $\frac{1}{3}$ of a pie to her brother and $\frac{1}{6}$ of a pie to her sister. How much of the pie did she give away all together?

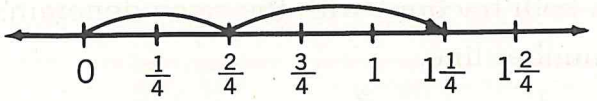
We can show whole numbers, fractions, or mixed numbers on a number line.



Tell what addition problem is shown on each number line.

1. 

2. 

3. 

Make a number line for each problem. Show the addition on the number line.

4. $\frac{4}{5} + \frac{2}{5} =$ ■

5. $1\frac{1}{2} + \frac{1}{2} =$ ■

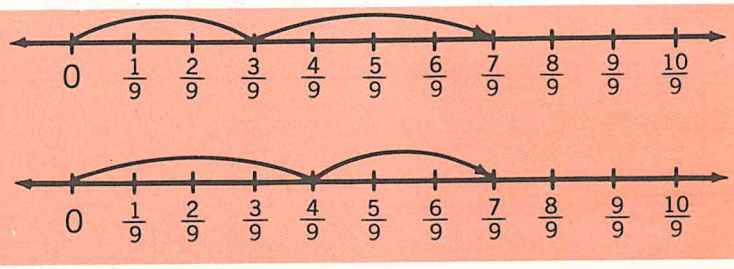
6. $\frac{3}{8} + \frac{7}{8} =$ ■

7. $\frac{3}{4} + \frac{2}{4} =$ ■

Use the number line to show that the commutative law for addition can be applied to each of these problems.

$\frac{3}{9} + \frac{4}{9} = \frac{7}{9}$

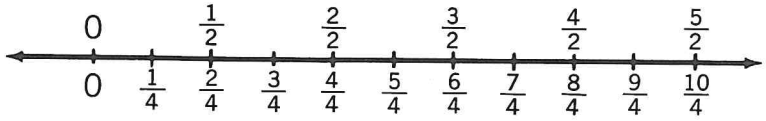
$\frac{4}{9} + \frac{3}{9} = \frac{7}{9}$


0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{6}{9}$ $\frac{7}{9}$ $\frac{8}{9}$ $\frac{9}{9}$ $\frac{10}{9}$
0 $\frac{1}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{6}{9}$ $\frac{7}{9}$ $\frac{8}{9}$ $\frac{9}{9}$ $\frac{10}{9}$

8. $\frac{2}{7} + \frac{3}{7} =$ ■

9. $\frac{3}{5} + \frac{4}{5} =$ ■

We can add fractions with unlike denominators. We can add $\frac{3}{4}$ and $\frac{1}{2}$.



We first find a common denominator.

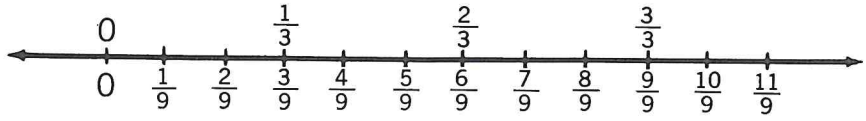
The point corresponding to $\frac{3}{4}$ does not also correspond to a number of halves. We cannot rename $\frac{3}{4}$ as a whole number of halves.

The point corresponding to $\frac{1}{2}$ does correspond to a whole number of fourths. We can rename $\frac{1}{2}$ as $\frac{2}{4}$.

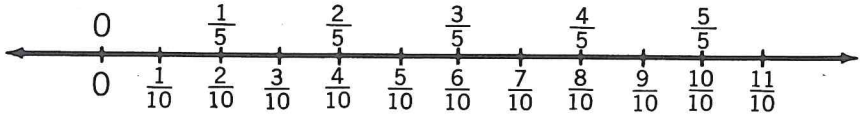
The problem becomes $\frac{3}{4} + \frac{2}{4} = \square$. The sum is $\frac{5}{4}$ or $1\frac{1}{4}$.

Copy each number line. Express both fractions with the same denominator. Then show the addition on the number line.

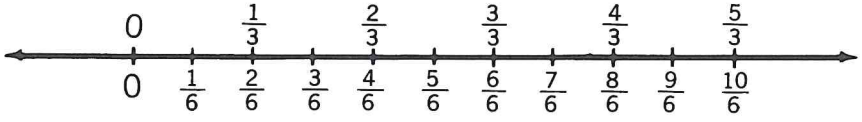
1. $\frac{2}{3} + \frac{4}{9} = \square$



2. $\frac{4}{5} + \frac{3}{10} = \square$



3. $\frac{1}{6} + \frac{4}{3} = \square$

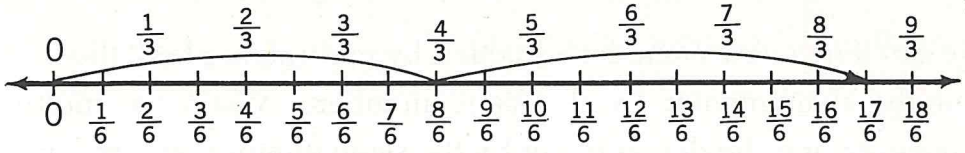


Make a number line for this problem. Show the answer on the number line. Then answer the question at the end of the story.

4. Martha was taking care of her little brother one afternoon. She took him to the school to play on the swings. That was $1\frac{1}{4}$ miles from their house. Then they went to their cousin's house. That was $\frac{1}{8}$ of a mile farther. How far did Martha take her brother that day?

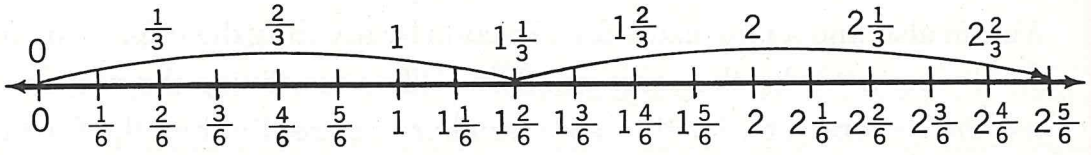
Addition of Mixed Numbers

We add fractions.



$\frac{4}{3} + \frac{9}{6} = \square$ becomes $\frac{8}{6} + \frac{9}{6} = \frac{17}{6}$

We can use mixed numbers to show the same problem.



$1\frac{1}{3} + 1\frac{3}{6} = \square$ becomes $1\frac{2}{6} + 1\frac{3}{6} = 2\frac{5}{6}$

Make two number lines for each pair of problems. Use fractions to correspond to points on the first line. Use mixed numbers to correspond to points on the second line. Then show each addition problem on the number line.

1. a. $\frac{7}{3} + \frac{10}{6} = \square$ 2. a. $\frac{6}{5} + \frac{13}{10} = \square$ 3. a. $\frac{5}{2} + \frac{7}{6} = \square$

   b. $2\frac{1}{3} + 1\frac{4}{6} = \square$ b. $1\frac{1}{5} + 1\frac{3}{10} = \square$ b. $2\frac{1}{2} + 1\frac{1}{6} = \square$

Find each sum. If you need help, make a number line.

4. $1\frac{3}{4} + 2\frac{1}{4} = \square$ 6. $4\frac{2}{5} + 2\frac{3}{5} = \square$ 8. $1\frac{1}{8} + 1\frac{2}{4} = \square$

5. $3\frac{2}{3} + 4\frac{2}{3} = \square$ 7. $1\frac{1}{10} + 1\frac{3}{5} = \square$ 9. $1\frac{1}{2} + 1\frac{3}{4} = \square$

Make a number line for this problem. Show the answer on the number line. Then answer the question at the end of the story.

10. Ben's house is $\frac{7}{10}$ of a mile from the school. Ben rides his bike so he can come home for lunch. How far does Ben go in one day, going between his home and the school?

Each fraction has many names.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

We can find a new name for a fraction by multiplying both the numerator and the denominator by the same number. When we multiply the numerator and the denominator by the same number, we are multiplying the fraction by one.

$$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8} \qquad \frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

We can also find a new name for a fraction by dividing the numerator and the denominator by the same number. When we divide the numerator and the denominator by the same number, we are dividing the fraction by one.

$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4} \qquad \frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

Find the new name.

1. $\frac{12}{24} = \frac{12 \div 12}{24 \div 12} = \square$
2. $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \square$
3. $\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \square$
4. $\frac{10}{25} = \frac{10 \div 5}{25 \div 5} = \square$
5. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \square$
6. $\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \square$
7. $\frac{3}{6} = \frac{3 \div \square}{6 \div 3} = \square$
8. $\frac{6}{9} = \frac{6 \div \square}{9 \div 3} = \square$
9. $\frac{6}{18} = \frac{6 \div \square}{18 \div 6} = \square$
10. $\frac{16}{24} = \frac{16 \div \square}{24 \div 8} = \square$
11. $\frac{3}{18} = \frac{3 \div \square}{18 \div 3} = \square$
12. $\frac{7}{21} = \frac{7 \div \square}{21 \div 7} = \square$
13. $\frac{12}{16} = \frac{12 \div 4}{16 \div \square} = \square$
14. $\frac{8}{14} = \frac{8 \div \square}{14 \div 2} = \square$
15. $\frac{6}{10} = \frac{6 \div 2}{10 \div \square} = \square$
16. $\frac{8}{28} = \frac{8 \div \square}{28 \div 4} = \square$
17. $\frac{3}{24} = \frac{3 \div 3}{24 \div \square} = \square$
18. $\frac{20}{25} = \frac{20 \div \square}{25 \div 5} = \square$

If the numerator and denominator of a fraction cannot both be divided by some number other than one, we say that the fraction name is in **lowest terms.**

$\frac{2}{8}$ is not in lowest terms because both 2 and 8 can be divided by 2.

$\frac{3}{5}$ is in lowest terms because 3 and 5 cannot both be divided by any number except one.

1. Which fraction names are in lowest terms?

a. $\frac{1}{2}$ c. $\frac{3}{24}$ e. $\frac{3}{2}$ g. $\frac{9}{10}$

b. $\frac{5}{15}$ d. $\frac{2}{3}$ f. $\frac{14}{17}$ h. $\frac{3}{12}$

Complete each problem.

2. $\frac{2}{8} = \frac{\square}{4}$
3. $\frac{3}{6} = \frac{\square}{2}$
4. $\frac{6}{15} = \frac{\square}{5}$
5. $\frac{8}{24} = \frac{\square}{3}$
6. $\frac{2}{2} = \frac{\square}{1}$
7. $\frac{4}{16} = \frac{\square}{4}$
8. $\frac{4}{12} = \frac{\square}{3}$
9. $\frac{12}{18} = \frac{\square}{3}$
10. $\frac{6}{9} = \frac{\square}{3}$
11. $\frac{5}{10} = \frac{\square}{2}$
12. $\frac{14}{21} = \frac{\square}{3}$
13. $\frac{3}{3} = \frac{\square}{1}$
14. $\frac{3}{12} = \frac{\square}{4}$
15. $\frac{2}{6} = \frac{\square}{3}$
16. $\frac{6}{12} = \frac{\square}{2}$

17. If we divide the denominator by 3 to find a new fraction name, we must divide the numerator by ■.

18. If we multiply the numerator by 4 to find a new fraction name, we must multiply the denominator by ■.

19. Cliff and Mike each caught a fish. Cliff measured his fish and said it was $\frac{8}{12}$ foot long. Mike's fish measured $\frac{2}{3}$ foot long. Was Cliff's fish longer?

Write each fraction name in lowest terms.

1. a. $\frac{10}{15}$ b. $\frac{2}{8}$

2. a. $\frac{7}{14}$ b. $\frac{9}{12}$

3. a. $\frac{6}{9}$ b. $\frac{15}{20}$

4. a. $\frac{9}{24}$ b. $\frac{8}{14}$

We may divide more than once.

$$\frac{80}{100} = \frac{80 \div 10}{100 \div 10} = \frac{8}{10}$$

$$\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$$

$\frac{80}{100}$ reduced to lowest terms is $\frac{4}{5}$.

5. a. $\frac{6}{12}$ b. $\frac{18}{24}$ c. $\frac{8}{20}$

6. a. $\frac{9}{27}$ b. $\frac{12}{18}$ c. $\frac{15}{45}$

7. a. $\frac{8}{12}$ b. $\frac{16}{24}$ c. $\frac{75}{100}$

8. a. $\frac{27}{36}$ b. $\frac{18}{42}$ c. $\frac{24}{36}$

Add or subtract. Give each fraction name in lowest terms.

9. $\frac{2}{3} + \frac{1}{12} = \frac{\square}{12} + \frac{1}{12} = \square$

10. $\frac{1}{4} + \frac{3}{100} = \frac{\square}{100} + \frac{3}{100} = \square$

11. $\frac{1}{2} - \frac{1}{10} = \frac{\square}{10} - \frac{1}{10} = \square$

12. $\frac{1}{2} - \frac{5}{18} = \frac{\square}{18} - \frac{5}{18} = \square$

13. $\frac{4}{3} - \frac{5}{6} = \frac{\square}{6} - \frac{5}{6} = \square$

14. $\frac{5}{12} + \frac{1}{3} = \frac{5}{12} + \frac{\square}{12} = \square$

15. $\frac{1}{9} + \frac{1}{3} = \frac{1}{9} + \frac{\square}{9} = \square$

16. $\frac{3}{20} + \frac{4}{5} = \frac{3}{20} + \frac{\square}{20} = \square$

17. $\frac{2}{3} - \frac{4}{9} = \frac{\square}{9} - \frac{4}{9} = \square$

18. $\frac{5}{6} - \frac{1}{18} = \frac{\square}{18} - \frac{1}{18} = \square$

19. Charley used $\frac{1}{3}$ of a bottle of glue to make a model plane and $\frac{1}{6}$ of the bottle to make a model car. How much of the glue did he use all together?

20. Cathy spent $\frac{1}{4}$ of an hour washing dishes and $\frac{1}{2}$ of an hour dusting. What part of an hour did she spend doing these chores?

Express each fraction either as a mixed number or as a whole number.

1. $\frac{8}{5}$
2. $\frac{5}{2}$
3. $\frac{9}{4}$
4. $\frac{10}{6}$
5. $\frac{11}{4}$
6. $\frac{25}{3}$
7. $\frac{13}{6}$
8. $\frac{19}{8}$
9. $\frac{34}{5}$
10. $\frac{29}{7}$
11. $\frac{15}{4}$
12. $\frac{17}{8}$

Find each sum.

13. $\frac{4}{8} + \frac{5}{8} =$ ■
14. $\frac{2}{3} + \frac{1}{3} =$ ■
15. $\frac{3}{5} + \frac{4}{5} =$ ■
16. $\frac{4}{6} + \frac{7}{6} =$ ■
17. $\frac{1}{3} + \frac{8}{9} =$ ■
18. $\frac{7}{5} + \frac{4}{5} =$ ■
19. $\frac{4}{5} + \frac{5}{5} =$ ■
20. $\frac{5}{6} + \frac{1}{3} =$ ■
21. $\frac{9}{10} + \frac{4}{5} =$ ■
22. $\frac{7}{6} + \frac{6}{6} =$ ■
23. $\frac{4}{3} + \frac{4}{9} =$ ■
24. $\frac{8}{10} + \frac{9}{10} =$ ■

Find each product.

25. $6 \times \frac{2}{5} =$ ■
26. $4 \times \frac{2}{3} =$ ■
27. $7 \times \frac{3}{4} =$ ■
28. $5 \times \frac{1}{3} =$ ■
29. $6 \times \frac{2}{3} =$ ■
30. $7 \times \frac{2}{5} =$ ■
31. $9 \times \frac{2}{9} =$ ■
32. $6 \times \frac{3}{7} =$ ■
33. $5 \times \frac{4}{7} =$ ■
34. $8 \times \frac{1}{2} =$ ■
35. $4 \times \frac{7}{9} =$ ■
36. $3 \times \frac{8}{9} =$ ■

37. Dean has $\frac{3}{5}$ of a pound of candy and Keith has $\frac{2}{5}$ of a pound. Together do they have a pound of candy?

38. Meg has 4 partly used spools of ribbon. If each spool contains $\frac{3}{5}$ of a yard, how many yards of ribbon does she have?

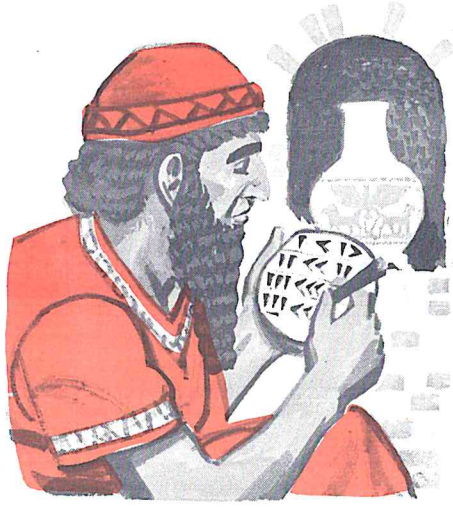
39. From the tip of Ben's little finger to the tip of his thumb is $\frac{1}{2}$ of a foot. From the tip of his middle finger to his shoulder is 3 times as long. How many feet is it from the tip of his middle finger to his shoulder?

40. Marla's bottle of glass beads is $\frac{2}{5}$ empty. If the bottle holds 50 beads when it is full, how many beads are left in the bottle?

Feedback

Choose $<$ or $=$ or $>$.

1. $\frac{7}{2}$ ● 1
2. $\frac{2}{3}$ ● $\frac{8}{12}$
3. $\frac{5}{12}$ ● $\frac{2}{6}$
4. $\frac{1}{4}$ ● $\frac{1}{3}$
5. $\frac{4}{7}$ ● $\frac{9}{21}$
6. $\frac{3}{4}$ ● $\frac{8}{12}$
7. $\frac{2}{7}$ ● $\frac{6}{14}$
8. $\frac{5}{5}$ ● 1
9. $\frac{4}{8}$ ● $\frac{3}{4}$
10. $\frac{3}{5}$ ● $\frac{5}{10}$
11. $\frac{3}{4}$ ● 1
12. $\frac{3}{9}$ ● $\frac{2}{3}$
13. $\frac{6}{10}$ ● $\frac{3}{5}$
14. $\frac{2}{7}$ ● $\frac{5}{7}$



Find each sum or difference.

15. $\frac{6}{7} + \frac{1}{7} =$ ■
16. $\frac{7}{8} - \frac{2}{8} =$ ■
17. $\frac{3}{4} + \frac{1}{8} =$ ■
18. $\frac{2}{3} + \frac{1}{6} =$ ■
19. $\frac{5}{8} - \frac{1}{4} =$ ■
20. $\frac{2}{3} - \frac{5}{12} =$ ■
21. $\frac{3}{4} + \frac{1}{12} =$ ■
22. $\frac{1}{3} + \frac{1}{4} =$ ■
23. $\frac{3}{4} - \frac{1}{8} =$ ■
24. $\frac{2}{5} + \frac{5}{6} =$ ■
25. $\frac{4}{5} + \frac{1}{10} =$ ■
26. $\frac{1}{2} + \frac{1}{3} =$ ■

Copy and complete each problem.

27. $4\frac{1}{2}$
$+2\frac{1}{2}$

28. $7\frac{4}{5}$
$-2\frac{1}{5}$

29. $6\frac{2}{7}$
$+9\frac{3}{7}$

30. $8\frac{7}{8}$
$-2\frac{3}{8}$

31. Doris had 3 pieces of ribbon each $\frac{5}{12}$ yd. in length. How many yards of ribbon did she have all together?

32. John had a piece of wood $7\frac{3}{4}$ ft. in length. He cut off and used a piece $4\frac{1}{4}$ ft. long. How long was the piece that remained?

Feedback

Choose $=$ or $\neq$.

1. $3 + 7 \bullet 7 + 3$
2. $8 - 4 \bullet 4 - 8$
3. $5 \times 9 \bullet 9 \times 5$
4. $6 \div 2 \bullet 2 \div 6$
5. $(9 - 7) - 2 \bullet 9 - (7 - 2)$
6. $3 \times (4 \times 5) \bullet (3 \times 4) \times 5$
7. $8 \div (4 \div 2) \bullet (8 \div 4) \div 2$
8. $(8 + 6) + 5 \bullet 8 + (6 + 5)$

Copy and complete each problem.

9. $\$1.23 + .54$
10. $\$7.63 + 2.35$
11. $\$18.19 + 21.00$
12. $\$23.45 + 5.33$
13. $\$3.98 - .35$
14. $\$7.26 - 3.15$
15. $\$19.37 - 2.15$
16. $\$27.54 - 15.52$

17. Round off each measurement to the nearest half dollar.
    a. \$3.75 b. \$9.52 c. \$4.86 d. \$5.21

18. Round off each measurement to the nearest half-hour.
    a. 7:03 b. 9:36 c. 1:17 d. 9:45

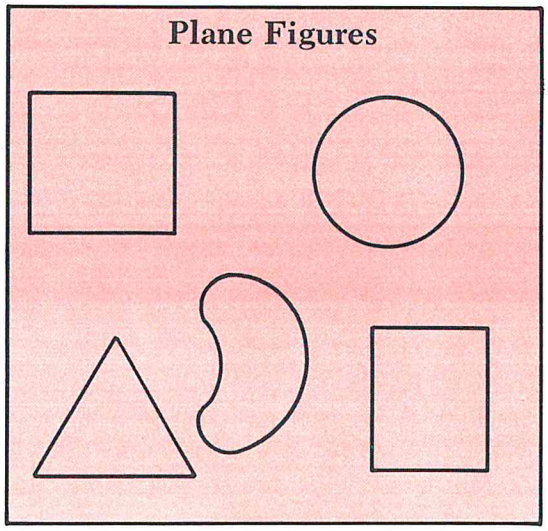
19. Change each time to 12-hour time.
    a. 0130 b. 0726 c. 1908 d. 2345

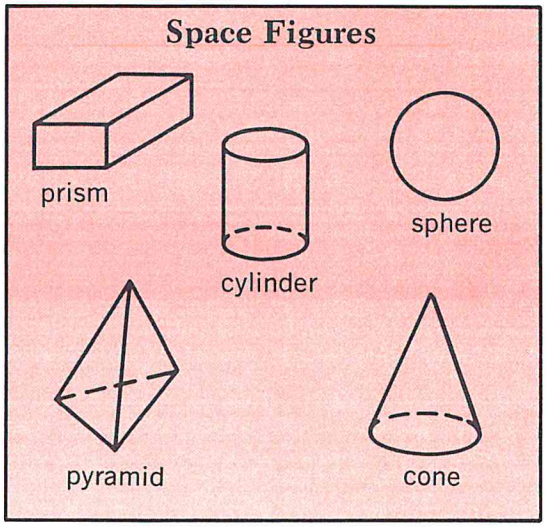
20. Change each time to 24-hour time.
    a. 2:27 A.M. b. 8:06 P.M. c. 11:56 P.M.

21. When it is 9 A.M. in London, is it earlier or later in New York?

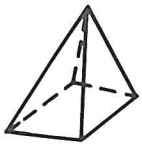
22. Which of these numbers are prime numbers?
    a. 31 b. 15 c. 23 d. 17 e. 6

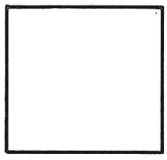
Part 9 Space Figures

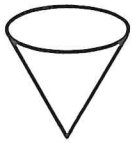


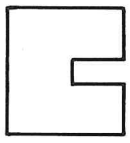


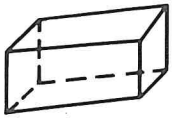
1. Which of the figures below are space figures?

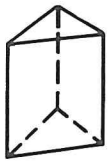
a. 

b. 

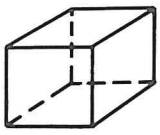
c. 

d. 

e. 

f. 

g.

h. 

2. Which of the figures above are plane figures?

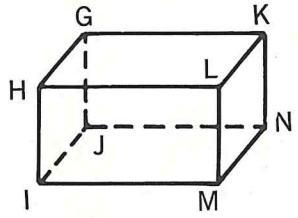
3. Write **plane** or **space**.

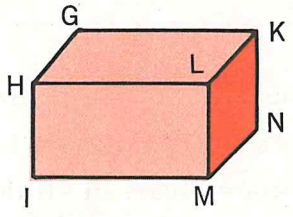
a. A prism is a ▢ figure.
b. A circle is a ▢ figure.
c. A rectangle is a ▢ figure.
d. A sphere is a ▢ figure.
e. A pyramid is a ▢ figure.
f. A cube is a ▢ figure.
g. A triangle is a ▢ figure.
h. A square is a ▢ figure.

Prisms

A **prism** has two bases of the same shape and size. Prisms are described by the shape of their bases.

This is a picture of a **rectangular prism.** Its bases are rectangular in shape.





A rectangular prism has 6 flat faces.

Two faces meet in an edge.

Two edges meet in a vertex.

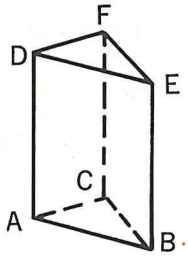
HIML is a face.

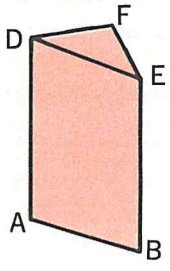
Line segment HI is an edge.

Point H is a vertex.

1. **a.** The rectangular bases are GHLK and ■.
   b. How many rectangular faces does a rectangular prism have?
   c. How many faces in all does a rectangular prism have?
   d. How many edges does a rectangular prism have?
   e. How many vertices does a rectangular prism have?

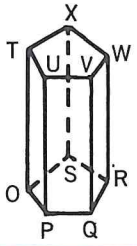
This is a picture of a **triangular prism.**

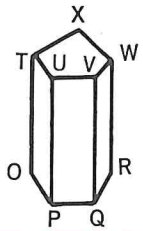




2. **a.** The triangular bases are ABC and ■.
   b. How many rectangular faces does a triangular prism have?
   c. How many faces in all does a triangular prism have?
   d. How many edges does a triangular prism have?
   e. How many vertices does a triangular prism have?

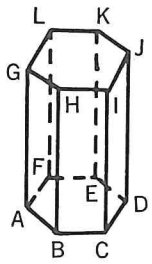
This is a picture of a **pentagonal prism.**

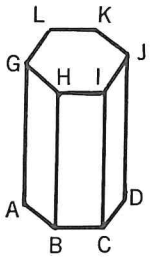




1. **a.** The pentagonal bases are OPQRS and ■.
   b. How many rectangular faces does a pentagonal prism have?
   c. How many faces in all does a pentagonal prism have?
   d. How many edges does a pentagonal prism have?
   e. How many vertices does a pentagonal prism have?

This is a picture of a **hexagonal prism.**





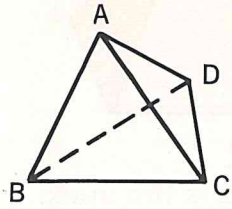
2. **a.** The hexagonal bases are ABCDEF and ■.
   b. How many rectangular faces does a hexagonal prism have?
   c. How many faces in all does a hexagonal prism have?
   d. How many edges does a hexagonal prism have?
   e. How many vertices does a hexagonal prism have?

3. Copy and complete this table.

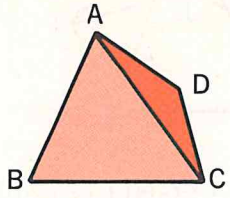
Type of prism	Triangular	Rectangular	Pentagonal	Hexagonal
Shape of bases			pentagon	
Number of bases				
Number of faces			7	
Number of edges	9			
Number of vertices				12

Pyramids

A **pyramid** has a base and triangular faces. These faces intersect at the **apex.** This is a picture of a **triangular pyramid.**



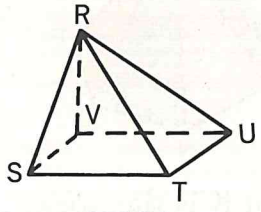
The base BCD is triangular.



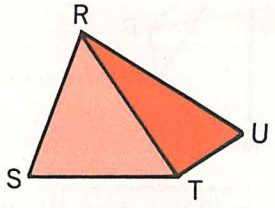
Point A is the apex.

1. **a.** How many bases does a triangular pyramid have?
   b. How many triangular faces does a triangular pyramid have?
   c. How many edges does a triangular pyramid have?
   d. How many vertices does a triangular pyramid have?
   e. How many faces meet at the apex?
   f. How many edges meet at the apex?

This is a picture of a **rectangular pyramid.**
A rectangular pyramid has a base and four triangular faces.



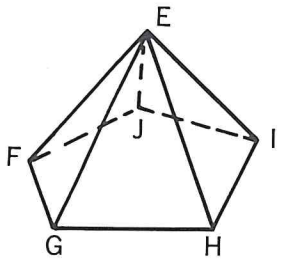
The base STUV is rectangular.

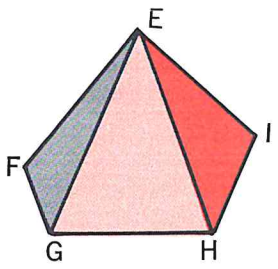


Point R is the apex.

2. **a.** How many bases does a rectangular pyramid have?
   b. How many triangular faces does a rectangular pyramid have?
   c. How many edges does a rectangular pyramid have?
   d. How many vertices does a rectangular pyramid have?
   e. How many faces meet at the apex?
   f. How many edges meet at the apex?

This is a picture of a **pentagonal pyramid.**



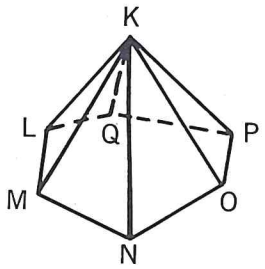


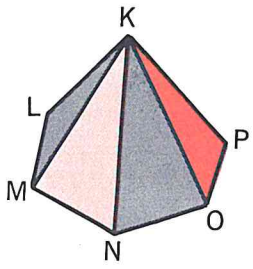
The base FGHIJ is pentagonal.

Point E is the apex.

1. **a.** How many bases does a pentagonal pyramid have?
   b. How many triangular faces does a pentagonal pyramid have?
   c. How many edges does a pentagonal pyramid have?
   d. How many vertices does a pentagonal pyramid have?
   e. How many faces meet at the apex?
   f. How many edges meet at the apex?

This is a picture of a **hexagonal pyramid.**





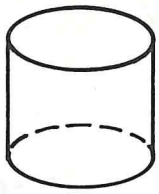
The base LMNOPQ is hexagonal.

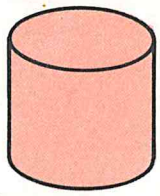
Point K is the apex.

2. **a.** How many bases does a hexagonal pyramid have?
   b. How many triangular faces does a hexagonal pyramid have?
   c. How many edges does a hexagonal pyramid have?
   d. How many vertices does a hexagonal pyramid have?
   e. How many faces meet at the apex?
   f. How many edges meet at the apex?

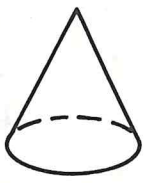
This is a picture of a **sphere.**

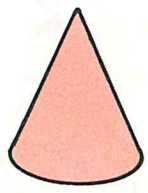
This is a picture of a **cylinder.**





This is a picture of a **cone.**

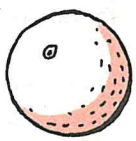




Below are pictured some physical objects.

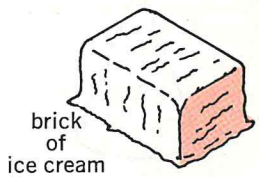
1. Which have shapes of prisms?
2. Which have shapes of pyramids?
3. Which have shapes of spheres?
4. Which have shapes of cylinders?
5. Which have shapes of cones?

can of corn

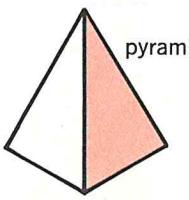


orange

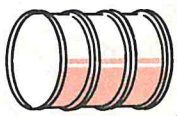
ice cream cone



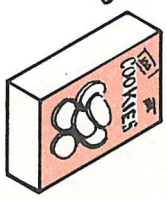
brick of ice cream



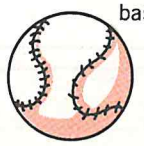
pyramid



oil drum

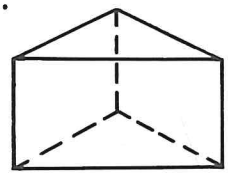


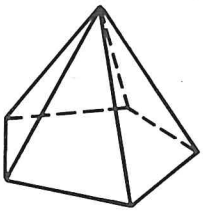
package of cookies

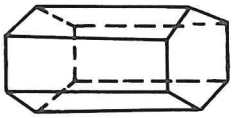


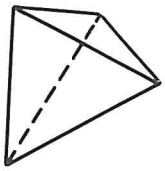
baseball

Pair each picture on the left with a picture on the right that shows the same space figure in the same position.

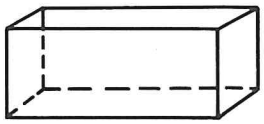
1. 

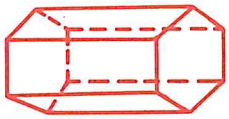
2. 

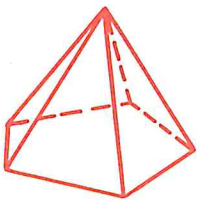
3. 

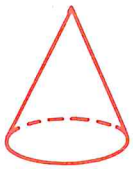
4. 

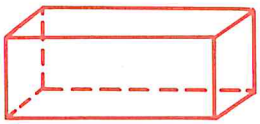
5.

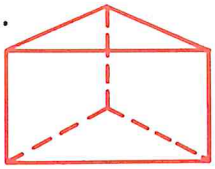
6. 

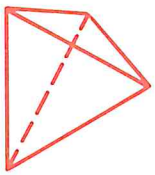
A. 

B. 

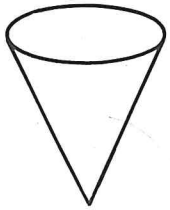
C. 

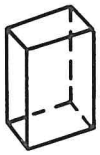
D. 

E. 

F. 

Pair each picture on the left with a picture on the right that shows the space figure a different size but in the same position.

7. 

8. 

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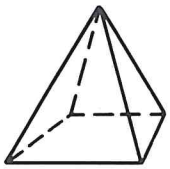
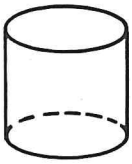
10. 

11. 

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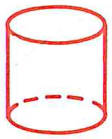
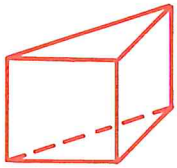
G. 

H. 

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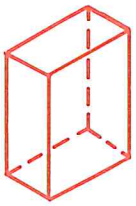
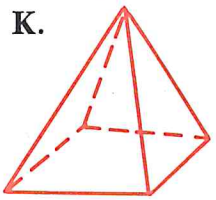
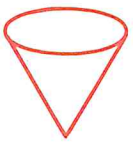
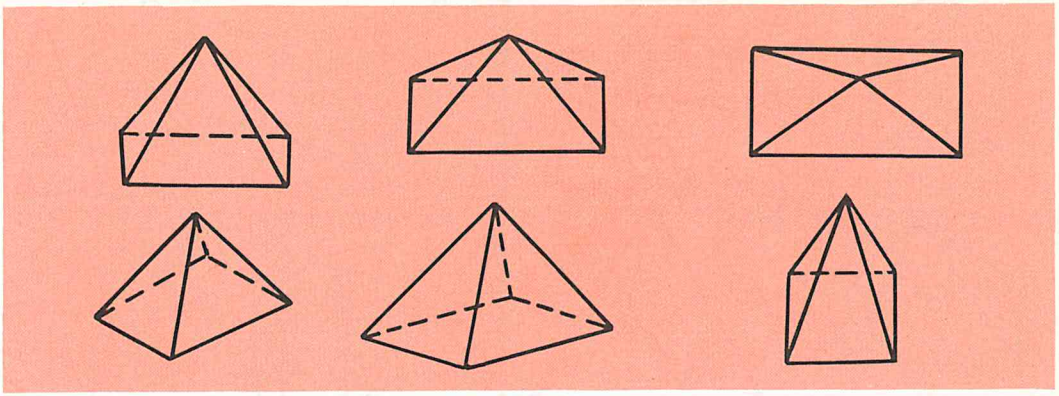
J. 

K. 

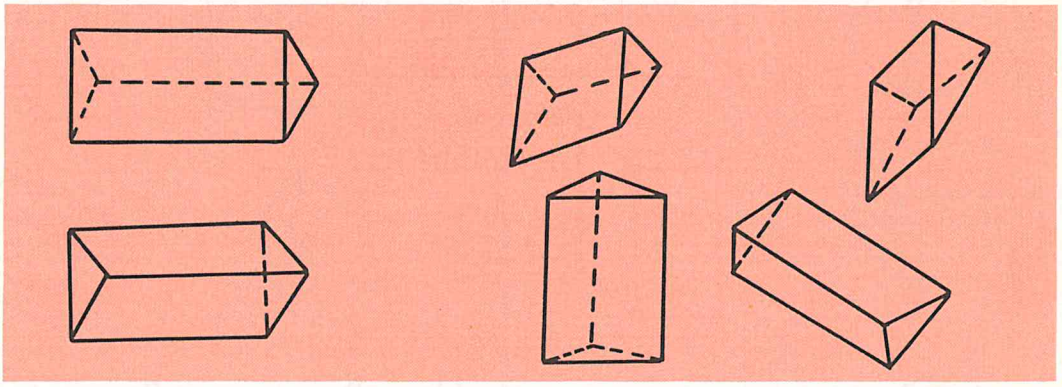
L. 

When the position of the space figure varies, the pictures look different.

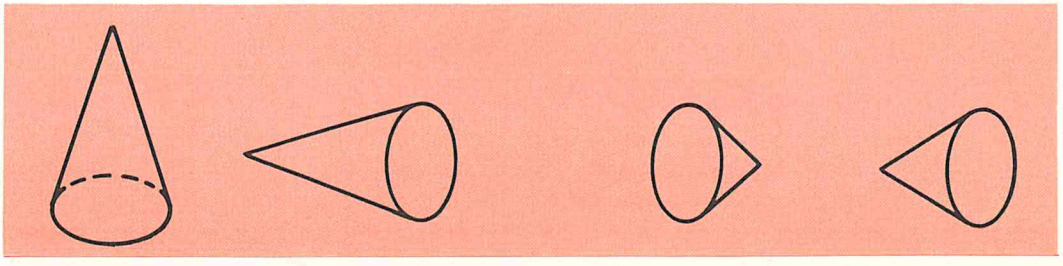
Different pictures of the same rectangular pyramid:



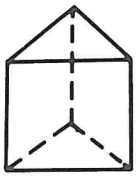
Different pictures of the same triangular prism:

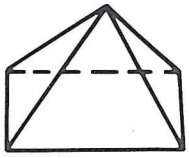


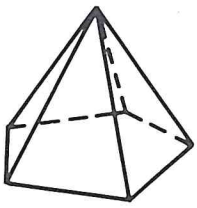
Different pictures of the same cone:

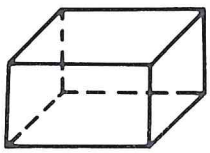


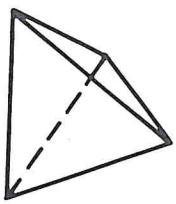
Pair each picture on the left with a picture on the right that shows the same space figure in a different position.

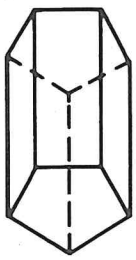
1. 

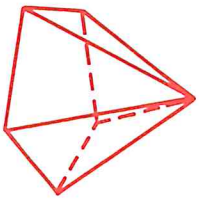
2. 

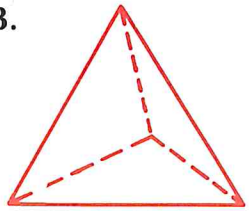
3. 

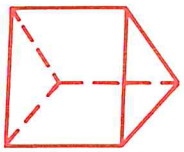
4. 

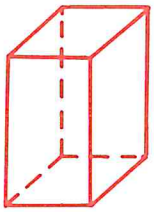
5. 

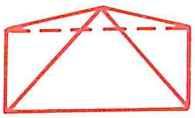
6. 

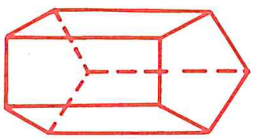
A. 

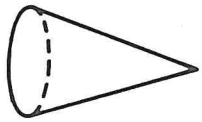
B. 

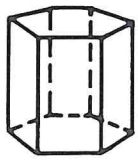
C. 

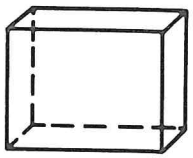
D. 

E. 

F. 

7. 

8. 

9. 

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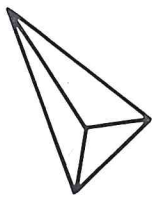
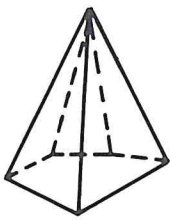
11. 

12. 

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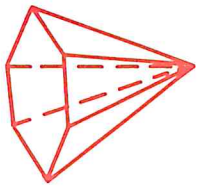
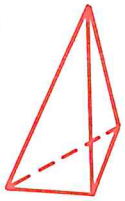
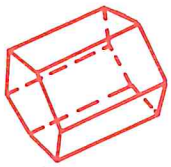
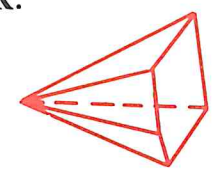
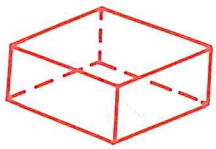
H. 

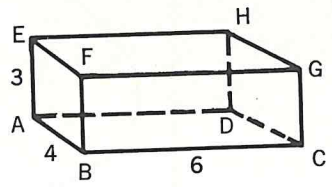
I. 

J. 

K. 

L. 

The **volume** of a figure is the measure of the space it encloses. Volume is given in cubic units.

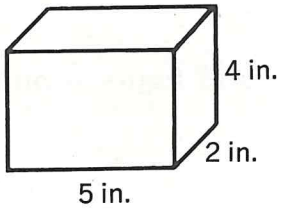


Volume = length × width × height
Volume = (6 × 4 × 3) cu. in.
= 72 cu. in.

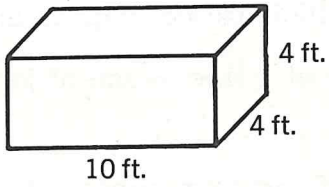
1. To find the volume of a rectangular solid whose length is l, width w, and height h, we can use the formula V = ___ × ___ × ___.

Find the volume of these rectangular solids.

2.



3.



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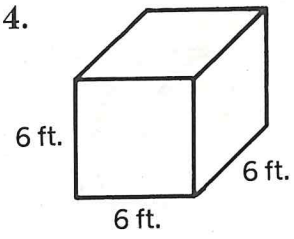

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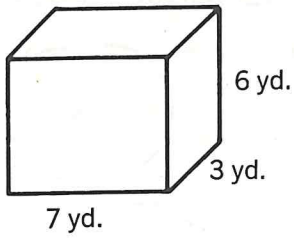
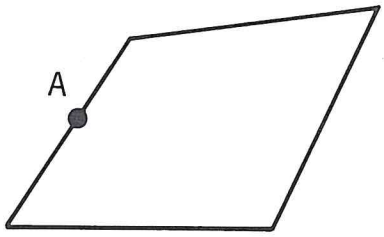
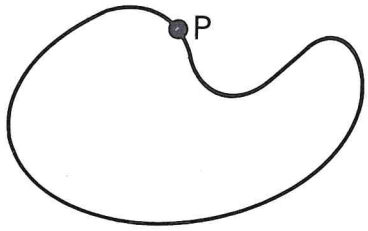


6. A cubic yard measures 3 ft. by 3 ft. by 3 ft. How many cu. ft. does it contain?

7. A cubic foot measures 12 in. by 12 in. by 12 in. How many cu. in. does it contain?

Convex and Concave Plane Figures





1. Can you find a point B on the quadrilateral so that part of $\overline{AB}$ is outside the figure?
   This quadrilateral is convex.

2. Can you find a point Q on the figure so that part of $\overline{PQ}$ is outside the figure?
   This figure is concave.

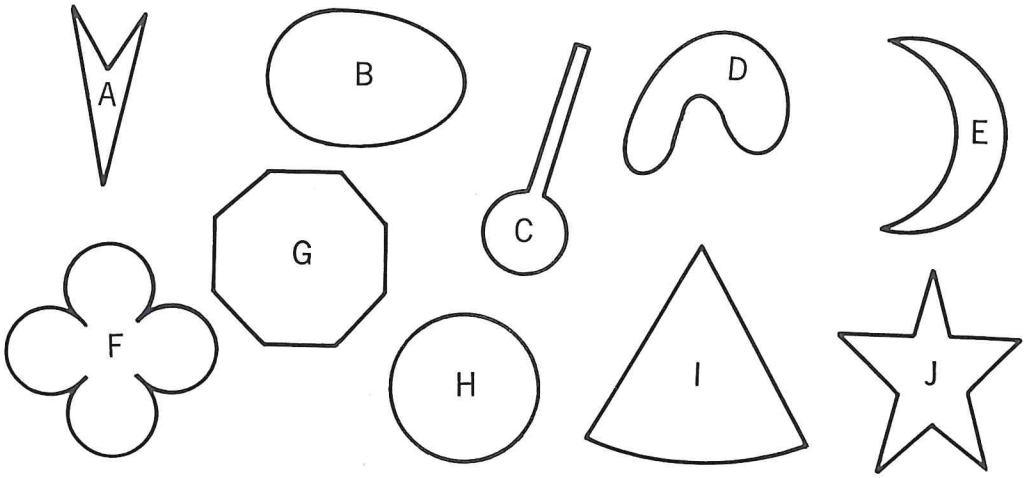
Choose **convex** or **concave**.

3. No part of a line segment joining two points on a ▪ figure is outside the figure.

4. Figures which are not convex are ▪ figures.

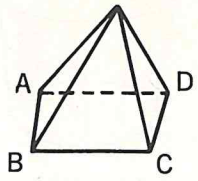
5. Some part of a line segment joining two points on a ▪ figure is outside the figure.

6. **a.** Which figures are concave?
   b. Which figures are convex?



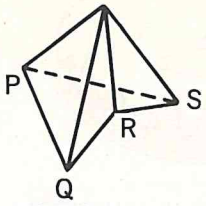
Convex and Concave Space Figures

Convex pyramid



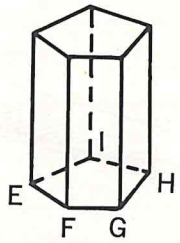
Base ABCD is a convex quadrilateral.

Concave pyramid



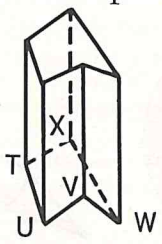
Base PQRS is a concave quadrilateral.

Convex prism



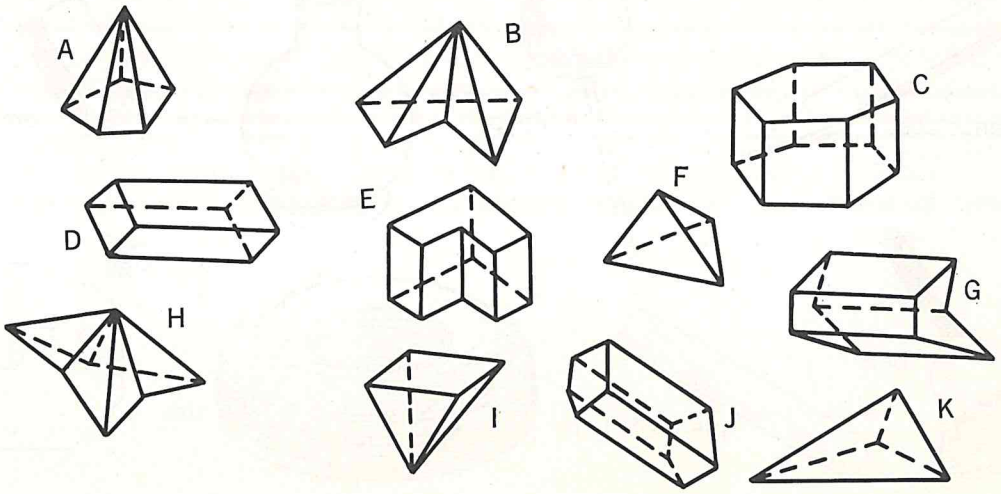
Base EFGHI is a convex pentagon.

Concave prism

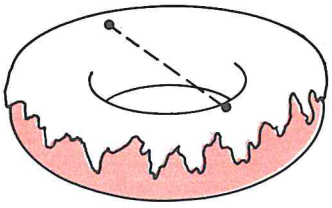


Base TUVWX is a concave pentagon.

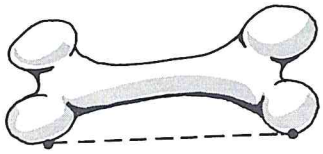
1. a. Which figures are concave?
b. Which figures are convex?
c. Which figures are pyramids?
d. Which figures are prisms?



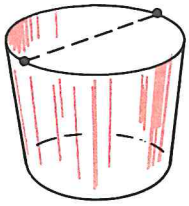
A figure is concave if part of some line segment connecting two points of the figure is outside the figure.



A doughnut is concave.

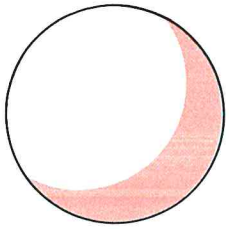


A bone is concave.

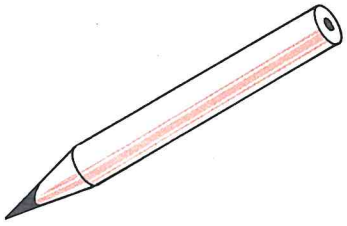


A glass is concave.

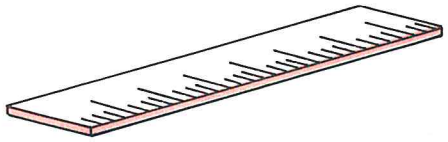
A figure is convex if no part of any line segment connecting any two points of the figure is outside the figure.



A ball is convex.



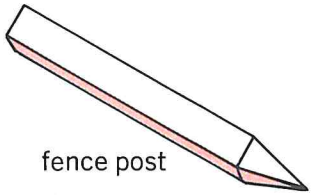
A pencil is convex.



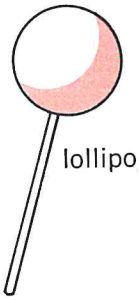
A ruler is convex.

1. a. Which figures are convex?

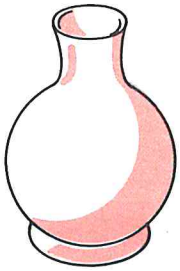
b. Which figures are concave?



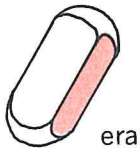
fence post



lollipop

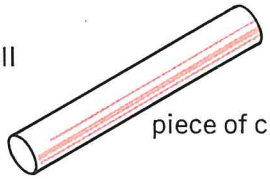


vase

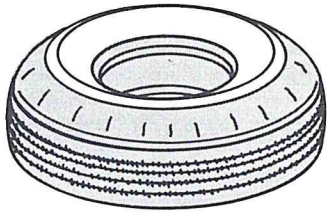


eraser

bell



piece of chalk



tire

road sign

Feedback

Find each sum or difference.

1. $\$183.76 + 204.13$
2. $\$439.42 - 326.32$
3. $\$475.05 + 304.70$
4. $\$767.95 - 345.53$

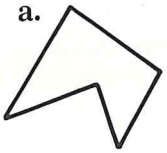
Find each product.

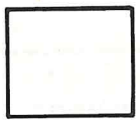
5. 846×3
6. 937×40
7. 4216×7
8. 876×45

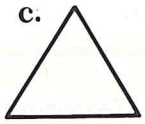
Find each quotient.

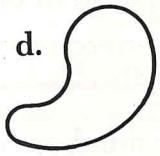
9. $8\overline{)749}$
10. $2\overline{)4986}$
11. $70\overline{)6852}$
12. $68\overline{)54321}$

13. Which figures below are convex?
14. Which figures below are concave?

a. 

b. 

c. 

d. 

True or false?

15. 5 is a factor of 53.
16. 496 is a multiple of 2.
17. 63 is a prime number.
18. 73 is a prime number.
19. 3 is a factor of 81,743.
20. 6 is a factor of 300.
21. 1902 is divisible by 3.
22. 0 is a multiple of 4.
23. 140 is a multiple of 15.
24. 23 is a prime number.
25. 384 is a multiple of 3.
26. 185 is divisible by 15.

Part 10 Systems of Numeration

Egyptian Numerals

The ancient Egyptians used pictures to represent numbers. Here are some of the pictures they used:

1 10 100 1000 10,000

These symbols could be arranged in any order in a numeral. For example, 234 could be written:

The system did not make use of **place value.** It had no zero.

A system or method of expressing numbers is called a system of numeration.

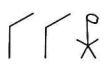
What number does each Egyptian numeral name?

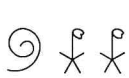
1.

2.

3.

4.

5. 

6. 

7.

8.

9.

Write an Egyptian numeral to express each number.

10. 24
11. 16
12. 39
13. 112
14. 1050
15. 3465
16. 26,000
17. 10,024

Choose = or ≠.

18.
19. 12 ● 21
20.
21. 102 ● 12
22. 121
23. 101 ●

Roman Numerals

The Romans used these symbols:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

In general the Romans repeated symbols and added the numbers for which each symbol stood. Although the early Romans wrote "4" as "IIII", the later Romans made use of subtraction and wrote "IV". Other exceptions were: "I" before "X", "X" before "L" or "C", and "C" before "D" or "M". In these cases they subtracted.

The Romans did not make use of place value. They had no zero.

Give the meaning of each symbol.

1. XI = 10 + 1 = ■
2. IX = 10 − 1 = ■
3. LX = 50 + 10 = ■
4. XL = 50 − 10 = ■
5. CM = 1000 − 100 = ■
6. MC = 1000 + 100 = ■

7. Copy and complete the table using Roman numerals.

I	II	■	IV	V
VI	VII	■	IX	X
■	XII	XIII	■	XV
XVI	■	XVIII	XIX	XX
XXI	XXII	■	XXIV	■
XXVI	■	XXVIII	■	XXX

8. What number does each Roman numeral express?

a. XXIV	d. CII	g. LXXV	j. CD	m. MCMLXX
b. XXVI	e. LIV	h. DX	k. MD	n. MCMII
c. XIX	f. LVI	i. MCC	l. MCM	o. DCLXVI

9. Write the Roman numerals for each number.

a. 59	d. 27	g. 90	j. 749
b. 1004	e. 48	h. 900	k. 1019
c. 79	f. 110	i. 905	l. 329

Decimal System of Numeration

Our system of numeration is sometimes called the Hindu-Arabic system because of its origin. It is a decimal system. It has ten different symbols which are often called digits. These digits are:

0 1 2 3 4 5 6 7 8 9

The numeral for any number can be written using only these ten symbols because our system of numeration has a zero and makes use of place value.

You are already familiar with the decimal system of numeration. You know that the value each digit expresses is determined by its face and place in the numeral. For instance, in the numeral "405" the "4" expresses 4 hundreds, while in the numeral "4065" the "4" expresses 4 thousands. The table below shows some number names.

trillions	billions	millions	thousands	ones
601,	842,	365,	221,	356

The number shown in the table is made up of the sum of these numbers:

356	is read "three hundred fifty-six".
221,000	is read "two hundred twenty-one thousand".
365,000,000	is read "three hundred sixty-five million".
842,000,000,000	is read "eight hundred forty-two billion".
601,000,000,000,000	is read "six hundred one trillion".
601,842,365,221,356	is read "601 trillion, 842 billion, 365 million, 221 thousand, 356".

Find each sum.

1.
$$\begin{array}{r} 10 \\ 20{,}000 \\ 610{,}000{,}000 \\ 3{,}000{,}000{,}000 \\ \underline{100{,}000{,}000{,}000{,}000} \end{array}$$

2.
$$\begin{array}{r} 215 \\ 140{,}000 \\ 231{,}000{,}000 \\ 10{,}000{,}000{,}000 \\ \underline{314{,}000{,}000{,}000{,}000} \end{array}$$

1. Practice reading aloud the following.

a. 18,310	e. 56,790,000	i. 3,613,497,002
b. 74,296	f. 600,000,000	j. 2,450,000,000
c. 156,392	g. 143,596,700	k. 1,075,010,000
d. 785,001	h. 4,821,601,325	l. 1,231,341,675,203

2. Write a numeral for each number.

a. fifty-four million four hundred fifty thousand two hundred six
b. twelve million three hundred fourteen thousand two hundred fifteen
c. four billion two million three hundred thousand one hundred eighteen
d. five billion three hundred fifteen million two hundred sixty thousand
e. twenty-two billion five hundred ten million
f. sixty-one billion seven hundred nine million two hundred three thousand
g. twenty trillion six billion seventy thousand
h. eighty-nine billion two thousand five hundred twenty-one
i. three hundred ten million six hundred thirty-one thousand ten
j. fifty-nine million three hundred sixty-nine thousand
k. six trillion nine hundred thousand
l. seventy-eight million five hundred fifteen thousand three hundred one
m. twenty-one million three hundred sixty-five thousand nine hundred
n. four million three hundred twenty-one thousand nine hundred
o. fifty-five billion fifty-nine thousand one hundred eleven
p. twelve trillion three hundred billion five hundred five thousand fifty

3. The population of the United States in 1940 was 131,669,275. In 1950 it was 150,697,361 and in 1960 it was 179,323,175.

a. What was the increase between 1940 and 1950?
b. What was the increase between 1950 and 1960?
c. What was the increase between 1940 and 1960?

1. The minimum and maximum distances of some of the planets from the sun are given in the chart below.

Planet	Maximum distance in miles	Minimum distance in miles
Venus	67,653,000	66,738,000
Earth	94,452,000	91,342,000
Mars	154,760,000	128,830,000
Saturn	935,570,000	836,700,000
Neptune	2,817,400,000	2,769,600,000
Pluto	4,600,000,000	2,760,000,000

   a. What is the difference in the maximum and minimum distances of Neptune from the sun?
   b. What is the difference in the maximum and minimum distances of Earth from the sun?
   c. How much farther does Saturn get from the sun than does Venus?
   d. How much farther does Pluto get from the sun than does Earth?

Find each sum or difference.

2. $\begin{array}{r} 864{,}275 \\ 156{,}381 \\ +\,427{,}926 \\ \hline \end{array}$

3. $\begin{array}{r} 1{,}236{,}455 \\ 4{,}521{,}308 \\ +\,6{,}072{,}983 \\ \hline \end{array}$

4. $\begin{array}{r} 4{,}118{,}236{,}400 \\ 2{,}012{,}045{,}615 \\ +\,1{,}421{,}110{,}023 \\ \hline \end{array}$

5. $\begin{array}{r} 872{,}204 \\ -\,158{,}196 \\ \hline \end{array}$

6. $\begin{array}{r} 8{,}040{,}154 \\ -\,2{,}136{,}289 \\ \hline \end{array}$

7. $\begin{array}{r} 6{,}034{,}662{,}041 \\ -\,1{,}428{,}540{,}192 \\ \hline \end{array}$

8. The areas in square miles of the four major oceans are: Pacific Ocean—63,801,668; Atlantic Ocean—31,839,306; Indian Ocean—28,356,276; and the Arctic Ocean—5,440,197.
   a. How much greater in area is the Pacific Ocean than the Atlantic Ocean?
   b. What is the total area in square miles of the four oceans?

1. A light-year is the distance light travels in a year. Light travels about 186,000 miles per second.

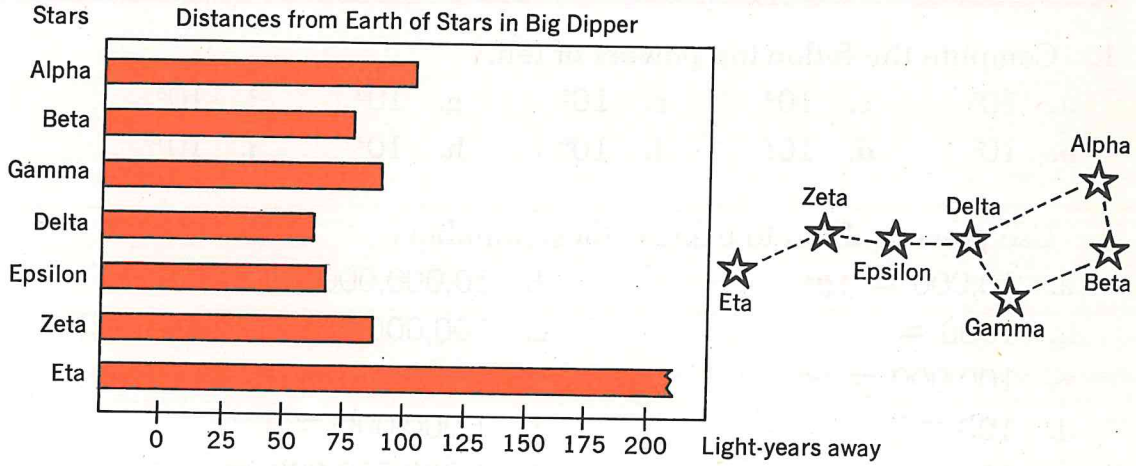
   a. Since there are 60 seconds in one minute, how far does light travel in one minute? Light travels about 60 × 186,000 or ■ miles in one minute.

   b. There are 60 minutes in one hour. Light travels about ■ miles in one hour.

   c. There are 24 hours in one day. Light travels about ■ miles in one day.

   d. There are 365 days in one year. Light travels about ■ miles in one year.

2. The stars of the Big Dipper have Greek letters for names. Although the stars all appear to be the same distance away, they actually are not. Their distances from Earth are shown in the graph below.



   a. Which is the most distant star?

   b. Is Alpha or Delta more distant?

   c. Are the stars in the Big Dipper more than 50 light-years away?

Find each product.

3. 186,265 × 3,600

4. 61,285 × 3,905

5. 8,261,452 × 8,090

Powers of Ten

Space engineers and scientists today must deal with numerals to represent very large quantities and very small quantities. They use a shorter notation making use of **powers** of **ten.**

$$10 = 10 = 10^1$$
$$100 = 10 \times 10 = 10^2$$
$$1000 = 10 \times 10 \times 10 = 10^3$$
$$10000 = 10 \times 10 \times 10 \times 10 = 10^4$$
$$100000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5$$

We read 10^2 as "10 to the second power" or "10 squared".
We read 10^3 as "10 to the third power" or "10 cubed".
We read 10^4 as "10 to the fourth power".
We read 10^5 as "10 to the fifth power".

1. Compute the following powers of ten.

a.	10^1	**c.**	10^4	**e.**	10^5	**g.**	10^2	**i.**	10^8
b.	10^3	**d.**	10^7	**f.**	10^9	**h.**	10^6	**j.**	10^{10}

2. Use powers of ten to express these numbers.

a. $10{,}000 = 10^4$
b. $1000 =$
c. $100{,}000 =$
d. $100 =$
e. $10 =$
f. $10{,}000{,}000{,}000 =$
g. $100{,}000{,}000{,}000{,}000 =$
h. $10{,}000{,}000 =$
i. $1{,}000{,}000 =$
j. $1{,}000{,}000{,}000 =$

3. Count the zeros in each numeral in Exercise 2. How does the number of zeros compare to the power of ten?

4. How do we read each of the following powers of ten?

a. 10^2 **b.** 10^4 **c.** 10^6 **d.** 10^8

We may express other numbers using powers of ten.

$$600 = 6 \times 100 = 6 \times 10^2$$
$$17{,}000{,}000 = 17 \times 1{,}000{,}000 = 17 \times 10^6$$

1. Express each number as a power of ten.

a. 40,000	f. sixty thousand	k. 300,000,000
b. 9,000,000	g. 8,000,000	l. twenty
c. 30	h. 70	m. 400,000
d. 500	i. forty billion	n. nine trillion
e. 20,000,000	j. 900,000	o. 70,000

2. The average distance from Earth to the moon is about 240,000 miles. This is $24 \times 10^{\blacksquare}$ miles.

3. The average distance between Earth and the sun is about 93,000,000 miles. This is $93 \times 10^{\blacksquare}$ miles.

4. Express each numeral in expanded notation using powers of ten. Follow the example.

$$57{,}431 = 50{,}000 + 7000 + 400 + 30 + 1$$
$$= (5 \times 10^4) + (7 \times 10^3) + (4 \times 10^2) + (3 \times 10^1) + 1$$

a. 4753	e. 26,980	i. 792,604
b. 9860	f. 17,000	j. 1,308,200
c. 8071	g. 88,888	k. 6,345,120,000
d. 5311	h. 79,406	l. 28,100,456,201

5. Write a decimal numeral for each expanded numeral.

a. $(6 \times 10^4) + (5 \times 10^3) + (4 \times 10^1) =$ 65,040

b. $(2 \times 10^8) + (6 \times 10^2) + (5 \times 10^1) + 6 =$ ■

c. $(7 \times 10^5) + (4 \times 10^3) + (3 \times 10^2) + 2 =$ ■

d. $(8 \times 10^{10}) + (3 \times 10^8) + (4 \times 10^4) + (2 \times 10^2) =$ ■

e. $(2 \times 10^7) + (5 \times 10^4) + (1 \times 10^3) + 6 =$ ■

Estimation

We have estimated a number to the nearest multiple of ten by looking at the digit in the ones' place. If this last digit was 1, 2, 3, or 4 we thought of it as a 0. If this last digit was 5, 6, 7, 8, or 9 we thought of it as a 0 and added 1 ten to the tens shown in the tens' place.

We know: 342 is 340 to the nearest multiple of ten.
126 is 130 to the nearest multiple of ten.

To estimate to the nearest multiple of a hundred, we look at the digit in the tens' place.

We think: 841 is 800 to the nearest multiple of a hundred.
863 is 900 to the nearest multiple of a hundred.
17,352 is 17,400 to the nearest multiple of a hundred.

To estimate to the nearest multiple of a thousand, look at the digit in the hundreds' place.

We think: 41,732 is 42,000 to the nearest multiple of a thousand.
9,754,586 is 9,755,000 to the nearest multiple of a thousand.

1. Estimate each number to the nearest multiple of ten.

a. 871	**d.** 6598	**g.** 2222	**j.** 93	**m.** 5,761,397
b. 9874	**e.** 2040	**h.** 1825	**k.** 96	**n.** 40,995
c. 507	**f.** 8933	**i.** 3949	**l.** 99	**o.** 99,999

2. Estimate each number to the nearest multiple of a hundred.

a. 8,700,001	**e.** 1,000,400	**i.** 4,197,500
b. 21,936,850	**f.** 3,851,996	**j.** 111,111,111,111
c. 601,347,395	**g.** 611,309,861	**k.** 83,094,720
d. 238,197,777	**h.** 27,634,912	**l.** 999,999,999

3. Estimate each number in Exercise 2 to the nearest multiple of a thousand.

Exponents

We use an **exponent** to show the number of times that a factor is to be repeated. The exponent 5 in 10^5 shows that the factor 10 is to be repeated 5 times. It tells us the power.

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10$$

1. Write each power as a product.

a. 10^6	c. 4^7	e. 16^2	g. 4^3	i. 1^6
b. 3^7	d. 7^4	f. 11^5	h. 2^2	j. 8^5

2. Write each product as a power.

a. $4 \times 4 \times 4 \times 4 \times 4$	e. $1 \times 1 \times 1 \times 1$	i. $5 \times 5 \times 5 \times 5$
b. 3×3	f. 12×12	j. $17 \times 17 \times 17$
c. $10 \times 10 \times 10 \times 10$	g. $8 \times 8 \times 8$	k. 24×24
d. $7 \times 7 \times 7$	h. 100×100	l. $25 \times 25 \times 25$

The number p^m has
the number p as the **base**
and the number m as the exponent.
3^5 has base 3 and exponent 5.

3. Name the base and the exponent in each expression.

a. 2^7	d. 5^3	g. 25^9	j. 27^2	m. 16^1
b. 9^2	e. 7^1	h. 6^2	k. 10^6	n. 1^{19}
c. 4^3	f. 1^8	i. 4^3	l. 100^3	o. 2^{10}

4. Nine is a square number because $9 = 3 \times 3$. Ten is not a square number because no whole number multiplied by itself equals ten. Why is each of these numbers a square number?

a. 25	c. 100	e. 1	g. 81	i. 144	k. 0
b. 4	d. 9	f. 64	h. 36	j. 121	l. 16

We can compute a power of any number.

$3^3 = 3 \times 3 \times 3$
$= 3 \times 3^2$
$= 3 \times 9$
$= 27$

$3^4 = 3 \times 3 \times 3 \times 3$
$= 3 \times 3^3$
$= 3 \times 27$
$= 81$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$
$= 3 \times 3^4$
$= 3 \times 81$
$= 243$

1. Compute each power.

a.	3^6	**f.**	4^2	**k.**	1^3	**p.**	6^3	**u.**	10^3
b.	2^3	**g.**	4^3	**l.**	1^4	**q.**	6^4	**v.**	10^4
c.	2^4	**h.**	4^4	**m.**	1^5	**r.**	9^3	**w.**	10^5
d.	2^5	**i.**	5^3	**n.**	1^{19}	**s.**	7^3	**x.**	10^6
e.	2^6	**j.**	5^4	**o.**	8^3	**t.**	10^2	**y.**	10^7

2. Compute each product of powers.

$2^3 \times 3^2 = 8 \times 9$
$= 72$

a.	$9^1 \times 9^1$	**g.**	$13^2 \times 3^1$	**m.**	$3^3 \times 10^6$	**s.**	$2^1 \times 3^1 \times 5^1$
b.	$9^1 \times 9^2$	**h.**	$4^1 \times 4^2$	**n.**	$43^1 \times 2^2$	**t.**	$3^4 \times 10^2 \times 17^1$
c.	$2^4 \times 3^1$	**i.**	$7^1 \times 5^1$	**o.**	$1^{10} \times 5^2$	**u.**	$4^2 \times 1^5 \times 3^2$
d.	$10^3 \times 7^2$	**j.**	$3^2 \times 2^2$	**p.**	$2^3 \times 7^1$	**v.**	$7^1 \times 29^1 \times 13^1$
e.	$29^1 \times 2^1$	**k.**	$10^6 \times 7^4$	**q.**	$3^3 \times 3^2$	**w.**	$5^2 \times 2^2 \times 10^3$
f.	$2^3 \times 2^2$	**l.**	$5^3 \times 2^3$	**r.**	$10^4 \times 8^2$	**x.**	$43^1 \times 2^1 \times 1^4$

3. a. Mary has one father and one mother. How many parents is this?

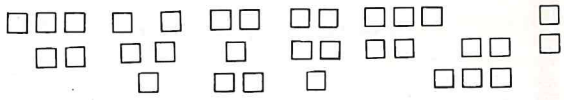
b. Mary's parents each have 2 parents. How many grandparents is this?

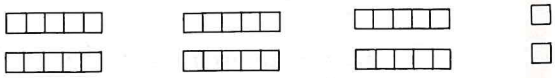
c. Each grandparent has 2 parents. How many great-grandparents is this?

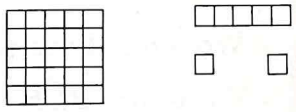
d. Each great-grandparent has 2 parents. How many great-great-grandparents is this?

Base Five

We can use any number for a base and group by powers of that number.
We usually group by tens.
However, we sometimes group by other bases, for example five.

We have a number of blocks 

First we group them by fives. 

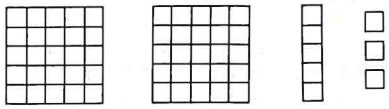
Then we group by 5 fives or twenty-fives. 

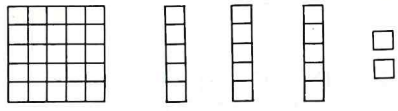
We have 1 twenty-five 1 five 2 ones.

We write: 112_{five} $112_{five} = (1 \times 5^2) + (1 \times 5^1) + 2$

We read "112_{five}" as "one one two to the base five".

1. What numeral to the base five represents each number of blocks?

a. 

b. 

2. Give the expanded notation in powers of five.

$2134_{five} = (2 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + 4$

a. 10_{five}
b. 100_{five}
c. 1000_{five}
d. 1342_{five}
e. 2144_{five}
f. 4231_{five}

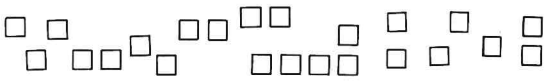
3. Group blocks in ones, fives, and twenty-fives to find the numeral in base five for each of these numbers.

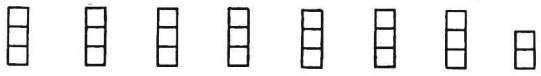
a. 6
b. 18
c. 41
d. 58
e. 38
f. 102
g. 84
h. 63

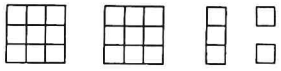
4. Which digits are used in base five numerals?

Base Three

We can group by powers of three.

We have a number of blocks. 

First we group them by threes. 

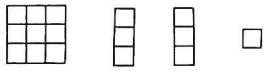
Then we group by 3 threes or nines. 

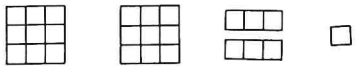
We have 2 nines, 1 three, and 2 ones.

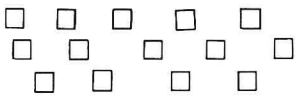
We write: 212_{three} $212_{three} = (2 \times 3^2) + (1 \times 3) + 2$

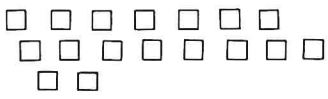
We read "212_{three}" as "two one two to the base three".

1. What numeral to the base three represents each number of blocks?

a. 

b. 

c. 

d. 

2. Give the expanded notation in powers of three.

$$2110_{three} = (2 \times 3^3) + (1 \times 3^2) + (1 \times 3) + 0$$

a. 10_{three}
b. 100_{three}
c. 1000_{three}
d. 211_{three}
e. 12_{three}
f. 120_{three}
g. 1200_{three}
h. 2211_{three}

3. Group in ones, threes, nines, and twenty-sevens to find the numeral in base three for each number.

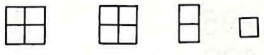
a. 6
b. 11
c. 18
d. 25
e. 30
f. 41
g. 56
h. 65

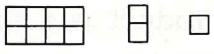
Base Two

We can group by powers of two.

We have a number of blocks.

First we group by twos.

Then we group by 2 twos or fours. 

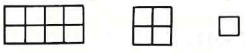
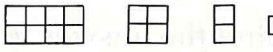
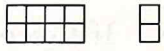
Then we group by 2 fours or eights. 

We have 1 eight 0 fours 1 two 1 one.

We write: 1011_{two} $1011_{two} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2) + 1$

We read "1011_{two}" as "one zero one one to the base two".

1. What numeral to the base two represents each number of blocks?

a.  **b.**  **c.** 

2. Give the expanded notation in powers of two.

$10111_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2) + 1$

a. 10_{two} **c.** 1000_{two} **e.** 1101_{two}

b. 100_{two} **d.** 10000_{two} **f.** 11110_{two}

3. Group by ones, twos, fours, eights, and sixteens to find the base-two numeral for each of these numbers.

a. 7 **c.** 16 **e.** 21 **g.** 28

b. 13 **d.** 9 **f.** 17 **h.** 29

Feedback

What number does each numeral name?

1.
2.
3.
4.
5. XXIV
6. LX
7. XC
8. MDCX
9. 11_{two}
10. 11_{three}
11. 11_{five}
12. 24_{five}

Find each sum or difference.

13. $\begin{array}{r} 846{,}295 \\ +\,197{,}408 \\ \hline \end{array}$
14. $\begin{array}{r} 286{,}975 \\ -\,149{,}298 \\ \hline \end{array}$
15. $\begin{array}{r} 7{,}469{,}004 \\ -\,2{,}327{,}969 \\ \hline \end{array}$

Write each product as a power.

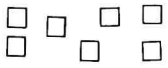
16. $4 \times 4 \times 4 \times 4 \times 4$
17. $3 \times 3 \times 3 \times 3$
18. $7 \times 7 \times 7$
19. $8 \times 8 \times 8 \times 8$

Compute each power.

20. 7^2
21. 9^3
22. 34^2
23. 10^3
24. 8^1
25. 2^5
26. 5^3
27. 48^2

28. If Joyce has 3 aunts, find the answer to each question.
    a. Each aunt has 3 children. How many cousins does Joyce have?
    b. Every cousin has 3 dogs. How many dogs do they have all together?
    c. Every dog has 3 puppies. How many puppies are there all together?
    d. Every puppy has 3 toys. How many toys do the puppies have all together?

29. John has the blocks pictured at the right.

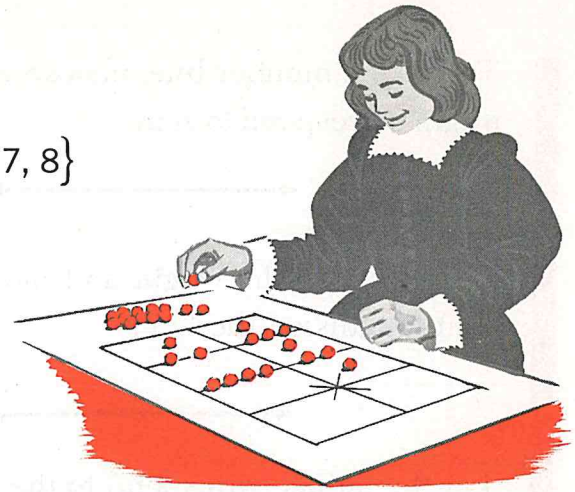


    a. If he groups them by fives, he has ___ five and ___ ones which he writes ___$_{five}$.
    b. If he groups by threes, he has ___ threes and ___ one which he writes ___$_{three}$.

Feedback

Choose $\cup$, $-$, $\cap$, $\subset$, or $\in$.

1. $\{1, 4, 7\}$ ● $\{2, 4, 8\} = \{4\}$
2. $\{1, 4, 7\}$ ● $\{2, 4, 8\} = \{1, 2, 4, 7, 8\}$
3. $\{1, 4, 7\}$ ● $\{2, 4, 8\} = \{1, 7\}$
4. 4 ● $\{1, 2, 4, 7, 8\}$
5. $\{4\}$ ● $\{1, 2, 4, 7, 8\}$



Find each product.

6. $\begin{array}{r} 1846 \\ \times\ 7 \\ \hline \end{array}$
7. $\begin{array}{r} 2305 \\ \times\ 9 \\ \hline \end{array}$
8. $\begin{array}{r} 865 \\ \times\ 48 \\ \hline \end{array}$
9. $\begin{array}{r} 967 \\ \times 309 \\ \hline \end{array}$

Find each quotient.

10. $8\overline{)1853}$
11. $23\overline{)5984}$
12. $46\overline{)2748}$
13. $60\overline{)56432}$

Complete each problem.

14. $\frac{2}{3} = \frac{\square}{12}$
15. $\frac{5}{8} = \frac{\square}{16}$
16. $\frac{9}{12} = \frac{\square}{4}$
17. $\frac{1}{2} + \frac{1}{4} = \square$
18. $\frac{3}{5} + \frac{1}{10} = \square$
19. $\frac{1}{2} + \frac{1}{3} = \square$
20. $\frac{1}{2} - \frac{1}{4} = \square$
21. $\frac{2}{3} - \frac{5}{12} = \square$
22. $\frac{7}{9} - \frac{2}{3} = \square$

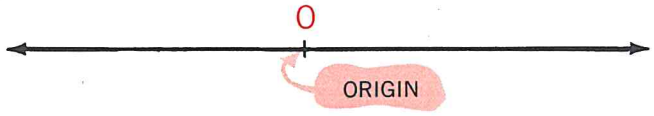
23. The traveling salesman drove 10,080 miles in 45 days. What was the average number of miles he drove each day?

24. Mrs. Davidson bought groceries which cost $8.17. How much change should she receive from a ten-dollar bill?

25. Mr. Gorski had 300 cans of tuna fish in his store. He sold 128 cans. How many cans did he have left?

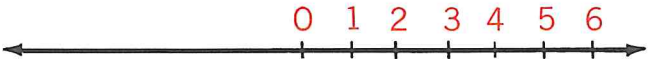
Part 11 Integers

To make a number line, choose a point of the line as the **origin.** Let the origin correspond to zero.

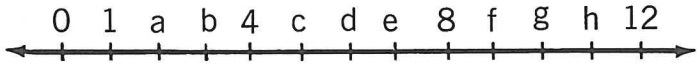


Beginning at the origin and moving to the right, mark other points at unit lengths on the line.

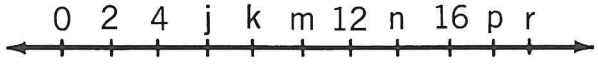
Let the points correspond to the **whole numbers.**



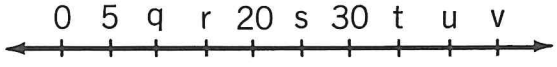
Each point marked on the number lines below corresponds to a number. Some of the numbers are represented by letters. What number does each letter represent?

1. 

a. a = ▢ **c.** c = ▢ **e.** e = ▢ **g.** g = ▢
b. b = ▢ **d.** d = ▢ **f.** f = ▢ **h.** h = ▢

2. 

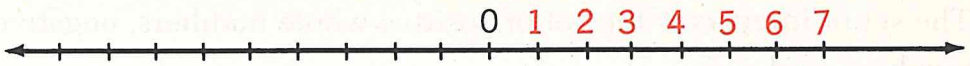
a. j = ▢ **c.** m = ▢ **e.** p = ▢
b. k = ▢ **d.** n = ▢ **f.** r = ▢

3. 

a. q = ▢ **c.** s = ▢ **e.** u = ▢
b. r = ▢ **d.** t = ▢ **f.** v = ▢

Positive and Negative Numbers

Points to the right of the origin correspond to numbers greater than zero.

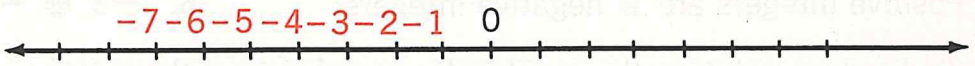


The numbers greater than zero are **positive numbers.**
On a number line a number corresponding to a point on the right is greater than a number corresponding to a point on the left.

Choose < or > .

1. **a.** 0 ● 5 **c.** 1 ● 0 **e.** 8 ● 17 **g.** 2 ● 1 **i.** 18 ● 8
b. 7 ● 3 **d.** 10 ● 11 **f.** 13 ● 0 **h.** 10 ● 3 **j.** 0 ● 5

Points to the left of the origin correspond to numbers less than zero.



The numbers less than zero are **negative numbers.** We read "−2" as "negative two."
On a number line a number corresponding to a point to the left is less than a number corresponding to a point to the right.

Choose < or >. Look at the number line to help you.

2. **a.** −4 ● 0
b. −10 ● −6
c. −2 ● −7
d. 0 ● 2
e. 0 ● −2
f. 2 ● −2

3. **a.** 0 ● −3
b. −10 ● −11
c. −5 ● −14
d. −4 ● 6
e. −1 ● −8
f. 6 ● 10

4. **a.** −9 ● −15
b. −12 ● −2
c. −8 ● 0
d. 7 ● −2
e. −8 ● 8
f. 3 ● −1

Integers

Zero is neither positive nor negative.

The set of **integers** is the set of positive whole numbers, negative whole numbers, and zero.

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

We speak of positive integers, negative integers, and zero. We sometimes speak of the non-negative integers. The **non-negative** integers include the positive integers and zero.

Choose $<$ or $>$.

1. Negative integers are ● zero.
2. Positive integers are ● zero.
3. Positive integers are ● negative integers.
4. -4 ● 3
5. 0 ● -6
6. -3 ● -2

7. To locate a point on the number line, we begin at the origin and go to the left or to the right a given number of units. Tell how to locate each of the following points.

   To locate -4, go four units to the left of the origin.
   To locate 4, go four units to the right of the origin.

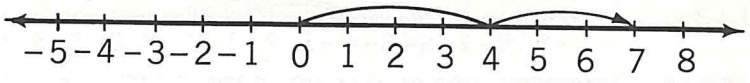
   a. 8 c. 10 e. 7 g. -11
   b. -8 d. 6 f. -7 h. -6

8. a. What numbers correspond to points 4 units from the origin?
   b. What numbers correspond to points 7 units from the origin?

9. a. Numbers corresponding to points to the right of the origin are ■ numbers. (positive, negative)
   b. Numbers corresponding to points to the left of the origin are ■ numbers. (positive, negative)
   c. What number is neither positive nor negative?

Addition of Integers

We can use the number line to show the addition of positive numbers.

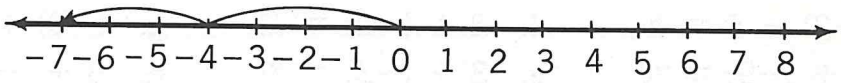


$4 + 3 = 7$

1. Make a number line for each problem.

a. $2 + 4 =$ ■ **b.** $6 + 3 =$ ■ **c.** $3 + 4 =$ ■

We can use the number line to show the addition of negative numbers.



$(-4) + (-3) = -7$

2. Make a number line for each problem.

a. $(-4) + (-3) =$ ■ **c.** $(-3) + (-1) =$ ■ **e.** $(-1) + (-5) =$ ■

b. $(-2) + (-5) =$ ■ **d.** $(-4) + (-2) =$ ■ **f.** $(-4) + (-4) =$ ■

3. Add. Use a number line if you need help.

a. $(-4) + (-2) =$ ■ **c.** $(-7) + (-4) =$ ■ **e.** $8 + 3 =$ ■

b. $(-8) + (-5) =$ ■ **d.** $6 + 7 =$ ■ **f.** $(-6) + (-3) =$ ■

4. Make a number line and follow the directions for each problem. Then write an addition sentence.

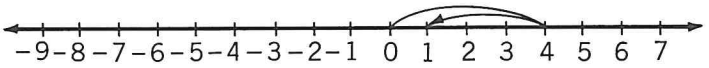
a. Start at the origin, move 5 units to the left, and then 11 more units to the left.

b. Start at the origin, move 8 units to the right, and then 12 more units to the right.

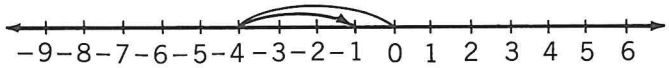
c. Start at the origin, move 7 units to the right, and then 9 more units to the right.

We can use the number line to show addition of integers.

$4 + (-3) = 1$



$(-4) + 3 = -1$



1. Make a number line for each problem.

a. $(-7) + 5 =$ ▢	d. $(-6) + 4 =$ ▢	g. $7 + (-9) =$ ▢
b. $4 + (-9) =$ ▢	e. $3 + (-2) =$ ▢	h. $(-10) + 4 =$ ▢
c. $(-3) + 8 =$ ▢	f. $8 + (-5) =$ ▢	i. $(-9) + 2 =$ ▢

2. Add. Use a number line if you need help.

a. $(-2) + 5 =$ ▢	d. $2 + (-2) =$ ▢	g. $(-9) + 9 =$ ▢
b. $6 + (-9) =$ ▢	e. $3 + (-3) =$ ▢	h. $8 + (-9) =$ ▢
c. $4 + (-4) =$ ▢	f. $(-4) + 5 =$ ▢	i. $6 + (-7) =$ ▢

3. Make a number line for each pair of problems.
With a red pencil, show the addition for the first problem of the pair.
With a black pencil, show the addition for the second problem of the pair.
Compare the answers. Are they the same?

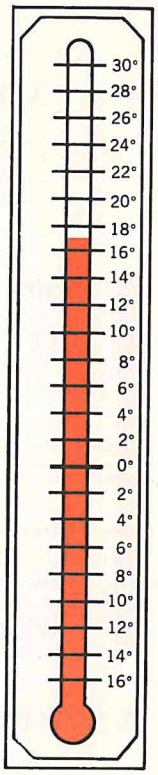
a. $4 + 6 =$ ▢ $6 + 4 =$ ▢	c. $(-5) + 3 =$ ▢ $3 + (-5) =$ ▢
b. $(-5) + (-3) =$ ▢ $(-3) + (-5) =$ ▢	d. $8 + (-2) =$ ▢ $(-2) + 8 =$ ▢

4. Make a number line and use it to check your sums.

a. $5 + (-8) =$ ▢	e. $8 + 3 =$ ▢	i. $0 + (-6) =$ ▢
b. $(-4) + (-2) =$ ▢	f. $(-3) + 8 =$ ▢	j. $8 + 0 =$ ▢
c. $9 + (-4) =$ ▢	g. $7 + (-4) =$ ▢	k. $(-5) + 5 =$ ▢
d. $(-11) + 6 =$ ▢	h. $(-6) + (-2) =$ ▢	l. $(-4) + 9 =$ ▢

Using Integers

The scale on a thermometer is a kind of number line.
Temperatures below zero correspond to negative numbers.
Temperatures above zero correspond to positive numbers.



Copy and complete each chart. Read across.

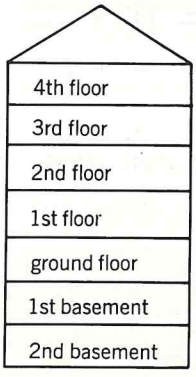
1.

Temp. at noon	Degrees temp. dropped between noon and midnight	Temp. at midnight
−7°	8°	
4°	12°	
−10°	6°	
−3°		−8°
−2°		−6°
	8°	−12°
	4°	−1°

2.

Temp. at 6 A.M.	Degrees temp. rose between 6 A.M. and noon	Temp. at noon
−4°	4°	
−6°	14°	
9°	5°	
−5°		2°
1°		6°
	5°	−8°
	3°	3°

3. In France, the floors of a building are numbered differently from those in the United States. The ground floor corresponds to 0, and the floors above the ground floor are numbered 1, 2, 3, and so on. The basements are called the first basement, second basement, and so on. The floors correspond to a number line.



a. How many flights of stairs will a person climb to get from the 1st basement to the 4th floor?

b. How many flights of stairs will a person go down to get from the 3rd floor to the ground floor?

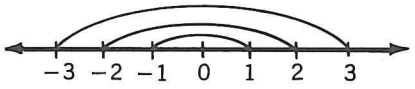
c. How many flights of stairs will a person climb to get from the 2nd basement to the 2nd floor?

d. How many flights of stairs will a person go down to get from the 4th floor to the 2nd basement?

Additive Inverse

1. Use a number line to show the addition of each of these pairs of numbers.
   a. $3 + (-3) =$ ▢
   b. $(-2) + 2 =$ ▢
   c. $(-6) + 6 =$ ▢
   d. $4 + (-4) =$ ▢
   e. $0 + 0 =$ ▢
   f. $1 + (-1) =$ ▢
   g. Did all these problems have the same answer?

Pairs of numbers like 3 and -3 or -2 and 2 are additive inverses.
For any number n, $n + (-n) = 0$.



The sum of any number and its additive inverse is zero.
The additive inverse of 6 is -6; $6 + (-6) = 0$.
The additive inverse of -3 is 3; $(-3) + 3 = 0$.

2. Give the additive inverse of each number.

a. 5	d. 12	g. -75	j. 23
b. -13	e. -11	h. 50	k. -31
c. -10	f. 101	i. -63	l. 40

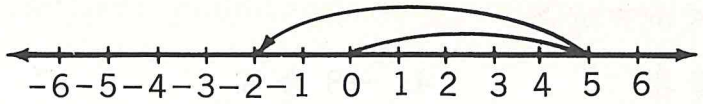
3. Complete each problem.

a. $(-12) +$ ▢ $= 0$	d. ▢ $+ 4 = 0$	g. $0 = (-3) +$ ▢
b. $6 +$ ▢ $= 0$	e. $(-7) +$ ▢ $= 0$	h. $0 = 16 +$ ▢
c. ▢ $+ (-8) = 0$	f. ▢ $+ 0 = 0$	i. $0 =$ ▢ $+ 2$

4. Scott's football team gained eight yards on its first play. On its second play it lost eight yards.
   a. What was the total yardage gained in the two plays?
   b. A loss of eight yards could be shown as ▢ yards.
   c. Write an equation for the yardage made in the two plays.

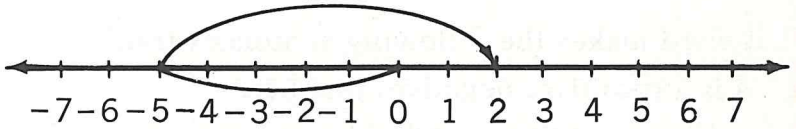
We can use a number line to find a missing addend.

$5 + \square = -2$



Begin at 0. Move to 5. Next move from 5 to -2. The answer is -7. Notice that from 5 you moved 7 spaces in a negative direction.

$(-5) + \square = 2$



Begin at 0. Move to -5. Next move from -5 to 2. The answer is 7. Notice that from -5 you moved 7 spaces in a positive direction.

Use a number line to find the missing addend. Then complete the mathematical sentence.

1. $5 + \square = 7$
2. $(-3) + \square = 2$
3. $5 + \square = -3$
4. $6 + \square = 8$
5. $(-3) + \square = -2$
6. $\square + (-4) = -6$

Make a number line. Use it to check your answers to these problems.

7. $3 + \square = -5$
8. $(-6) + \square = 10$
9. $\square + (-5) = 0$
10. $(-7) + (-3) = \square$
11. $0 + \square = -4$
12. $\square + (-6) = 6$
13. $8 + (-5) = \square$
14. $6 + \square = 2$
15. $7 + (-3) = \square$
16. $\square + 4 = 3$
17. $(-5) + 4 = \square$
18. $\square + (-2) = 5$

19. Tim had a score of -4. Bob had a score of 7. Bob scored how many more points than Tim?

20. The temperature yesterday was 5°F. Today the temperature is -3°F. How many degrees drop in temperature was there?

Feedback

Choose $<$ or $=$ or $>$.

1. $7 \bullet 4$
2. $0 \bullet \frac{2}{3}$
3. $-2 \bullet 0$
4. $-8 \bullet 6$
5. $-4 \bullet -6$
6. $\frac{4}{10} \bullet \frac{5}{10}$
7. $-6 \bullet 8$
8. $\frac{7}{100} \bullet \frac{9}{100}$
9. $2 \bullet 0$

What word makes the following sentences true?

10. 4 is a (positive, negative) number.

11. -4 is a (positive, negative) number.

12. What number is neither positive nor negative?

13. The set of positive whole numbers, and their additive inverses, and zero is the set of ■.

Find each sum.

14. $(-4) + (-3) =$ ■

15. $(-2) + (+5) =$ ■

16. $9 + 4 =$ ■

17. $8 + (-5) =$ ■

18. $(-6) + 7 =$ ■

19. $0 + (-4) =$ ■

20. $(-8) + (-7) =$ ■

21. $3 + 0 =$ ■

22. $9 + (-2) =$ ■

23. $(-5) + 0 =$ ■

24. Give the additive inverse of each of these numbers.

a. 6 **b.** -4 **c.** 0 **d.** 32

25. Terry had a score of -3 and Bill had a score of 8. If they were partners in a game, what was their total score?

26. The temperature today is 4°F. Yesterday the temperature was -2°F. How many degrees did the temperature rise?

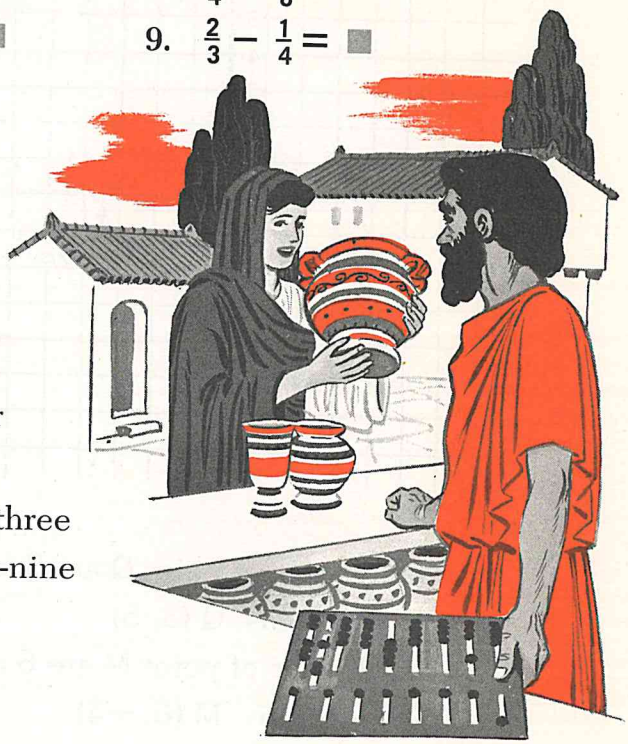
Feedback

Complete each problem.

1. $\frac{4}{9} = \frac{\square}{18}$
2. $\frac{4}{10} = \frac{\square}{5}$
3. $\frac{3}{5} = \frac{\square}{100}$
4. $\frac{2}{9} + \frac{1}{3} = \square$
5. $\frac{5}{12} + \frac{1}{4} = \square$
6. $\frac{5}{16} + \frac{3}{4} = \square$
7. $\frac{3}{5} - \frac{4}{15} = \square$
8. $\frac{3}{4} - \frac{5}{8} = \square$
9. $\frac{2}{3} - \frac{1}{4} = \square$

Find each quotient.

10. $3\overline{)742}$
11. $40\overline{)1749}$
12. $83\overline{)9146}$
13. $29\overline{)6452}$



Write the Arabic numeral for each number described.

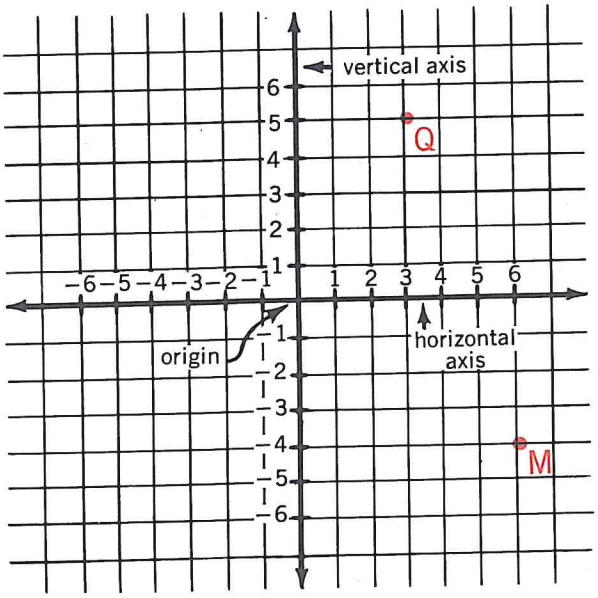
15. forty-nine thousand six hundred fifty-three
16. seventy thousand five hundred eighty-nine
17. ninety-eight thousand seven hundred
18. sixty-seven thousand nine

19. How many sides does each of the following figures have?
    a. rectangle
    b. square
    c. pentagon
    d. equilateral triangle
    e. isosceles triangle
    f. quadrilateral

20. Dick has 6 yards of fishline, Jim has 19 feet, and Terry has 210 inches. Which boy has the most?

21. The bottling machine can fill 24,138 bottles in 9 hours. How many bottles can it fill in one hour?

Part 12 A Coordinate System



Points can be located using a **coordinate system.**
A point is located by giving the two numbers which are its **coordinates.**
The horizontal coordinate is measured on the **horizontal axis.**
The vertical coordinate is measured on the **vertical axis.**

The coordinates of point Q are 3 and 5.
We write this: Q (3, 5)
The coordinates of point M are 6 and −4.
We write this: M (6, −4)

We write the horizontal coordinate first.

1. Name the horizontal coordinate in each pair of coordinates.

a. (5, 2)	**c.** (6, 7)	**e.** (8, −2)	**g.** (6, 0)
b. (0, −1)	**d.** (−3, 6)	**f.** (1, 6)	**h.** (−4, −3)

2. Name the vertical coordinate in each pair of coordinates.

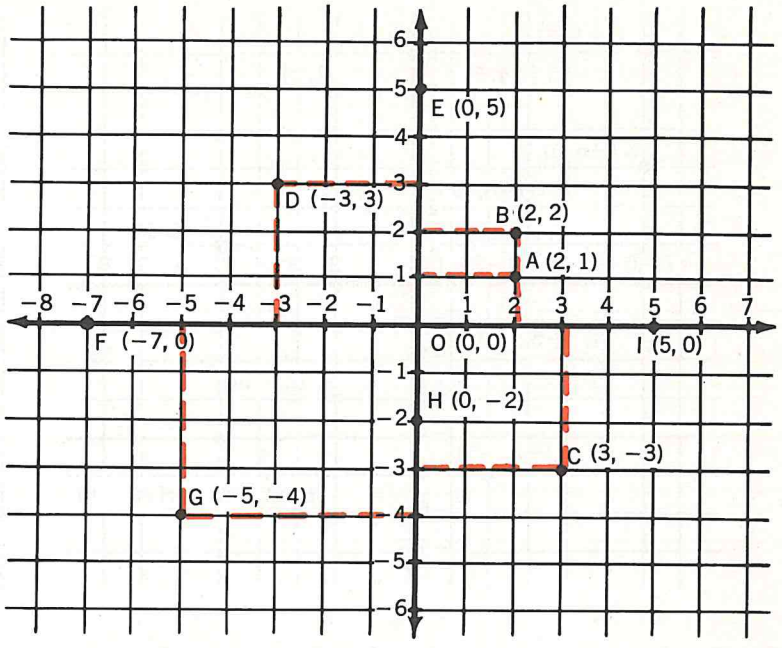
a. (3, 5)	**c.** (6, 2)	**e.** (3, −5)	**g.** (0, 2)
b. (−2, −5)	**d.** (−4, 0)	**f.** (2, 9)	**h.** (6, −3)

3. Do (0, 4) and (4, 0) have the same horizontal coordinate?

Points on a Coordinate System

1. Copy the table. Fill in the coordinates of each point shown.

point	horizontal coordinate	vertical coordinate
A	2	1
B		2
C		−3
D	−3	
E	0	
F		0
G		
H		
I		



2. a. What is the name of the point where the axes intersect?
   b. What are the coordinates of this point?

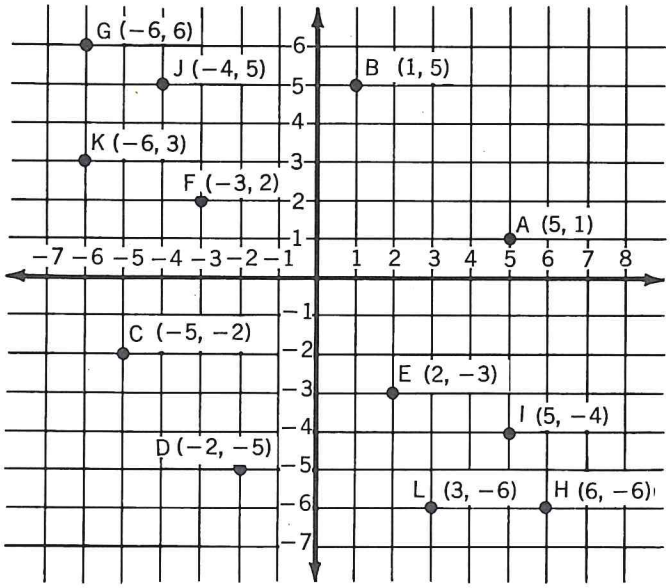
3. a. Which of the above points have zero as the horizontal coordinate?
   b. On which axis do they lie?

4. a. Which of the above points have zero as the vertical coordinate?
   b. On which axis do they lie?

5. Choose **left** or **right.**
   a. If the horizontal coordinate of a point is a positive number, the point lies to the ___ of the vertical axis.
   b. If the horizontal coordinate of a point is a negative number, the point lies to the ___ of the vertical axis.

Ordered Pairs

1. Copy the table and fill in the coordinates for each point.



point	horizontal coordinate	vertical coordinate
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		

2. Is the point given by the first pair of coordinates the same as the point given by the second pair of coordinates?

   a. (5, 1) (1, 5) c. (−3, 2) (−3, 2) e. (−6, 3) (3, −6)

   b. (−6, 6) (6, −6) d. (−5, −2) (−2, −5) f. (5, −4) (5, −4)

3. a. Does it matter in which order we give the pair of numbers?

   b. In a coordinate system is the point (2, 3) the same as the point (3, 2)?

We call (x, y) an **ordered pair** because it matters in which order we give the numbers x and y.

4. For each problem, tell whether the ordered pairs give the same point.

   a. (3, −2) (−2, 3) c. (0, 3) (3, 0) e. (−4, −4) (−4, −4)

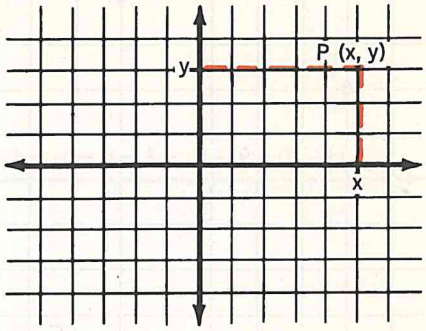
   b. (4, 1) (4, −1) d. (7, −6) (−6, 7) f. (2, 5) (−2, −5)

Finding Ordered Pairs

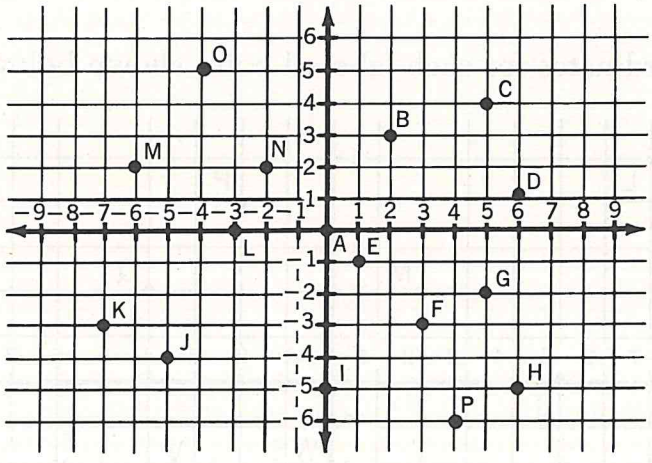
Given a point P, we can find the ordered pair (x, y) which locates it.

The first coordinate, x, is given by the number assigned to the point on the horizontal axis nearest P.

The second coordinate, y, is given by the number assigned to the point on the vertical axis nearest P.



1. Jim found the coordinates of the points on the coordinate system shown below. Sometimes he made mistakes and gave the coordinates in the wrong order.

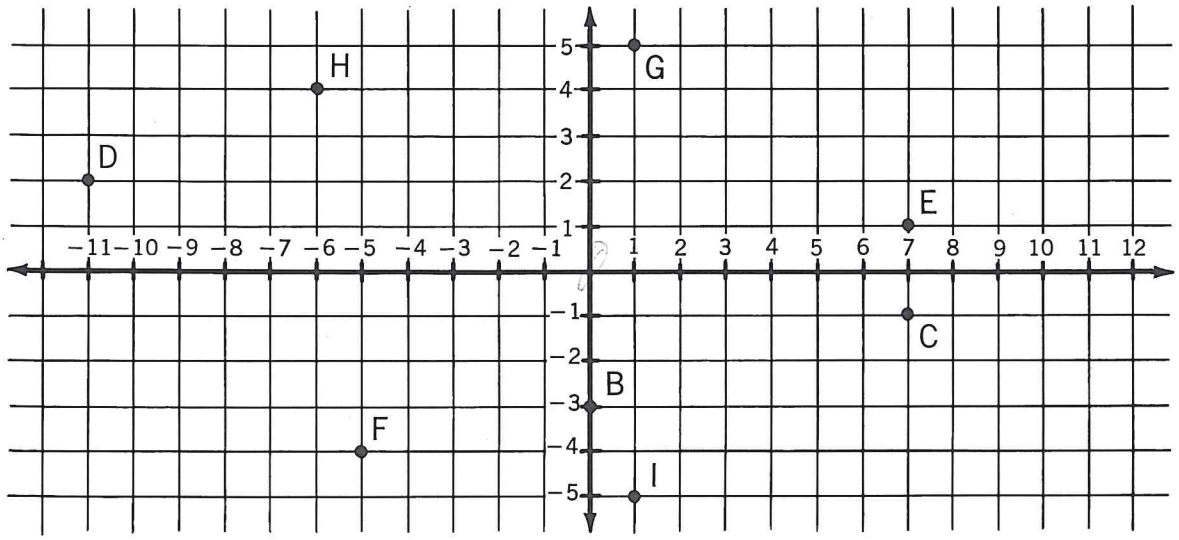


a. For which points did he make mistakes?

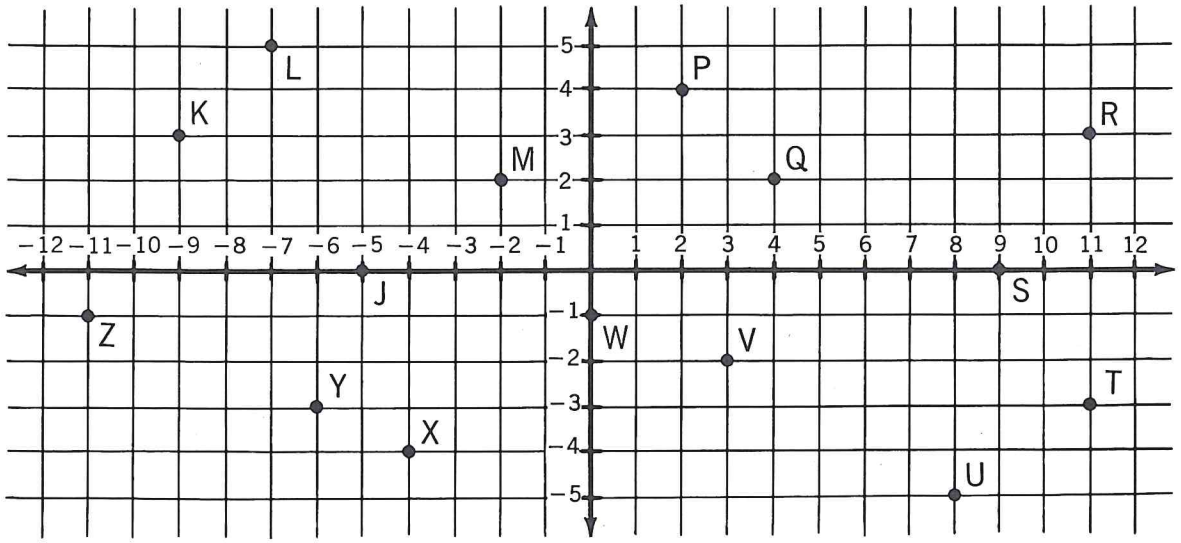
A (0, 0)	E (−1, 1)	I (0, −5)	M (2, −6)
B (2, 3)	F (3, −3)	J (−4, −5)	N (2, −2)
C (4, 5)	G (−2, 5)	K (−7, −3)	O (−4, 5)
D (1, 6)	H (6, −5)	L (−3, 0)	P (4, −6)

b. Write the correct coordinates for the points for which Jim made mistakes.

1. Give the coordinates for each labeled point shown below.



2. Give the coordinates for each labeled point shown below.

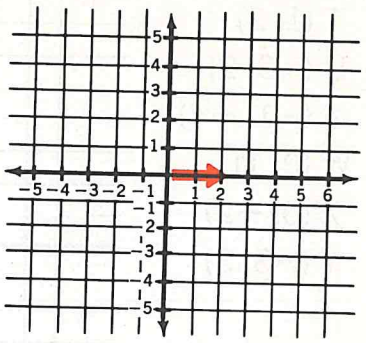


3. In problems 1 and 2 which points fall on the vertical axis?
4. In problems 1 and 2 which points fall on the horizontal axis?

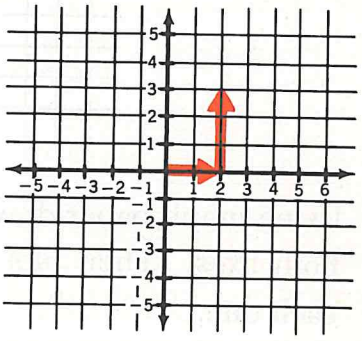
Plotting Points

Given an ordered pair (2, 3), we can plot the point P that corresponds to it.

Step 1 Count 2 units to the right of the origin on the horizontal axis.



Step 2 Count up 3 units.



Choose **left** or **right** to fill each blank.

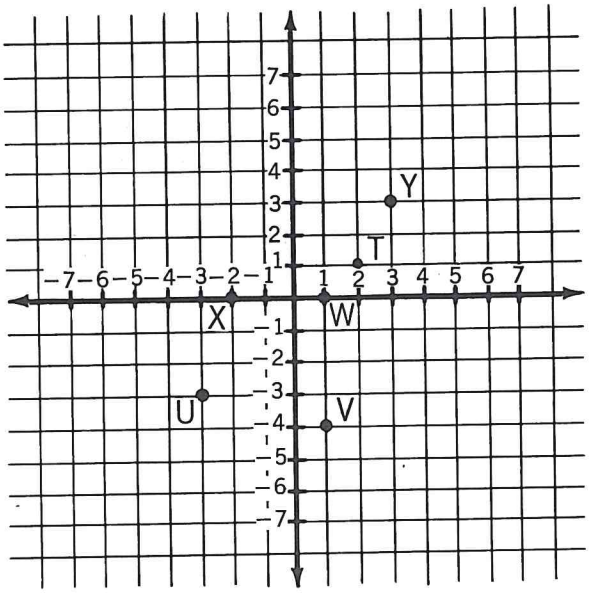
1. To plot (4, 1), count 4 units to the ■ on the horizontal axis.
2. To plot (−2, 3), count 2 units to the ■ on the horizontal axis.
3. To plot (2, −3), count 2 units to the ■ on the horizontal axis.

Choose **up** or **down** to fill each blank.

4. To plot (4, 1), count ■ 1 unit on the vertical axis.
5. To plot (−2, 3), count ■ 3 units on the vertical axis.
6. To plot (2, −3), count ■ 3 units on the vertical axis.

1. Alice plotted some points incorrectly. Use the coordinate system and list of ordered pairs to find which ones.

T (1, 2)
U (−3, 2)
V (−1, −4)
W (0, 1)
X (0, −2)
Y (−3, 3)



2. Using graph paper draw a coordinate system showing from −10 to 10 on both axes. Then plot the points given by these ordered pairs. Label each one.

A (4, 7)	F (0, −4)	K (−1, 3)	Q (−6, −7)	V (5, 5)
B (6, 8)	G (3, −7)	L (0, 7)	R (−9, −5)	W (−5, −4)
C (3, 3)	H (8, −2)	M (−10, 6)	S (−2, −6)	X (10, −3)
D (9, 6)	I (6, −10)	N (−8, 8)	T (0, −3)	Y (−6, 2)
E (10, 0)	J (4, −5)	P (−3, 4)	U (−10, −9)	Z (9, −9)

3. a. Draw a coordinate system on graph paper, showing both axes from −10 to 10. On it, plot and label these points:
V (−5, 9) W (3, 1) X (7, −3) Y (0, 4)

b. Draw segment VX.

c. Are points W and Y on segment VX? If not, check your work.

1. a. Draw a coordinate system on graph paper, showing both axes from -10 to 10. On it, plot and label these points:

A $(-2, -10)$	D $(5, -2)$	G $(0, 5)$	J $(4, 10)$	M $(-6, 9)$
B $(8, 0)$	E $(-5, -4)$	H $(-8, -5)$	K $(-10, 6)$	N $(9, -6)$
C $(-7, 0)$	F $(10, 2)$	I $(-2, 5)$	L $(4, -4)$	P $(-4, 0)$

   b. Draw segment MN.
   c. Do points D and I lie on segment MN? If not, check to make sure points M, N, D, and I were plotted correctly.
   d. Draw segment HJ.
   e. Do points P and G lie on segment HJ?
   f. Draw segment AF.
   g. Which other two points that you plotted lie on segment AF?
   h. Draw segment KA.
   i. Which other points that you plotted lie on this segment?

Draw a coordinate system on graph paper, showing both axes from -10 to 10. Use this coordinate system for Exercises 2-5.

2. a. Plot these points: M (1, 2), N (5, 9), P (8, 1).
   b. Draw segments MN, NP, and MP.
   c. What kind of figure have you drawn?

3. a. Plot these points: Q $(2, -2)$, R $(2, -4)$, S $(2, -9)$.
   b. Connect the points.
   c. What kind of figure is figure QRS?

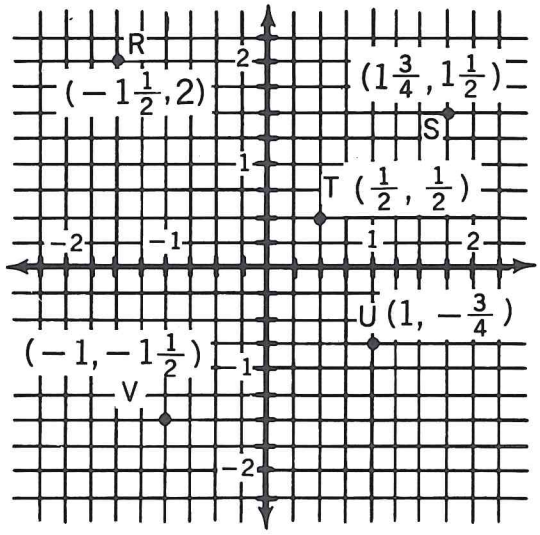
4. a. Plot these points: T $(-2, -2)$, U $(-4, -5)$, V $(-5, -1)$.
   b. Connect the points in all possible ways.
   c. What kind of figure is figure TUV?

5. a. Plot these points: W $(-7, 2)$, X $(-4, 5)$, Y $(-2, 7)$.
   b. Draw segments WX, XY, and YW.
   c. What kind of figure have you drawn?

Fractions as Coordinates

We can plot points whose coordinates are not whole numbers. We let the distance between two marks on the axis be less than 1 unit.

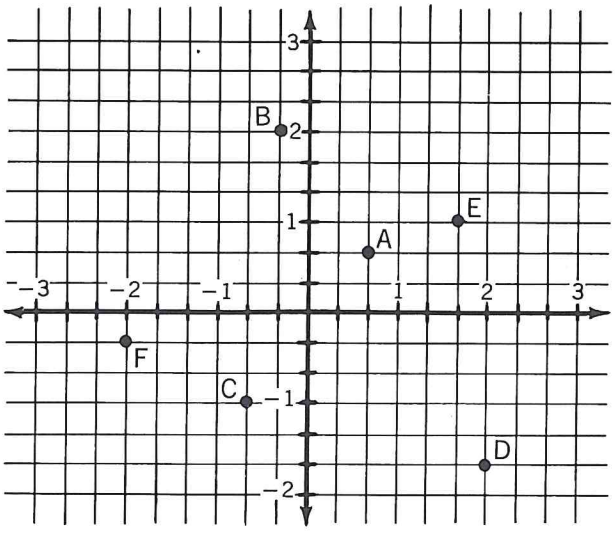
For example, the axes of this coordinate system are marked in fourths.



1. Look at the graph above and give the coordinates for these points: R, S, T, U, V.

2. On the graph above, how many squares equal 1 unit?
3. On the graph above, how many squares equal 2 units?
4. On the graph above, how many squares equal $1\frac{1}{2}$ units?
5. On the graph below, how many squares equal 1 unit?
6. On the graph below, how many squares equal 2 units?
7. On the graph below, how many squares equal $1\frac{2}{3}$ units?

8. Jack plotted some points incorrectly. Use the coordinate system and the list of ordered pairs to find which ones.



A $\left(\frac{2}{3}, \frac{2}{3}\right)$ D $\left(-1\frac{2}{3}, 2\right)$

B $\left(-\frac{1}{3}, 2\right)$ E $\left(1\frac{2}{3}, -1\right)$

C $\left(-1, -\frac{2}{3}\right)$ F $\left(-2, -\frac{1}{3}\right)$

1. a. Draw a coordinate system on graph paper, showing both axes from -8 to 8. Let 2 squares each be one unit. Plot and label these points:

A $\left(4\frac{1}{2}, 3\frac{1}{2}\right)$	D $\left(-3, 1\frac{1}{2}\right)$	G $\left(-5\frac{1}{2}, 6\right)$	J $\left(-4\frac{1}{2}, 2\frac{1}{2}\right)$
B $\left(1\frac{1}{2}, -1\frac{1}{2}\right)$	E $\left(\frac{1}{2}, 4\frac{1}{2}\right)$	H $\left(-7\frac{1}{2}, -5\right)$	K $\left(7\frac{1}{2}, -\frac{1}{2}\right)$
C $\left(-1\frac{1}{2}, 5\right)$	F $\left(0, -\frac{1}{2}\right)$	I $\left(2\frac{1}{2}, -2\right)$	L $\left(-2\frac{1}{2}, -3\frac{1}{2}\right)$

b. Draw segment AG. Point C and what other point lies on $\overline{AG}$?

c. Draw segment BJ. Which other two points plotted lie on $\overline{BJ}$?

d. Draw segment HK. Which other points plotted lie on this segment?

Draw a coordinate system showing both axes from -3 to 3. Let 3 squares equal 1 unit. Show Exercises 2-6 on this coordinate system.

2. a. Plot these points: $A\left(2, 2\frac{1}{3}\right)$, $B(2, 1)$, $C\left(\frac{2}{3}, 1\right)$, $D\left(\frac{2}{3}, 2\frac{1}{3}\right)$.

b. Draw segments AB, BC, CD, and DA.

c. What kind of figure is figure ABCD?

3. a. Plot these points: $E\left(1\frac{2}{3}, -\frac{2}{3}\right)$, $F\left(2\frac{2}{3}, -\frac{2}{3}\right)$, $G\left(2\frac{2}{3}, -3\right)$, $H\left(1\frac{2}{3}, -3\right)$.

b. Draw segments EF, FG, GH, and HE.

c. What kind of figure have you drawn?

4. a. Plot these points: $I\left(-3, -1\frac{1}{3}\right)$, $J\left(-2, -2\frac{2}{3}\right)$, $K\left(-\frac{1}{3}, -1\frac{2}{3}\right)$, $L\left(-1\frac{1}{3}, -\frac{1}{3}\right)$.

b. Draw segments IJ, JK, KL, and LI.

c. What kind of figure is figure IJKL?

5. a. Plot these points: $M\left(-2\frac{2}{3}, 1\frac{2}{3}\right)$, $N\left(-2\frac{1}{3}, \frac{2}{3}\right)$, $P\left(-1\frac{1}{3}, 1\right)$, $Q\left(-1\frac{2}{3}, 2\right)$.

b. Draw segments MN, NP, PQ, and QM.

c. What kind of figure is figure MNPQ?

6. a. Plot these points: $R\left(\frac{2}{3}, 0\right)$, $S\left(0, -\frac{2}{3}\right)$, $T\left(-\frac{2}{3}, 0\right)$, $U\left(0, \frac{2}{3}\right)$.

b. Draw segments RS, ST, TU, and UR.

c. What kind of figure is figure RSTU?

1. **a.** Draw a coordinate system showing both axes from -5 to 5 and let 2 squares equal 1 unit. Plot and label these points:

A $(4\frac{1}{2}, -3\frac{1}{2})$	F $(4\frac{1}{2}, 3)$	K $(3\frac{1}{2}, 5)$	P $(-4, 5)$
B $(2\frac{1}{2}, 0)$	G $(-\frac{1}{2}, 5)$	L $(4\frac{1}{2}, 0)$	Q $(4\frac{1}{2}, -1\frac{1}{2})$
C $(-5, -4\frac{1}{2})$	H $(2, -4\frac{1}{2})$	M $(-3\frac{1}{2}, 0)$	R $(-3\frac{1}{2}, 2\frac{1}{2})$
D $(1\frac{1}{2}, 5)$	I $(-1, -4\frac{1}{2})$	N $(-1\frac{1}{2}, 0)$	S $(5, -4\frac{1}{2})$
E $(-3\frac{1}{2}, 4\frac{1}{2})$	J $(-3\frac{1}{2}, -1)$	O $(0, 0)$	T $(-4\frac{1}{2}, 0)$

b. Draw lines AF, BT, CS, EJ, and KP.

c. Which lines are horizontal? Which lines are vertical?

2. **a.** Choose **horizontal** or **vertical.**

a. On a horizontal line, all points have the same ___ coordinate.

b. On a vertical line, all points have the same ___ coordinate.

3. Which sets of ordered pairs determine a straight line when plotted?

a.	**b.**	**c.**	**d.**
$(-4, -1)$	$(-2, 3)$	$(-8, -8)$	$(7, 5)$
$(0, -1)$	$(-2, -4)$	$(-3, 0)$	$(-4, 5)$
$(4, -1)$	$(-2, 7)$	$(3, 4)$	$(5, 6)$
$(5, -1)$	$(-2, 5)$	$(7, 8)$	$(5, 7)$
$(9, -1)$	$(-2, -9)$	$(9, 9)$	$(5, 5)$

4. **a.** Which of the sets in Exercise 3 determine vertical lines? (Answer without plotting the points.)

b. Which of the sets in Exercise 3 determine horizontal lines?

On vertical lines, one horizontal coordinate occurs with many different vertical coordinates.

On horizontal lines, one vertical coordinate occurs with many different horizontal coordinates.

1. Without plotting, tell which of these sets of ordered pairs determines a horizontal line.

a.	b.	c.	d.
(4, −2)	(3, 4)	(3, −4)	(2, 1)
(5, −2)	(2, −3)	(3, −2)	(3, 2)
(7, −2)	(6, 3)	(3, 4)	(4, 3)
(8, −2)	(1, 2)	(3, 6)	(5, 4)

2. Without plotting, tell which of the sets in Exercise 1 determines a vertical line.

3. **a.** Give a set of 4 ordered pairs which determines a horizontal line.
   b. Give a set of 4 ordered pairs which determines a vertical line.

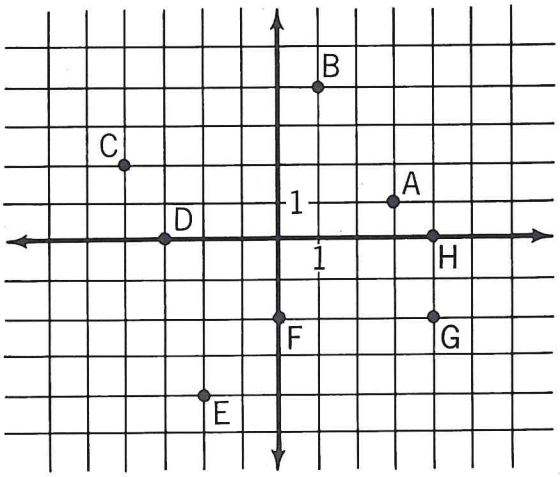
4. On graph paper draw a coordinate system showing both axes from −10 to 10.
   a. Plot these points: U (0, 10), V (8, −4), W (−8, −4).
   b. Connect these points in all possible ways.
   c. On the same coordinate system, plot these points: X (−8, 6), Y (0, −8), Z (8, 6).
   d. Draw segments XY, YZ, and XZ.
   e. What does your drawing look like?

5. **a.** Plot these points: H (1, 1), I (1, 5), J (5, 5), K (5, 1), L (7, 3), M (7, 7) N (3, 7).
   b. Draw segments HI, IJ, JK, KH, KL, JM, IN, NM, and ML.
   c. What does your drawing look like?

6. **a.** Plot these points: A (1, 3), B (−1, −2), C (3, −2), D (4, 0).
   b. Draw segments AB, AC, AD, BC, and CD.
   c. What does your drawing look like?

Feedback

1. Give the coordinates for each point.



2. Draw a coordinate system showing both axes from -4 to 4. Let 2 squares equal 1 unit. Plot these points.

A $\left(-3, 1\frac{1}{2}\right)$	C $\left(0, 2\frac{1}{2}\right)$	E $\left(-2, 3\frac{1}{2}\right)$	G (2, 0)
B $\left(2\frac{1}{2}, 0\right)$	D $\left(-3\frac{1}{2}, -2\right)$	F $\left(1\frac{1}{2}, 2\right)$	H (0, -2)

3. Which is the horizontal coordinate in the ordered pair $(3, -2)$?

4. Which is the vertical coordinate in the ordered pair $(3, -2)$?

5. What is the name of the point where the axes intersect?

Complete each problem.

6. $-3 + 7 =$ ■
7. $-2 + (-4) =$ ■
8. $8 + 9 =$ ■
9. $0 + (-4) =$ ■
10. $6 + (-6) =$ ■
11. $5 +$ ■ $= -1$
12. $-2 +$ ■ $= 3$
13. ■ $+ (-5) = -7$
14. ■ $+ 3 = 0$
15. $-8 +$ ■ $= 0$

Feedback

Find each quotient.

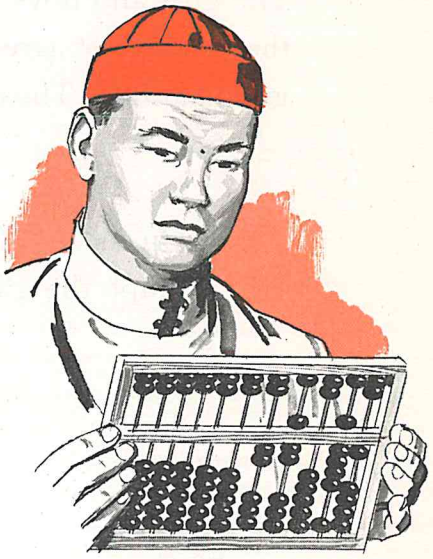
1. $40\overline{)732}$
2. $5\overline{)5205}$
3. $28\overline{)3876}$
4. $76\overline{)3248}$
5. $50\overline{)7698}$
6. $34\overline{)3408}$
7. $39\overline{)18254}$
8. $15\overline{)71094}$

Find each product.

9. $\begin{array}{r} 846 \\ \times\ 7 \\ \hline \end{array}$
10. $\begin{array}{r} 9206 \\ \times\ 8 \\ \hline \end{array}$
11. $\begin{array}{r} 83 \\ \times 45 \\ \hline \end{array}$
12. $\begin{array}{r} 936 \\ \times\ 78 \\ \hline \end{array}$

Find each sum or difference.

13. $8 + 7 + 6 =$ ■
14. $9 + 2 + 4 =$ ■
15. $5 + 7 + 3 =$ ■
16. $11 - 4 =$ ■
17. $21 - 4 =$ ■
18. $31 - 4 =$ ■



Complete each problem.

19. $\{\text{Mars}\} \cup \{\text{Mars, Venus}\} =$ ■
20. $\{n: n < 5 \text{ and } n \text{ is an odd positive number}\} =$ ■
21. $\{n: 7 < n < 11 \text{ and } n \text{ is a whole number}\} =$ ■
22. $\{\text{Ohio, Utah}\} \cap \{\text{Utah, Texas}\} =$ ■
23. $\{2, 4, 6, 8, 10\} - \{2, 4, 8, 16\} =$ ■

24. The Girl Scouts packed 4080 mints in 68 boxes. How many mints were packed in each box?

25. What is the perimeter of an equilateral triangle whose sides are 3 inches long?

26. What is the perimeter of a square whose sides are $\frac{3}{4}$ ft. long?

Part 13 Graphs

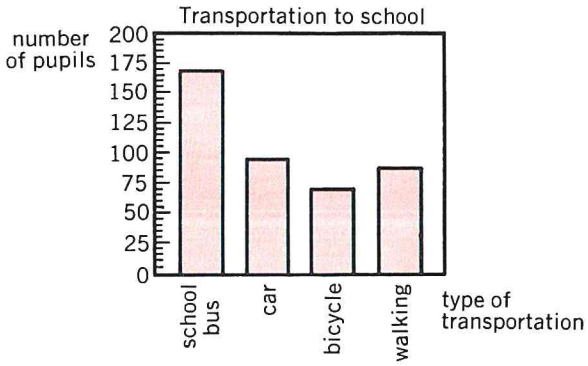
Bar Graphs

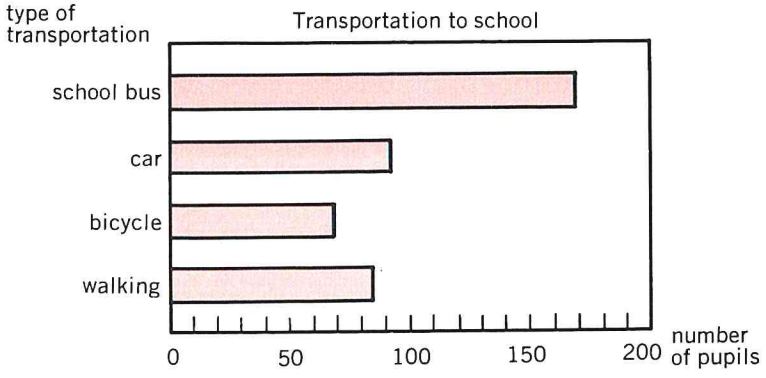
The fifth grade made a survey to find out the different ways in which the 420 pupils in their school get to school. This is what they found.

type of transportation	school bus	car	bicycle	walking
number of pupils using it	170	95	70	85

The girls and boys wanted to make a bar graph to show this information, but they could not agree on how to do it. Finally the boys made one graph, the girls another. These are the graphs.

On the right is the graph the girls made.





On the left is the graph the boys made.

1. Let's see if the graphs on page 208 give the same information.

   a. Copy this chart and fill in the blanks.

number of pupils coming by:	bus	car	bicycle	walking
girls' graph				
boys' graph				

   b. Jane said that the bars were wider on the girls' graph. Does this change the information given? Ted noticed that the boys left more space between their bars than the girls did. Does this change the information given?

   c. The girls said their graph was easier to read because they marked off smaller units on the number scale. How many pupils were represented by each unit on the girls' scale? How many pupils were represented by each unit on the boys' scale?

   d. In reading the boys' graph we had to estimate for two of the groups. Which two groups were these?

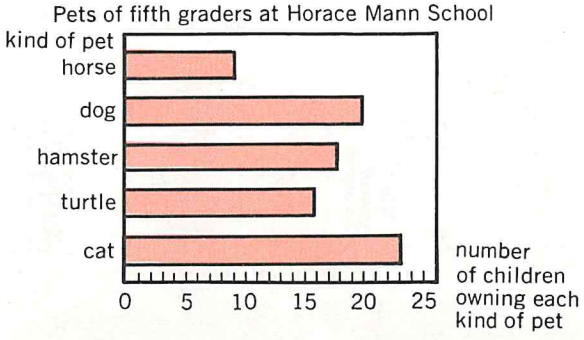
   e. The boys said their graph was easier to read because the names were written on the **vertical** axis, while the numbers were marked on the **horizontal** axis. On which axis were the names written on the girls' graph? On which axis did the girls mark the numbers?

   f. Do the graphs give the same information?

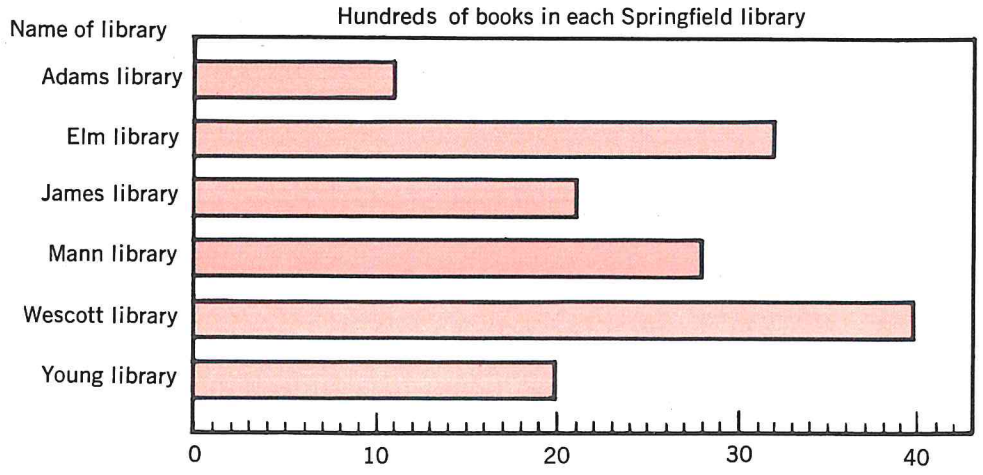
2. a. How many children have horses for pets?

   b. How many children have turtles for pets?

   c. How many more children own cats than own dogs?

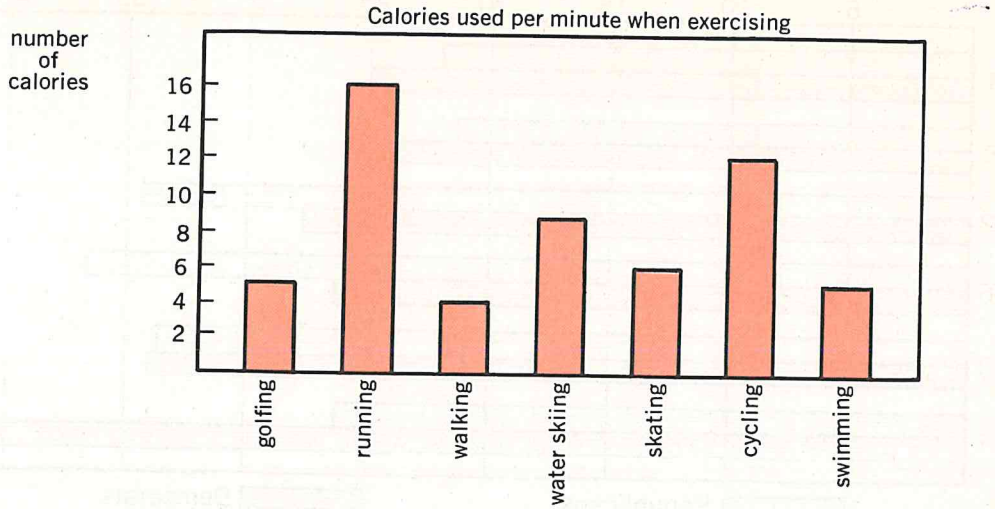


Springfield has six libraries. The bar graph below shows how many hundreds of books each library contains. Using the graph, answer the questions below.



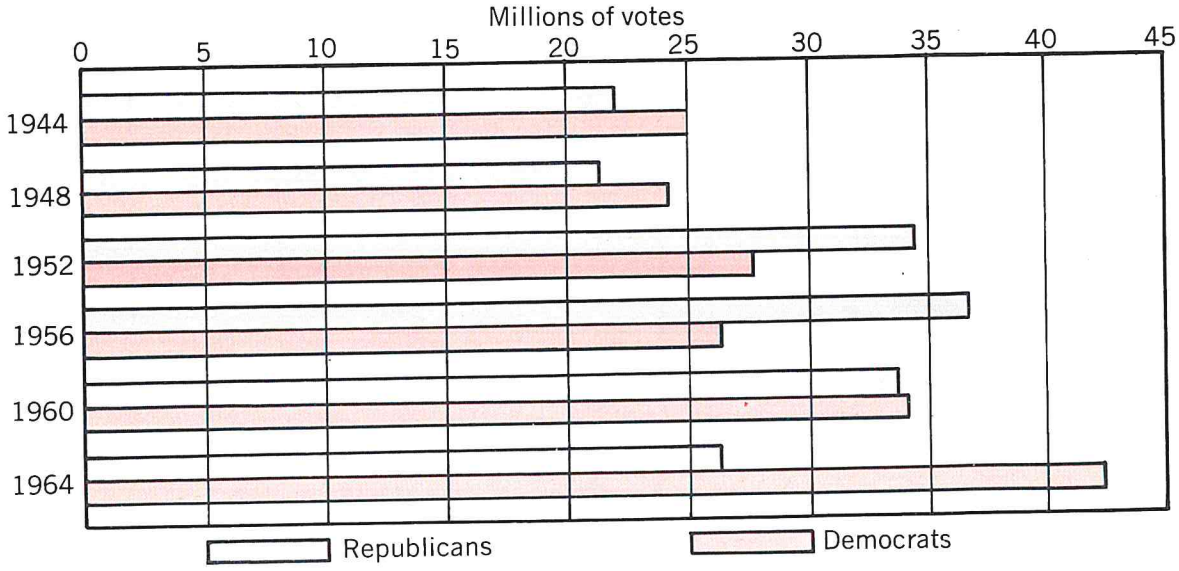
1. What is the title of the graph?
2. Give the approximate number of books in each library.
3. Which two libraries have almost the same number of books?
4. Which library has the most books? Which has the fewest?
5. Which library has about 3200 books?
6. Does the horizontal axis give the names or numbers?
7. How many books does each unit on the number scale represent?
8. About how many more books has Mann Library than James Library?
9. a. Do Adams Library and Wescott Library together have more or less books than Young Library and James Library together?
   b. How many more or less books?

The bar graph below shows the number of calories used per minute by a person when exercising different ways.

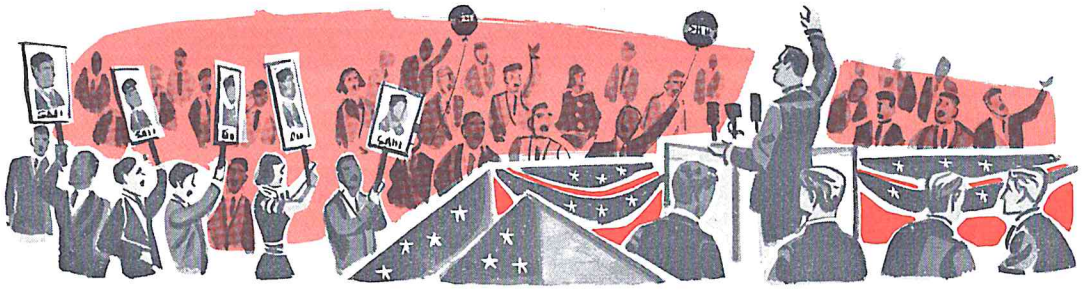


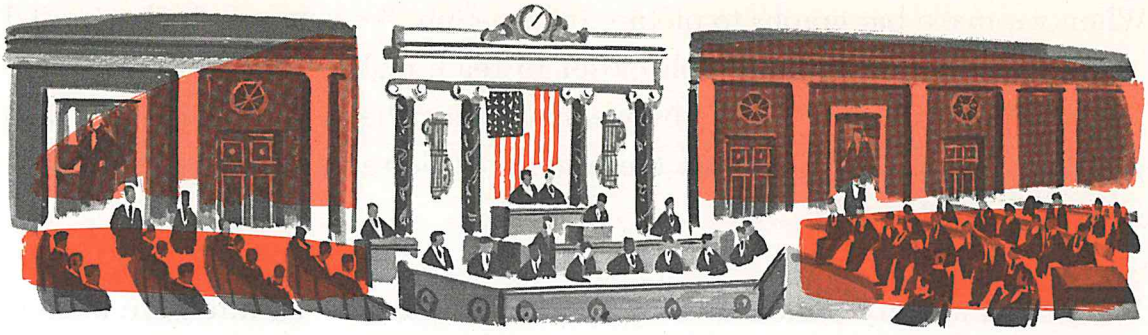
1. What is the title of the graph?
2. How many calories are burned per minute when walking?
3. How many calories are burned per minute when running?
4. How many calories are burned per minute when golfing?
5. How many calories are burned per minute when cycling?
6. How many calories are burned per minute when water skiing?
7. How many calories are burned per minute when skating?
8. How many calories are burned per minute when swimming?
9. Are more calories used per minute when swimming or skating?
10. Are more calories used per minute when golfing or cycling?
11. Are more calories used per minute when running or swimming?
12. Are more calories used per minute when walking or golfing?

Republican and Democratic votes in Presidential elections
1944 to 1964



1. In which of the years did the Democrats win the election?
2. In which years was a Republican President elected?
3. In which year did about 27,500,000 Democrats vote?
4. About how many Republicans voted in 1948?
5. About how many Democrats voted in 1956?
6. In which year did about 22,000,000 Republicans vote?
7. In which year was the total number of Republican and Democratic votes the smallest?
8. In which year did the Republicans get more than 35 million votes?
9. In which year did the Democrats get more than 35 million votes?





1. Fred and Kay know there is a total of 435 members in the House of Representatives. Make a bar graph to show the following information which Fred and Kay learned while working on a history project.

Number of members in the House of Representatives in 1960

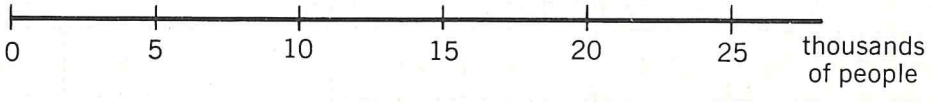
State	Calif.	Fla.	Ill.	Mich.	N. J.	N. Y.	Pa.	Tex.
Number of Representatives	38	12	24	19	15	41	27	23

2. To show on a bar graph the information given in the table, the axis must show numbers from 0 to 25,000.

1960 Populations in nearby towns

Town	Ashland	Camden	Campbell	Milford	San Pablo	Weston
Population	6,530	10,050	15,388	8,768	23,276	18,495

We need not mark off all 25,000 units. Instead, let one mark represent 5000 people.



Using this scale, make a bar graph showing the approximate population of the cities listed in the above table.

When we make bar graphs to picture information, the bars may be horizontal or vertical. To make the graph easier to read, make all the bars the same width. Also make the spaces between the bars the same width. For each problem, tell what one unit on the number scale represents. Remember to give your graph a title.

1. During activity period in Mrs. Manning's class, 10 children are making pottery, 8 are making maps, 3 are building model villages, and 7 are working on science fair projects.
   a. Show this on a bar graph.
   b. How many children are working during the activity period?

2. In the election for president of the fifth grade, Ann received 14 votes, Bob 17 votes, Ed 30 votes, Judy 19 votes, Mike 24 votes, and Tony 27 votes.
   a. Show this on a bar graph.
   b. Who received the most votes?
   c. How many votes were cast all together?

3. In Rice School there are 150 first-graders, 125 second-graders, 140 third-graders, 130 fourth-graders, 150 fifth-graders, and 160 sixth-graders.
   a. Show this on a bar graph.
   b. What is the total number of pupils in grades one through six at Rice School.

4. a. Show the information given below on a bar graph.

Height in inches of seven fifth-graders

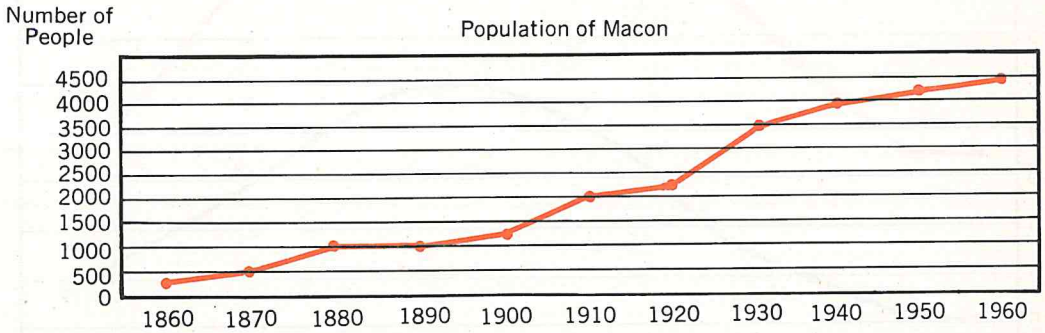
Name	Anne	Bob	Elaine	Jack	Mary	Pete	Sam
Height	58	56	56	60	60	61	59

   b. Is the average height of the girls or the boys greater?

Line Graphs

A line graph is very much like a bar graph. The scale of the line graph is like the scale used for the bar graph. Line graphs are used when showing data such as temperatures for a year, or daily scores where every point on the graph has meaning. Two or more lines can be shown on the same graph.

1. The graph below shows the population of Bill's home town from 1860 to 1960.



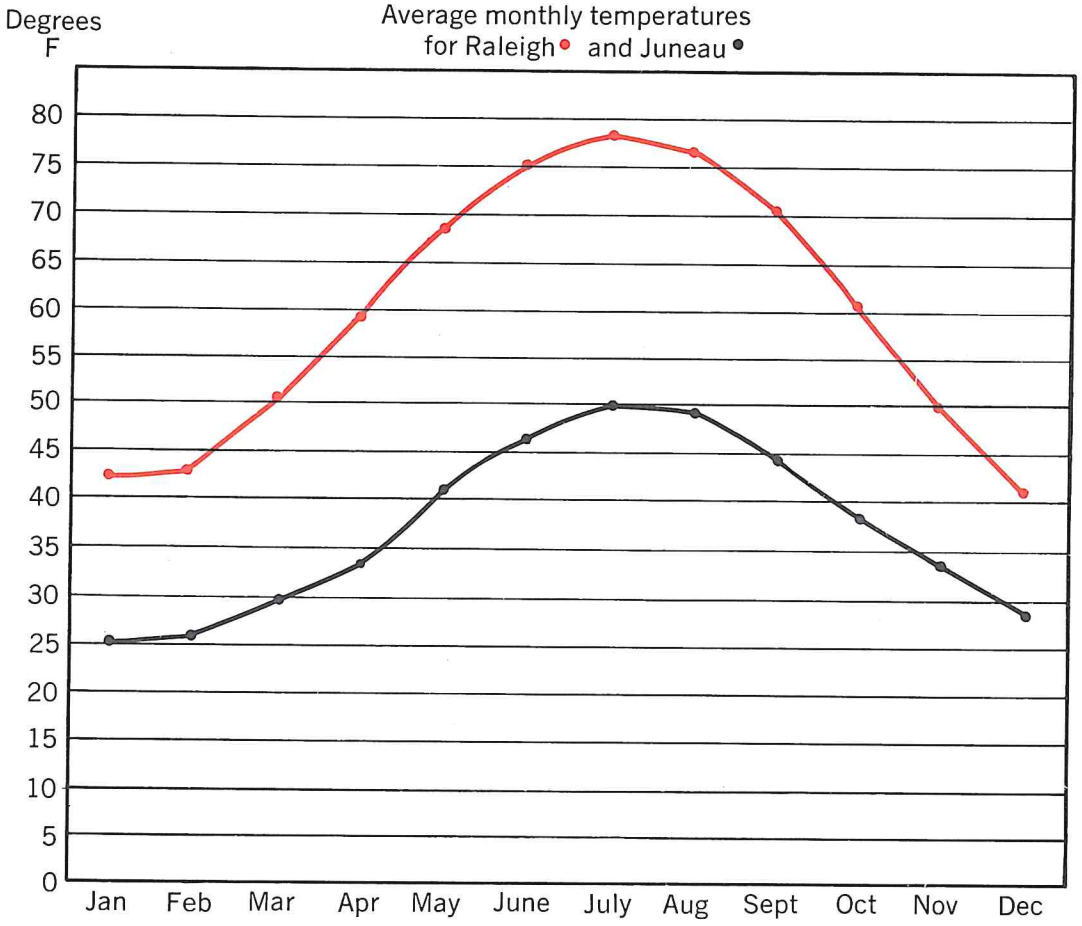
a. During which ten-year period (decade) did the population of Macon increase the most?

b. During which decade did the population not increase?

c. Copy and complete the table below.

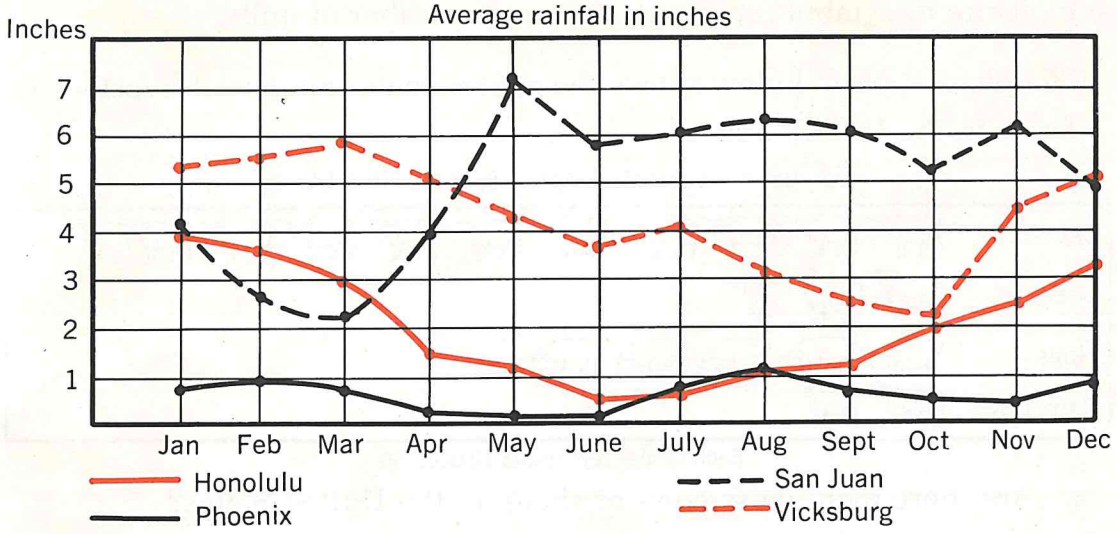
Year	Population of Macon	Increase in Population
1860	400	—
1880	1000	600
1900		
1920		
1940		
1960		

The average monthly temperatures for Raleigh, North Carolina and Juneau, Alaska are given in the graph below.



1. During which month did Juneau, Alaska have the lowest average temperature?
2. During which month did Raleigh, North Carolina have the lowest average temperature?
3. What was the highest average monthly temperature for Juneau?
4. What was the highest average monthly temperature for Raleigh?
5. Does Juneau or Raleigh have the greater average temperature during the year?

The average rainfall in inches during the last thirty years for the cities of Vicksburg, Mississippi; Phoenix, Arizona; Honolulu, Hawaii; and San Juan, Puerto Rico are shown in the graph below.



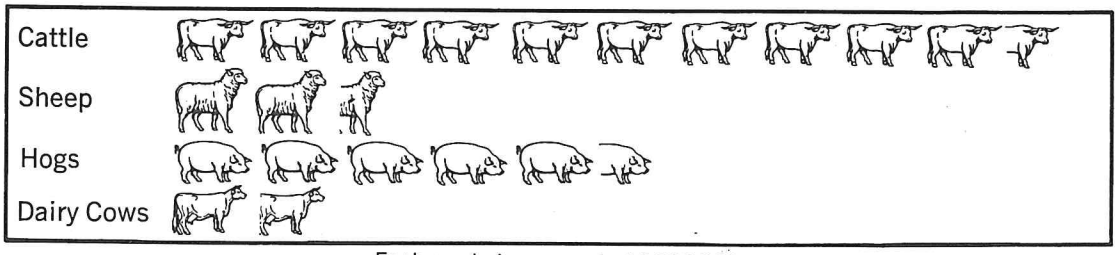
1. Which city had the greatest rainfall during June?
2. Which city had the greatest rainfall during April?
3. Which city had the least amount of rainfall during July?
4. Which city had the least amount of rainfall during December?
5. During which month was the rainfall greatest in Honolulu?
6. During which two months was the rainfall greatest in Phoenix?
7. During which month was the rainfall greatest in San Juan?
8. During which month was the rainfall greatest in Vicksburg?

Picture Graphs

A picture graph or a pictograph uses pictures or symbols instead of bars. Each picture or symbol represents the same number of units.

1. The picture graph below shows the approximate number of livestock on farms in the United States.

Number of livestock on farms in the United States

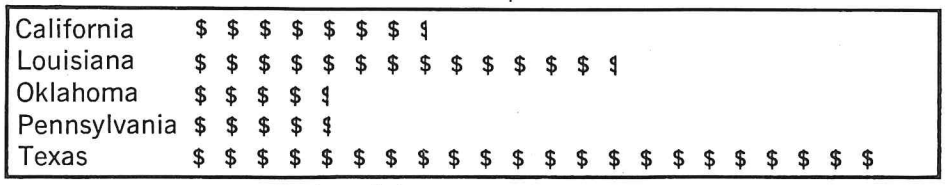


Each symbol represents 10,000,000

   a. Are there more dairy cows or sheep in the United States?
   b. Approximately how many sheep are there in the United States?
   c. Approximately how many hogs are there in the United States?
   d. Approximately how many cattle are there in the United States?

2. The picture graph below shows the approximate value of mineral production in five states.

Value of mineral production



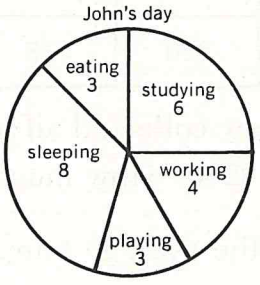
Each symbol represents $200,000,000

   a. Which state had the greatest mineral production?
   b. Which state was second in mineral production?
   c. Was the value of mineral production in Texas greater than $4,000,000,000?
   d. Was the value of mineral production in California greater than $2,000,000,000?

Circle Graphs

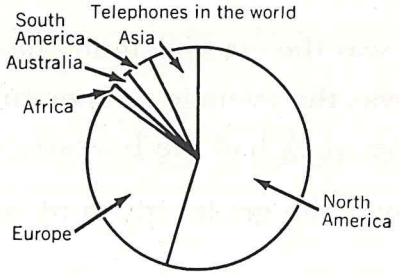
A circle graph uses a circle instead of a bar, line, or symbol. The circle makes it possible to compare each part with the whole.

1. The circle graph below shows how John spends his day.



   a. What fractional part of the time does John sleep?
   b. What fractional part of the time does John study?
   c. Does John spend more time playing or working?
   d. Does John spend more time eating or sleeping?

2. The graph below shows the approximate number of telephones in the world.



   a. Are there more telephones in Europe or in Asia?
   b. Are there more telephones in North America or in South America?
   c. Are there more telephones in Africa or in Asia?
   d. Are there more telephones in Australia or in Europe?
   e. Are more than half the telephones in the world in North America?

Feedback

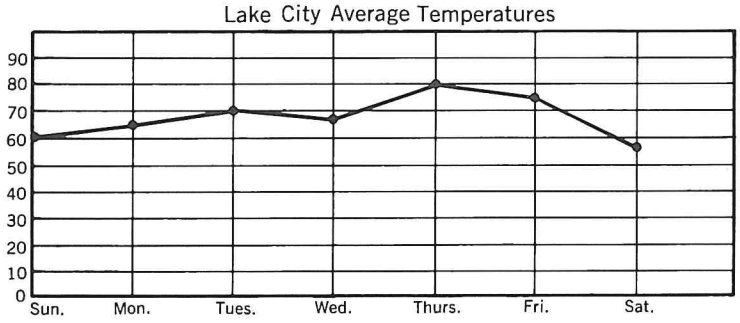
1. Make a bar graph to show the following information. The graph may have vertical or horizontal bars.

Lincoln School's Library Fund Drive

Grade Level	1	2	3	4	5	6
Dollars collected for library fund	$20	$28	$28	$26	$35	$38

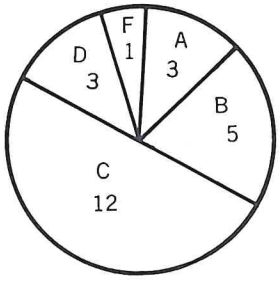
   a. How much have they collected all together?
   b. The school goal is $200. How much more do they need?

2. The graph below shows the average temperatures in Lake City for one week.



   a. On what days was the average temperature above 70°?
   b. On what day was the average temperature 70°?
   c. What day of the week had the lowest average temperature?

3. The graph below shows the grades given by Miss McCosky on a recent test.

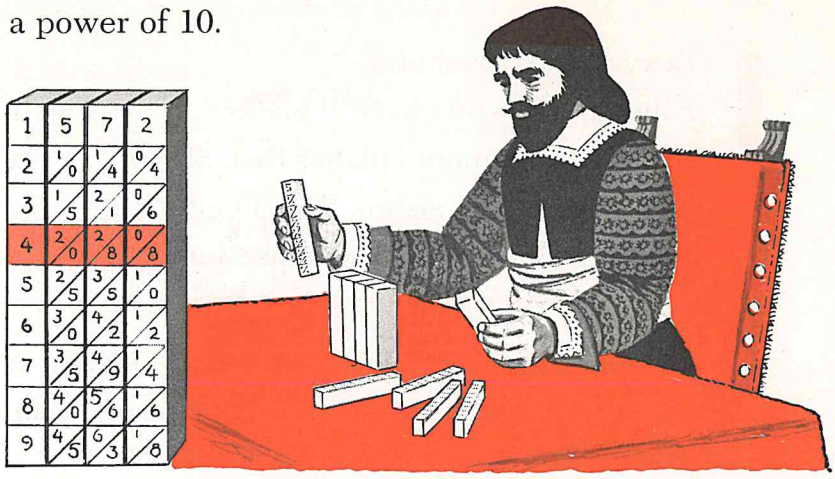


   a. How many children were in her class?
   b. What grade did most of the students receive on the test?
   c. Did more students get A's or F's?

Feedback

1. Write each number as a power of 10.

   a. 10
   b. 10,000
   c. 1,000,000
   d. 100,000
   e. 10,000,000,000
   f. 100



2. Compute each power.

   a. 4^5 b. 2^7 c. 5^1 d. 9^3 e. 19^2 f. 10^4

Solve each equation.

3. $2 \times 3 = g$
4. $12 \div 6 = m$
5. $12 + 11 = n$
6. $15 \div 5 = r$
7. $d \times 6 = 42$
8. $e \div 5 = 1$
9. $c + 0 = 9$
10. $b \times 7 = 63$
11. $9 \div m = 9$
12. $3 + q = 3$
13. $5 \times v = 60$
14. $96 \div y = 8$

Pair each set description with a figure.

15. $\overline{MN} \cup \overline{NQ} \cup \overline{QR} \cup \overline{RM}$

16. $\overline{MN} \cap \overline{QR} = \{T\}$

17. $\overline{MN} \cup \overline{NQ} \cup \overline{QR} \cup \overline{RT}$

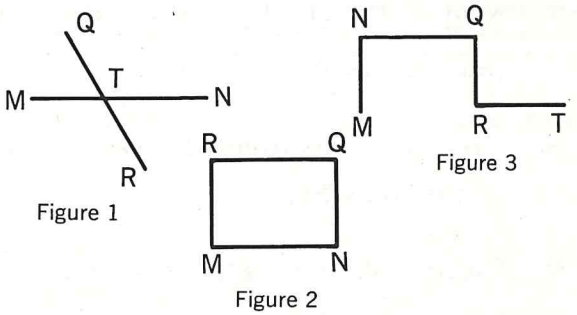


Figure 1 Figure 3

Figure 2

18. One row of fig trees contains 11 trees and produces 2860 figs. In this row what is the average number of figs a tree produces?

Part 14 Mathematical Sentences

Rex has two brothers.	$r = 2$
Ellen and Brenda have 5 books.	$e + b = 5$
Ben has more model planes than Steve.	$b > s$
Karen has fewer sisters than Peggy.	$k < p$
Ed has half as many books as Mark.	$e = \frac{1}{2} \times m$ or $e = m \div 2$

Choose the mathematical sentence that matches each English sentence.

1. Ann and Jack together have less money than Quentin.
2. Max is twelve years old.
3. Dan weighs twice as much as Elsie
4. Fran and Rich together have as many books as Steve.
5. Penny eats twelve times as many cherries as Maurice.
6. Ben has half as many books as Carl.
7. Ned and Vicky have three times as many pennies saved as Tim.
8. Fred and Rex together weigh four times as much as Sam.
9. Cathy has more apples than Bart.
10. Sidney is not as heavy as Floyd and Rachel together.

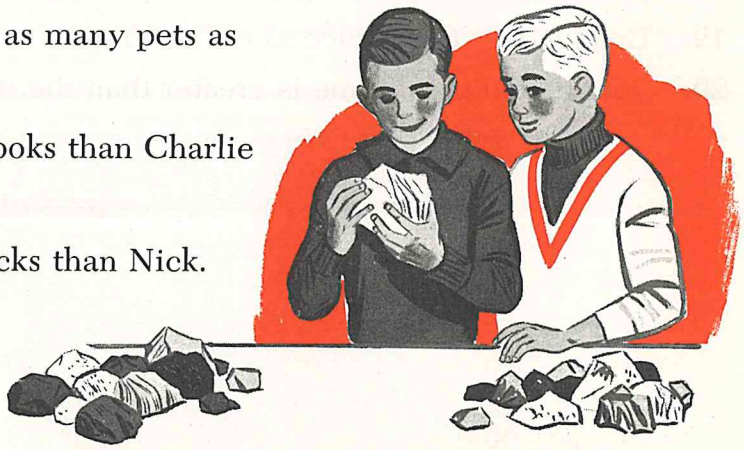
A. $f + r = s$
B. $n + v = 3 \times t$
C. $p = 12 + m$
D. $c > b$
E. $a + j < q$
F. $d = 2 \times e$
G. $s < f + r$
H. $m = 12$
I. $p = 12 \times m$
J. $f + r = 4 \times s$
K. $b = c \div 2$
L. $s > f + r$

John weighs more than Bill.	$j > b$
John weighs four pounds more than Bill.	$j = b + 4$
Ron weighs less than Ted.	$r < t$
Ron weighs six pounds less than Ted.	$r = t - 6$

Choose the mathematical sentence that matches each English sentence.

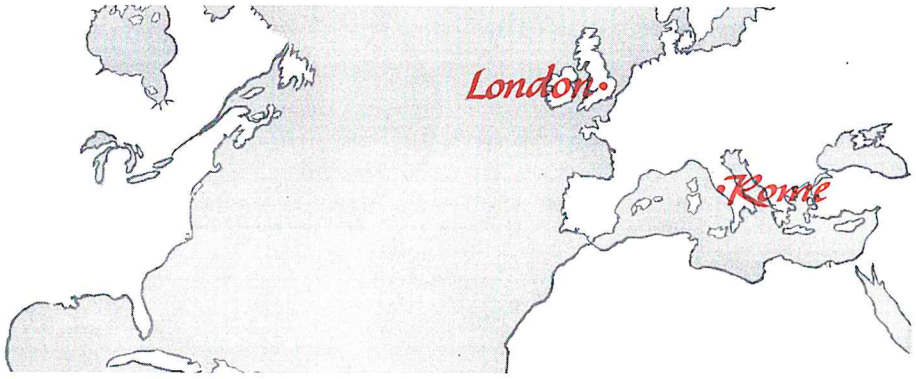
1. Kay and Celia together have three times as many dolls as Nora and Fay together.
2. Cathy and Flo together have five fewer books than Debbie and Ellen.
3. Carol and Fred together scored less than Dora and Ed.
4. Connie and Todd together have the same amount of money as Rod.
5. Karen has six more books than Carl.
6. Teresa and Rachel have collected more stamps than Carla.
7. Cleon has four fewer books than Ian.
8. Roger has three times as many pets as Ned.
9. Isabel has ten more books than Charlie and Vic together.
10. Ray has three more rocks than Nick.

A. $c = i - 4$
B. $c + f < d + e$
C. $i = (c + v) + 10$
D. $k + c = 3 \times (n + f)$
E. $t + r > c$
F. $c + t = r$
G. $r = n + 3$
H. $k = c + 6$
I. $c + f = (d + e) - 5$
J. $c = i + 4$
K. $k < c + 6$
L. $r = 3 \times n$



Write a mathematical sentence for each English sentence.

1. The population of California is greater than the population of Nevada and Utah together.
2. Gerry is twice as tall as David.
3. Charlie drops seven catches more than Larry.
4. Tom and Becky together ate more apples than Huck.
5. Alice is three times as old as Billy.
6. Homer is older than Bill.
7. Steve has less money than Carl and Max together.
8. Henry and Stan together have more stamps than Dick and Ed.
9. George is more than three times as heavy as Vicky.
10. Punch eats as much as Judy and Toby together.
11. Buddy and Paul together are younger than Jay.
12. Ted is eight years older than Abby.
13. Don, Max, and Rod together are more than twice as heavy as Bart.
14. Bill and Mary together have more than three times as many books as Jim.
15. John weighs five times as much as Robin and Marian together.
16. Lance is six years older than Art.
17. Cleon and Mark together are older than Bill and John together.
18. John, Ted, and Paul together are younger than Henry.
19. Lynn is less than twice as old as Tom.
20. The distance to Rome is greater than the distance to London.



Pair each English sentence with the corresponding mathematical sentence. Tell what each letter in the mathematical sentence stands for.

Ed and Al together have fewer than 16 books. $e + a < 16$
e is the number of books Ed has.
a is the number of books Al has.

1.	Pam has more money than Karen.	**A.**	$a = m + 2$
2.	April had half as many rainy days as March.	**B.**	$b = 2 \times (t + r)$
3.	September had four times as many warm days as December.	**C.**	$p + j > 3 \times k$
		D.	$b < 2 \times (t + r)$
4.	Arizona has two fewer counties than Maine.	**E.**	$b = (t + r) \div 2$
5.	Paul is younger than Karl.	**F.**	$s = 4 \times d$
6.	Bob has half the number of books that Ted and Ron have together.	**G.**	$p > k$
		H.	$p < k$
7.	Arlington has two more libraries than Milton.	**I.**	$s = d + 4$
8.	Barbie has less than twice as much money as Terry and Ruth together.	**J.**	$b > 2 \times (t + r)$
		K.	$a = m \div 2$
9.	Sandy has four fewer pets than Dave.	**L.**	$p + j < 3 \times k$
10.	The population of Peterson and Johnstown together is less than three times the population of Kelsey.	**M.**	$s = d - 4$
		N.	$a = m - 2$
11.	The population of Boston is more than twice the population of Townsend and Roxbury together.		
12.	Susan has four more brothers than Denise.		
13.	Penny and Joy together weigh more than three times as much as Kay.		
14.	Bradley Lake is twice as large as Tiny Lake and Ruby Lake together.		

If the meaning is clear, we may omit the multiplication sign.

$5 + 5 + 5 = 3 \times 5$ We cannot omit the multiplication sign or we might read "thirty-five" instead of "three times five."

$k + k + k = 3 \times k$
$= 3k$ We can omit the multiplication sign because the meaning is clear: "3k" and "3 × k" both mean "multiply k by 3." We write "3 × k" instead of "k × 3" because there are 3 k's.

Write each sum as a product. Omit the multiplication sign if the meaning is clear. Follow the examples in the box above.

1. $6 + 6$
2. $j + j$
3. $r + r + r$
4. $m + m + m + m$
5. $7 + 7 + 7 + 7$
6. $s + s + s + s + s$
7. $4 + 4 + 4 + 4 + 4$
8. $y + y + y + y + y$
9. $c + c + c + c + c$

Choose $=$ or $\neq$.

10. $5f \bigcirc 5 \times f$
11. $77 \bigcirc 7 \times 7$
12. $6g \bigcirc 6 + g$
13. $f + 8 \bigcirc 8f$
14. $3 + 3 \bigcirc 2 \times 3$
15. $2t \bigcirc t + t$
16. $2 + 2 + 2 + 2 \bigcirc 42$
17. $v + v + v + v \bigcirc 3v$
18. $5 \times r \bigcirc 5r$

In which expressions is the meaning clear if we omit the multiplication sign?

$6 \times c \times a = 6ca$

19. 6×4
20. $a \times b$
21. $5 \times c$
22. 4×2
23. $m \times n$
24. $3 \times k$
25. $c \times d$
26. $2 \times t$
27. $8 \times y$
28. 7×4
29. $h \times m \times r$
30. $4 \times e \times g$

Rewrite each expression omitting the multiplication sign.

31. $4 \times k$
32. $7 \times b$
33. $6 \times d$
34. $3 \times t$
35. $2 \times c$
36. $8 \times m$

Choose the mathematical sentence that matches each English sentence. You will not need to use all the mathematical sentences that are given.

$f + g > 2e$	$b = (c + d) + 3$
$m = j$	$e + f < 2g$
$(2e) + (2f) < g$	$b = 3(c + d)$
$b < 3\ (c + d)$	$m = j + 2$
$e = (f + g) - 2$	$m < 2j$
$b = (3c) + d$	$b - 3 = c + d$
$m > 2j$	$e < 2(f + g)$
$b > 3(c + d)$	$c + d = 3b$

1. Mary is more than twice as old as Judy.
2. Bob has three times as many books as Carl and Don together.
3. Connie and Dan together have three times as many cousins as Ben.
4. Mars is less than twice as large as Jupiter.
5. Betty has more than three times as much money as Carol and Debbie together.
6. May had two more rainy days than June.
7. Bonnie has three more brothers than Cindy and Donna together.
8. Ed has less than twice as many apples as Fred and George together.
9. Boston has less than three times as many people as Chicago and Detroit together.
10. March has the same number of days as July.
11. Evan has two fewer marbles than Frank and Gene together.
12. Ellen and Fran together have less than twice as many fish as Gale.

Write a mathematical sentence to match each English sentence.

1. Karen has eight more books than Janet.
2. Jeff is twice as old as Pete.
3. The population of Cincinnati is greater than that of Pierre.
4. December has one more day than April.
5. Pluto is farther from the sun than Venus.
6. Linda is two inches taller than Nancy.
7. February has fewer days than November.
8. Alaska has a larger area than Texas.

Write an equation for each sentence and complete the problem.

9. a.	Joyce is three years older than Cathy.	$j = c + 3$
b.	Cathy is twenty-one years older than Helen.	$c = h + 21$
c.	Helen is two years old.	$h = 2$
d.	How old is Joyce?	$j =$ ▢
10. a.	Sandra and Kate weigh the same.	$s = k$
b.	Kate is fifteen pounds heavier than Judy.	$k = j + 15$
c.	Judy weighs one hundred ten pounds.	$j = 110$
d.	How heavy is Sandra?	$s =$ ▢
11. a.	Henry saved twice as much money as Dick.	$h =$ ___ $\times$ d
b.	Dick saved fifty dollars more than Ed.	$d =$ ___ $+$ ___
c.	Ed saved nothing.	$e =$ ___
d.	How much did Henry save?	$h =$ ▢
12. a.	Rutledge is fifty miles beyond Carterville.	$r =$ ___ $+$ ___
b.	Carterville is sixteen miles away.	$c =$ ___
c.	How far is Rutledge?	$r =$ ▢

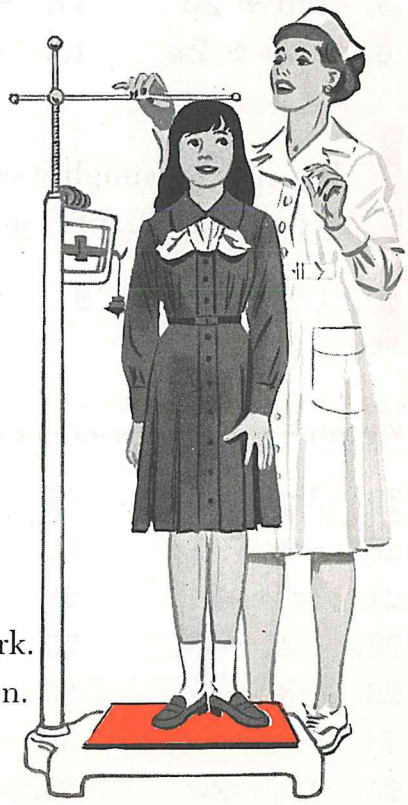
Write an equation for each sentence and complete the problem.

a.	John has four times as many rocks as Peter.	$j = 4 \times p$
b.	Peter has twenty-seven rocks.	$p = 27$
c.	How many rocks has John?	$j = 4 \times 27$
	John has 108 rocks.	$j = 108$

1. **a.** Phyllis rode a mile and one half.
b. Jay rode twenty-seven miles farther than Phyllis.
c. How far did Jay ride?

2. **a.** Jenny has thirty cookies.
b. She gives the same number of cookies to each of her six friends.
c. How many does each friend get?

3. **a.** Priscilla and Helen are the same age.
b. Mary is seven years older than Helen.
c. Mary is thirty-five.
d. How old is Priscilla?



4. **a.** Jane is four inches shorter than Pam.
b. Pam is 61 inches tall.
c. How tall is Jane?

5. **a.** Alan is twenty-five years older than Carl.
b. Carl is eight years older than Leslie.
c. Leslie is eight.
d. How old is Alan?

6. **a.** Chicago is 1000 miles nearer than New York.
b. New York is 3000 miles nearer than London.
c. London is 6000 miles away.
d. How far away is Chicago?

The distributive law tells us that $ac + bc = (a + b)c$. We may omit the multiplication signs because the meaning is clear.
We can use the distributive law to simplify expressions.

$$6t + 3t + 4t = (6 + 3 + 4)t \qquad -7m + 3m = (-7 + 3)m$$
$$= 13t \qquad = -4m$$
$$4a - 2a = (4 - 2)a$$
$$= 2a$$

Simplify each expression by using the distributive law.

1. $3r + 5r$
2. $2t + 7t$
3. $-4x + 4x$
4. $-2y + 8y$
5. $13p + 2p$
6. $-8w + 2w$
7. $7a - 3a$
8. $9e - 2e$
9. $4f - 6f$
10. $q - 9q$
11. $-6k + 3k$
12. $-t + 7t$
13. $s + 3s$
14. $-7r + 6r$
15. $8g - 8g$
16. $-4h + 2h$
17. $6j - j$
18. $9n - 5n$

We can not simplify expressions if the distributive law does not apply.
We can not simplify any of these expressions:

$$2t + 3 \qquad 4m + 5x \qquad 9c - 2d \qquad -7 + 3p$$

Simplify the expressions to which we can apply the distributive law.

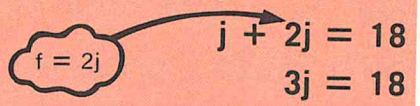
19. $3m + 4p$
20. $-9x + 2x$
21. $3g - 4$
22. $-2j - 6k$
23. $-3t - 4$
24. $-7s - 5g$
25. $16 - 2w$
26. $-8q - 9q$
27. $3x + 7x$
28. $2w - 6$
29. $p + 5p$
30. $-2r - 2r$
31. $9v + 8x$
32. $-7m + 7m$
33. $4i - 8i$
34. $9c + 3d$
35. $-11k + 4j$
36. $-3e + 6e$
37. $-5y + 7y$
38. $10m - 4n$
39. $2p + c$

Write an equation for each sentence and complete each problem.

a. Fred is twice as old as Joe. $f = 2j$

b. Together Joe and Fred are 18. $j + f = 18$

c. How old is each?



$j + 2j = 18$

$3j = 18$

$j = 6$

Joe is 6 and Fred is 12. $f = 2 \times 6 = 12$

1. a. Tony has four times as many books as Bob. $t =$ ___

b. Together they have 45 books. $t + b =$ ___

c. How many books does each have?

2. a. Mary is three times as old as Helen. $m =$ ___

b. Together their ages total 24. $m +$ ___ $=$ ___

c. How old is each?

3. a. Jean has twice as much money as Pat. $j =$ ___

b. Brenda has three times as much money as Pat. $b =$ ___

c. Together the girls have 90¢. $j + b + p =$ ___

d. How much money does each girl have?

4. a. Don has the same number of books as Bob.

b. Rick has twice as many books as Bob.

c. Together the boys have 40 books.

d. How many books does each boy have?

5. a. Mary has the same number of pets as Wendy.

b. Wendy has two times as many pets as Pete.

c. Together they have 10 pets.

d. How many pets does each person have?

We can simplify expressions like these:

$3m + 2m = (3 + 2)m = 5m$
$4a - 2a = (4 - 2)a = 2a$

We can also simplify expressions like these:

$3m + 2m + 1 = 5m + 1$
$4a - 2a - 6 = 2a - 6$

We see that $3m + 2m + 1$ is not the same as $3m + 2m + m$.

$3m + 2m + 1 = (3 + 2)m + 1 = 5m + 1$
$3m + 2m + m = (3 + 2 + 1)m = 6m$
$5m + 1 \neq 6m$

Simplify each expression.

1. $4a - 3a + 6$
2. $2x + 3x + 5x$
3. $6y + 2 + 4y$
4. $n + 3n - 4$
5. $2b + 5 + 7b$
6. $3c - c + 2$
7. $4b + 2b - b$
8. $5a + a + 2$
9. $2y + 6 + 8y$
10. $3x + 2x + x + 4$
11. $7a - 4a + 5$
12. $3y + 3y + 4y$

Choose $=$ or $\neq$.

13. $6a + 3 + 4a \bigcirc 13a$
14. $3y + 4y + 2y \bigcirc 9y$
15. $n + 5n + 2 \bigcirc 6n + 2$
16. $n + n + n \bigcirc 3n$
17. $r + r + 5 \bigcirc 7r$
18. $2t + 3s \bigcirc 5st$
19. $3r + 2r + 7r \bigcirc 12r$
20. $r + 6 + 2 \bigcirc r + 8$
21. $t + 5 + 4t \bigcirc 10t$
22. $y + 1 + y \bigcirc 2y + 1$
23. $m - 2 + m \bigcirc 0$
24. $a + 6a - 4a \bigcirc 11a$
25. $3m - 1 + m \bigcirc 4m - 1$
26. $b + 3b - 2b \bigcirc 2b$
27. $7p + 4a \bigcirc 11pa$
28. $r + k + 7 \bigcirc rk + 7$
29. $2m \bigcirc m + m$
30. $3v + 3 + v \bigcirc 4v + 3$

Write an equation for each sentence. Solve each problem.

a. Margie is 3 years older than Ruth.
b. Together they are 19.
c. How old is each?

$m = r + 3$
$m + r = 19$
$r + 3 + r = 19$
$2r + 3 = 19$ — Subtract 3 from both sides.
$2r = 16$
$r = 8$
$m = 8 + 3 = 11$

Ruth is 8 years old and Margie is 11.

1. a. March had 2 more rainy days than April.
b. Together the months had 14 rainy days.
c. How many rainy days did each have?

$m = \_\_ + \_\_$
$m + a = \_\_$

2. a. Jenny has 8 more books than Carla.
b. Together they have 24 books.
c. How many books does each girl have?

$j = \_\_ + \_\_$
$\_\_ + c = \_\_$

3. a. Bill has collected 4 fewer rocks than Joe.
b. Together they have collected 38 rocks.
c. How many rocks has each boy collected?

4. a. Tom has 2 fewer pets than Dick.
b. Together they have 6 pets.
c. How many pets does each boy have?

5. a. Kathy is twice as old as Ann.
b. Together their ages total 18.
c. How old is each girl?

Feedback

Choose the mathematical sentence that matches each English sentence.

1.	Dora weighs 2 pounds less than Fay.	**A.**	$d = 2f$
2.	Dick and Frank have 34 model planes.	**B.**	$d < f$
3.	Dolly has fewer sisters than Flora.	**C.**	$d = f + 34$
4.	Donna has half as many dolls as Frances.	**D.**	$d = f - 2$
5.	Don has twice as many books as Fred.	**E.**	$d = f \div 2$
6.	David weighs 34 pounds more than Freda.	**F.**	$d + f = 34$

Simplify each expression.

7. $s + 4 + 2 =$ ■

8. $y + 2y + 6 =$ ■

9. $a + 4a - 2a =$ ■

10. $4b + 3b - 5 =$ ■

Write an equation for each sentence and complete the problem.

11. **a.** Pierre has 8 more marbles than Sid.
b. Together they have 42 marbles.
c. How many does each boy have?

12. **a.** Don has 1 more sister than Bill.
b. Together they have 5 sisters.
c. How many sisters does each boy have?

13. James has twice as many arrowheads as Michael. James and Shawn together have 24 arrowheads. Shawn has 8 arrowheads. How many arrowheads does each boy have?

14. Betty has 3 more butterflies than Andra. Carol has four times as many butterflies as Andra. Betty has 9 butterflies. How many butterflies does each girl have?

Feedback

List the members of each set if each member is a whole number.

1. $\{n: n < 8\}$
2. $\{n: 15 < n < 19\}$
3. $\{n: 27 < n < 31\}$
4. $\{n: 137 < n < 142\}$
5. $\{n: 599 < n < 604\}$
6. $\{n: 1998 < n < 2003\}$

Find the sums or differences.

7. $\frac{2}{3} + \frac{3}{4}$
8. $\frac{1}{2} + \frac{1}{3}$
9. $\frac{2}{9} + \frac{4}{5}$
10. $\frac{7}{8} + \frac{5}{6}$
11. $\frac{6}{7} - \frac{1}{3}$
12. $\frac{4}{5} - \frac{3}{4}$
13. $\frac{7}{13} - \frac{1}{2}$
14. $\frac{5}{6} - \frac{2}{7}$

Solve each equation.

15. $6 + (-4) = \square$
16. $(-7) + (-5) = \square$
17. $(-8) + 7 = \square$
18. $5 + \square = 3$
19. $(-8) + \square = -4$
20. $\square + (-4) = -7$

21. Miss Thomas bought 27 yards of ribbon for a play. How many inches of ribbon was this?

22. Mr. Murphy received 12 boxes of books. Each box contained 36 books. How many books did he receive?

23. Tom needed some pieces of balsa 6 inches in length for a bridge he was building. If he had 20 strips of balsa that were 36 inches in length, how many 6-inch pieces could he cut?

24. Today there were 950 pints of milk sold at the West City High School cafeteria. If 494 pints of milk were sold to boys, how many pints of milk were sold to girls?

Part 15 Using Fractions

To add fractions we may need all these steps:

1) Rename the fractions so that they have a common denominator.
2) Add these fractions.
3) Express any improper fraction either as a whole number or as a mixed number.
4) Express the resulting fraction in lowest terms.

$$\begin{array}{r} \frac{5}{6} = \frac{25}{30} \\ +\ \frac{3}{10} = \frac{9}{30} \\ \hline \end{array}$$

$$\frac{34}{30} = 1\frac{4}{30} = 1\frac{2}{15}$$

Find each sum.

1. $\begin{array}{r} \frac{2}{3} \\ +\ \frac{2}{5} \\ \hline \end{array}$
2. $\begin{array}{r} \frac{2}{5} \\ +\ \frac{1}{2} \\ \hline \end{array}$
3. $\begin{array}{r} \frac{1}{6} \\ +\ \frac{1}{12} \\ \hline \end{array}$
4. $\begin{array}{r} \frac{8}{9} \\ +\ \frac{1}{2} \\ \hline \end{array}$
5. $\begin{array}{r} \frac{7}{15} \\ +\ \frac{5}{6} \\ \hline \end{array}$
6. $\begin{array}{r} \frac{1}{2} \\ +\ \frac{2}{3} \\ \hline \end{array}$
7. $\begin{array}{r} \frac{5}{6} \\ +\ \frac{7}{9} \\ \hline \end{array}$
8. $\begin{array}{r} \frac{1}{2} \\ +\ \frac{4}{7} \\ \hline \end{array}$
9. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{2}{5} \\ \hline \end{array}$
10. $\begin{array}{r} \frac{2}{9} \\ +\ \frac{5}{6} \\ \hline \end{array}$

To subtract fractions we may need all these steps:

1) Rename the fractions so that they have a common denominator.
2) Subtract these fractions.
3) Express the resulting fraction in lowest terms.

$$\begin{array}{r} \frac{5}{6} = \frac{25}{30} \\ -\ \frac{3}{10} = \frac{9}{30} \\ \hline \end{array}$$

$$\frac{16}{30} = \frac{8}{15}$$

Find each difference.

11. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{6} \\ \hline \end{array}$
12. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{3}{7} \\ \hline \end{array}$
13. $\begin{array}{r} \frac{3}{5} \\ -\ \frac{1}{3} \\ \hline \end{array}$
14. $\begin{array}{r} \frac{5}{9} \\ -\ \frac{1}{4} \\ \hline \end{array}$
15. $\begin{array}{r} \frac{5}{7} \\ -\ \frac{2}{3} \\ \hline \end{array}$
16. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{1}{4} \\ \hline \end{array}$
17. $\begin{array}{r} \frac{3}{7} \\ -\ \frac{2}{5} \\ \hline \end{array}$
18. $\begin{array}{r} \frac{5}{6} \\ -\ \frac{11}{15} \\ \hline \end{array}$
19. $\begin{array}{r} \frac{7}{12} \\ -\ \frac{1}{4} \\ \hline \end{array}$
20. $\begin{array}{r} \frac{3}{5} \\ -\ \frac{3}{8} \\ \hline \end{array}$

Add or subtract.

1. $\frac{5}{6} - \frac{1}{4}$

2. $\frac{1}{4} + \frac{3}{7}$

3. $\frac{2}{3} - \frac{4}{7}$

4. $\frac{2}{5} + \frac{1}{2}$

5. $\frac{1}{6} + \frac{5}{8}$

6. $\frac{7}{11} - \frac{1}{2}$

7. $\frac{3}{4} - \frac{7}{10}$

8. $\frac{7}{8} + \frac{1}{12}$

9. $\frac{1}{4} + \frac{9}{10}$

10. $\frac{4}{7} + \frac{2}{3}$

11. $\frac{5}{6} + \frac{7}{8}$

12. $\frac{7}{6} - \frac{4}{5}$

13. $\frac{3}{9} + \frac{7}{9}$

14. $\frac{5}{8} + \frac{2}{3}$

15. $\frac{4}{5} - \frac{2}{3}$

16. $\frac{5}{6} - \frac{2}{9}$

17. Gary caught a trout that was $\frac{3}{4}$ ft. long. Bud's trout was $\frac{11}{12}$ ft. long. Whose trout was the longer? How much longer?

18. Sophie walked $\frac{5}{8}$ mi. one way to the store and walked $\frac{1}{2}$ mi. back another way. How far did she walk all together?

19. Alice practices her music $\frac{3}{4}$ hr. in the morning and $\frac{3}{4}$ hr. in the afternoon. How long does she practice all together?

20. Mrs. Dixon had a piece of material that was $\frac{7}{8}$ yd. in length. She made a small apron and used $\frac{3}{4}$ yd. of the material. How much material was left?

21. Kent, Steve, and David decided to tile a floor. If Kent and Steve each tiled $\frac{3}{8}$ of the floor, how much of the floor was left for David to tile?

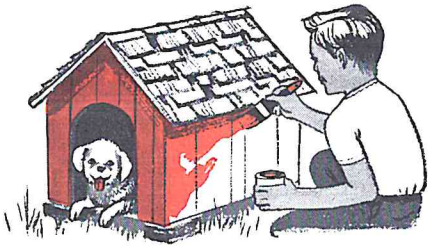
22. Terry's belt is $\frac{2}{3}$ yd. in length. Walt's belt is $\frac{5}{9}$ yd. in length. Whose belt is the longer? How much longer?

Add or subtract. Watch the signs. Give each answer in lowest terms.

1. $\frac{1}{2} - \frac{1}{7}$	2. $\frac{2}{7} - \frac{1}{8}$	3. $\frac{6}{7} + \frac{1}{3}$	4. $\frac{5}{14} + \frac{10}{21}$	5. $\frac{4}{5} + \frac{3}{7}$
6. $\frac{4}{5} - \frac{2}{3}$	7. $\frac{9}{10} - \frac{5}{6}$	8. $\frac{5}{12} + \frac{5}{6}$	9. $\frac{5}{21} - \frac{1}{6}$	10. $\frac{6}{7} - \frac{3}{5}$
11. $\frac{8}{11} + \frac{1}{3}$	12. $\frac{1}{6} + \frac{3}{14}$	13. $\frac{7}{8} - \frac{2}{3}$	14. $\frac{2}{9} + \frac{1}{4}$	15. $\frac{1}{3} + \frac{1}{5}$
16. $\frac{3}{10} + \frac{1}{6}$	17. $\frac{2}{3} - \frac{2}{9}$	18. $\frac{5}{6} - \frac{5}{18}$	19. $\frac{3}{8} - \frac{1}{5}$	20. $\frac{2}{7} + \frac{1}{4}$

21. Judy was given a cake. She ate one third of it yesterday and one half of it this morning. How much of the cake has she eaten? How much is left?

22. Charlie painted the doghouse red and blue. He used $\frac{5}{8}$ of a can of red paint and $\frac{2}{3}$ of a can of blue paint. How much paint did he use?



23. Marcy's school is $\frac{9}{10}$ of a mile from her house. If she takes a bus for $\frac{7}{8}$ of a mile, how far does she have to walk?

24. Carla wrote a story that was $\frac{3}{5}$ of a page long. She drew pictures to go with the story that took up $\frac{4}{7}$ of a page. Are the story and pictures together longer than one page? If so, how much longer?

25. Bruce's dog is $\frac{8}{9}$ of a foot tall. His cat is $\frac{7}{12}$ of a foot tall. Which is taller? How much taller?

Addition of Mixed Numbers

Sometimes we must express a mixed number another way when we add.

$$\begin{array}{r} 2\frac{5}{8} \\ +5\frac{7}{8} \\ \hline 7\frac{12}{8} = 8\frac{4}{8} \\ = 8\frac{1}{2} \end{array} \qquad \begin{array}{r} 3\frac{1}{4} \\ +2\frac{3}{4} \\ \hline 5\frac{4}{4} = 6 \end{array}$$

Copy and complete each problem.

1. $6\frac{7}{4} = 7\frac{\blacksquare}{4}$
2. $4\frac{6}{5} = 5\frac{\blacksquare}{5}$
3. $3\frac{8}{3} = \blacksquare\frac{2}{3}$
4. $2\frac{3}{2} = \blacksquare\frac{1}{2}$
5. $7\frac{10}{8} = 8\frac{\blacksquare}{8} = 8\frac{1}{\blacksquare}$
6. $9\frac{6}{4} = 10\frac{\blacksquare}{4} = 10\frac{1}{\blacksquare}$
7. $4\frac{20}{12} = 5\frac{\blacksquare}{12} = 5\frac{\blacksquare}{3}$
8. $3\frac{12}{10} = 4\frac{\blacksquare}{10} = 4\frac{\blacksquare}{5}$

Find each sum.

9. $4\frac{2}{6} + 3\frac{1}{6}$
10. $7\frac{5}{12} + 2\frac{3}{12}$
11. $6\frac{4}{6} + 3\frac{1}{6}$
12. $7\frac{3}{8} + 4\frac{3}{8}$
13. $6\frac{7}{10} + 2\frac{7}{10}$
14. $1\frac{2}{5} + 9\frac{3}{5}$
15. $2\frac{7}{8} + 1\frac{3}{8}$
16. $4\frac{1}{7} + 5\frac{6}{7}$
17. $7\frac{5}{9} + 2\frac{7}{9}$
18. $8\frac{9}{10} + 4\frac{9}{10}$
19. $6\frac{1}{4} + 3\frac{3}{4}$
20. $4\frac{3}{8} + 5\frac{7}{8}$

21. Sara bought $1\frac{1}{2}$ pounds of pecans, $2\frac{1}{4}$ pounds of English walnuts, and $\frac{5}{8}$ pound of almonds. How many pounds of nuts did she buy all together?

Subtraction of Mixed Numbers

Sometimes we must express a mixed number or a whole number another way when we subtract.

$8\frac{1}{4} = 7 + 1\frac{1}{4}$ $\qquad$ $9 = 8 + 1$

$= 7\frac{5}{4}$ $\qquad$ $= 8\frac{4}{4}$

Each time the fraction is less than 2. When we subtract, we can write:

$$\begin{array}{r} 8\frac{1}{4} = 7\frac{5}{4} \\ -2\frac{3}{4} = -2\frac{3}{4} \\ \hline 5\frac{2}{4} = 5\frac{1}{2} \end{array} \qquad \begin{array}{r} 9 = 8\frac{4}{4} \\ -4\frac{1}{4} = -4\frac{1}{4} \\ \hline 4\frac{3}{4} \end{array}$$

Complete each problem.

1. $6\frac{2}{3} = 5\frac{\square}{3}$
2. $7\frac{2}{9} = 6\frac{\square}{9}$
3. $6 = 5\frac{\square}{3}$
4. $3\frac{1}{4} = 2\frac{\square}{4}$
5. $8\frac{1}{3} = 7\frac{\square}{3}$
6. $4 = 3\frac{\square}{5}$
7. $4\frac{5}{8} = 3\frac{\square}{8}$
8. $9\frac{7}{9} = 8\frac{\square}{9}$
9. $6\frac{5}{6} = 5\frac{\square}{6}$
10. $8\frac{2}{7} = 7\frac{\square}{7}$
11. $6 = 5\frac{\square}{10}$
12. $3\frac{3}{8} = 2\frac{\square}{8}$

Find each difference.

13. $4\frac{1}{3} - 1\frac{2}{3}$
14. $6\frac{2}{9} - 4\frac{3}{9}$
15. $4 - \frac{3}{4}$
16. $7\frac{1}{11} - 2\frac{3}{11}$
17. $8\frac{2}{5} - 3\frac{4}{5}$
18. $6\frac{1}{5} - 5\frac{2}{5}$
19. $6 - 1\frac{5}{7}$
20. $2\frac{1}{12} - \frac{8}{12}$
21. $7\frac{1}{8} - 4\frac{7}{8}$
22. $9 - 3\frac{5}{6}$
23. $7\frac{2}{12} - 6\frac{9}{12}$
24. $3 - 2\frac{5}{6}$

Add or subtract. Give each answer in lowest terms.

1. $4\frac{3}{4} + 2\frac{3}{4}$

2. $6\frac{5}{7} - 4\frac{6}{7}$

3. $7\frac{4}{6} - 4\frac{1}{6}$

4. $2\frac{3}{24} + 7\frac{5}{24}$

5. $9\frac{5}{9} - 7\frac{2}{9}$

6. $3\frac{1}{3} + 5\frac{2}{3}$

7. $6\frac{2}{7} + 3\frac{6}{7}$

8. $6\frac{2}{10} - 5\frac{7}{10}$

9. $6\frac{4}{11} + 1\frac{7}{11}$

10. $1\frac{11}{20} + 8\frac{9}{20}$

11. $2\frac{1}{6} + 2\frac{3}{6}$

12. $9\frac{1}{3} - 6\frac{2}{3}$

13. $8\frac{3}{15} - 3\frac{2}{15}$

14. $8\frac{1}{4} - 5\frac{3}{4}$

15. $1\frac{3}{14} + 8\frac{4}{14}$

16. $2\frac{1}{9} + 6\frac{2}{9}$

17. Mike's kite string was only thirteen and three-quarters feet long. He tied another piece of string to it to make the total length twenty feet. What was the length of the second piece of string?

18. Margot's frog jumped $16\frac{5}{8}$ feet. Tim's frog jumped $14\frac{3}{8}$ feet. How much farther did Margot's frog jump?

19. Mrs. Carson put $3\frac{1}{6}$ cups of brown sugar and $2\frac{1}{6}$ cups of powdered sugar in a bowl. How much sugar was in the bowl all together?

20. Joel ran around the track $5\frac{2}{10}$ times this morning and $3\frac{3}{10}$ times this afternoon. How many times did he run around the track today?

21. Carol rode her bicycle $2\frac{3}{10}$ miles on Monday and $4\frac{1}{10}$ miles on Tuesday. How far did she bicycle on both days?

22. Mr. Erwin had $3\frac{7}{8}$ gallons of paint. He used $2\frac{1}{2}$ gallons. How much paint did he have left?

Add or subtract. Give each answer in lowest terms.

1. $7\frac{13}{18} + 4\frac{7}{18}$

2. $6\frac{2}{3} - 4\frac{2}{3}$

3. $2\frac{4}{18} + 5\frac{5}{18}$

4. $8\frac{11}{12} - 6\frac{5}{12}$

5. $3\frac{2}{5} - 2\frac{4}{5}$

6. $5\frac{1}{4} - 2\frac{3}{4}$

7. $6\frac{3}{4} - 2\frac{3}{8}$

8. $5\frac{1}{3} + 3\frac{2}{3}$

9. Alex is building a tree-house $6\frac{3}{8}$ ft. wide. He has a board $8\frac{5}{8}$ ft. long. How much must he cut off to make the board fit?

10. Lee has $2\frac{5}{16}$ yd. of red ribbon. Laura has $3\frac{3}{16}$ yd. of ribbon. How much ribbon do they have together?

11. Chris swam $8\frac{3}{4}$ ft. under water. Sylvia swam $6\frac{1}{4}$ feet. How much farther did Chris swim under water than Sylvia?

12. Kay and Dick are riding their horses to their cousins' house which is $6\frac{7}{10}$ miles away. They have already ridden for $4\frac{3}{10}$ miles. How much farther must they go?

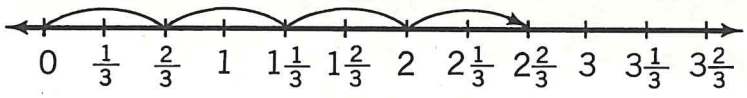
13. Danny has $2\frac{7}{8}$ lb. of hamburger, but he will need $4\frac{3}{8}$ lb. for his barbecue party. How many pounds must he buy?

14. One truck carried $3\frac{1}{4}$ tons of dirt from the space where the new hotel will be built. A second truck carried away $4\frac{3}{4}$ tons. How much dirt did both trucks carry?

We can multiply a fraction by a whole number.

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

We can show this on a number line.



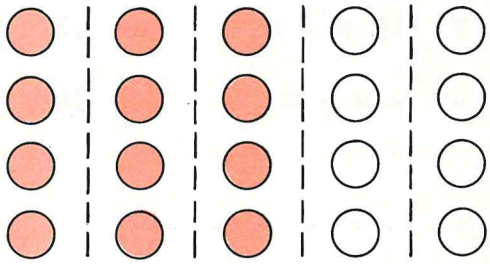
$$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 4 \times \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

Give each problem as a multiplication equation and find the product.

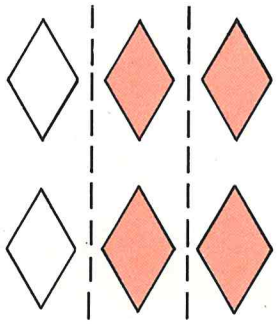
1. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$
2. $\frac{5}{6} + \frac{5}{6}$
3. $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$
4. $\frac{6}{7} + \frac{6}{7} + \frac{6}{7}$
5. $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
6. $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

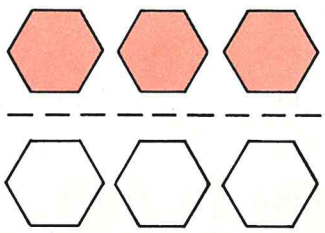
There are 20 balls divided into 5 small sets. The balls in 3 of the small sets are shaded. The balls in $\frac{3}{5}$ of the whole set are shaded.

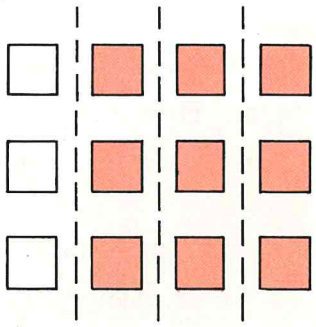
$$\frac{3}{5} \times 20 = 12$$



Give a multiplication equation for each problem.

7. 

8. 

9. 

To multiply a fraction and a whole number:

1) Multiply the numerator of the fraction by the whole number.
2) Simplify the product.

$$2 \times \frac{5}{6} = \frac{2 \times 5}{6} = \frac{10}{6} = 1\frac{4}{6} = 1\frac{2}{3}$$

$$\frac{4}{7} \times 3 = \frac{4 \times 3}{7} = \frac{12}{7} = 1\frac{5}{7}$$

Find each product.

1. $3 \times \frac{6}{7} =$ ■
2. $1 \times \frac{8}{9} =$ ■
3. $4 \times \frac{3}{5} =$ ■
4. $7 \times \frac{7}{10} =$ ■
5. $8 \times \frac{3}{4} =$ ■
6. $12 \times \frac{1}{3} =$ ■
7. $16 \times \frac{3}{8} =$ ■
8. $9 \times \frac{5}{6} =$ ■
9. $\frac{1}{2} \times 8 =$ ■
10. $\frac{3}{7} \times 21 =$ ■
11. $\frac{2}{3} \times 18 =$ ■
12. $\frac{3}{8} \times 16 =$ ■
13. $\frac{5}{6} \times 12 =$ ■
14. $\frac{1}{3} \times 8 =$ ■
15. $\frac{5}{6} \times 9 =$ ■
16. $\frac{1}{4} \times 20 =$ ■
17. $5 \times \frac{2}{5} =$ ■
18. $18 \times \frac{1}{3} =$ ■
19. $20 \times \frac{3}{4} =$ ■
20. $6 \times \frac{2}{3} =$ ■
21. $16 \times \frac{5}{8} =$ ■
22. $\frac{5}{6} \times 30 =$ ■
23. $20 \times \frac{2}{5} =$ ■
24. $\frac{2}{5} \times 9 =$ ■

Choose $<$ or $=$ or $>$.

25. 2×12 ● 12
26. $\frac{1}{2} \times 12$ ● 12
27. 3×12 ● 12
28. 4×30 ● 30
29. $\frac{1}{3} \times 30$ ● 30
30. 1×30 ● 30

31. Mr. Watson had 18 melons. He sold $\frac{2}{3}$ of them.
    a. How many melons did he sell?
    b. How many melons did he have left?

32. Helen practiced her music $\frac{4}{5}$ of an hour. If there are 60 minutes in an hour, how many minutes did she practice?

Find each product.

1. $\frac{4}{5} \times 9 = \square$
2. $\frac{3}{4} \times 21 = \square$
3. $\frac{9}{10} \times 15 = \square$
4. $\frac{5}{6} \times 24 = \square$
5. $\frac{3}{5} \times 11 = \square$
6. $8 \times \frac{7}{9} = \square$
7. $\frac{7}{10} \times 1 = \square$
8. $\frac{3}{8} \times 12 = \square$
9. $7 \times \frac{5}{6} = \square$
10. $0 \times \frac{2}{5} = \square$
11. $\frac{4}{7} \times 8 = \square$
12. $1 \times \frac{3}{4} = \square$

Choose $<$ or $=$ or $>$.

13. $\frac{3}{4} \times 20 \bigcirc 20$
14. $1 \times 20 \bigcirc 20$
15. $0 \times 20 \bigcirc 20$
16. $4 \times 20 \bigcirc 20$
17. $2 \times 200 \bigcirc 20$
18. $\frac{5}{8} \times 200 \bigcirc 200$
19. $1 \times 200 \bigcirc 200$
20. $0 \times 200 \bigcirc 200$

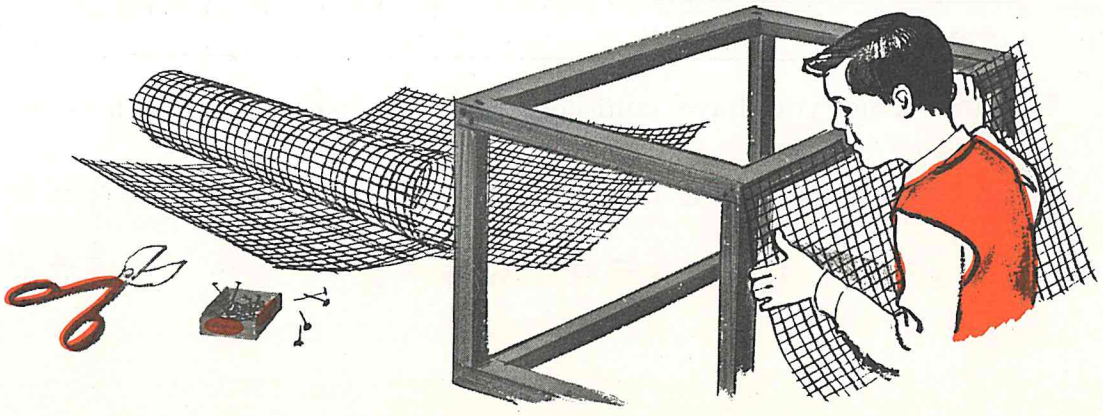
21. Mrs. Keller made 6 doll dresses. If she used $\frac{2}{3}$ yd. of material for each dress, how many yards of material did she need?

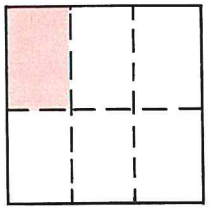
22. Dan used $\frac{3}{4}$ yd. of paper to make a kite. How much paper did he use to make 4 kites?

23. There are 60 minutes in an hour. How many minutes are there in $\frac{5}{6}$ of an hour?

24. Harry weighed 106 pounds. Harley weighed only $\frac{2}{3}$ as much as Harry. How much did Harley weigh?

25. Mark built a cage for his pet squirrel. He used 8 strips of wire each $\frac{3}{4}$ yd. long. How many yards of wire did he need?



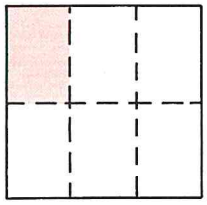


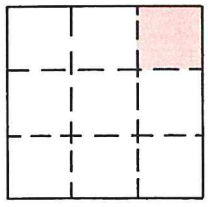
The area of this square is one square inch. We can see that $\frac{1}{6}$ of the square is shaded.
We know that length times width equals area.

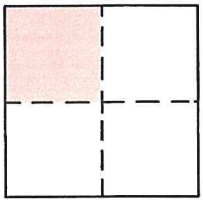
The length of the shaded area is $\frac{1}{2}$ inch.
The width of the shaded area is $\frac{1}{3}$ inch.
So we know:

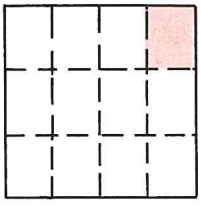
$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

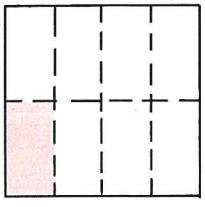
1. The area of each of these squares is one square inch. Use the squares to complete the table below.

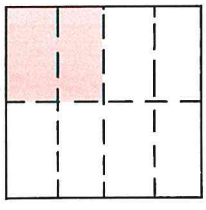
a. 

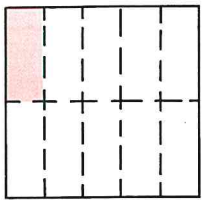
b. 

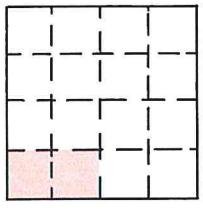
c. 

d. 

e. 

f. 

g. 

h. 

Square	a	b	c	d	e	f	g	h
Length of shaded part (in inches)	$\frac{1}{2}$							
Width of shaded part (in inches)	$\frac{1}{3}$							
Area of shaded part (in sq. inches)	$\frac{1}{6}$							

2. Use the table you have completed in Exercise 1 to complete these equations.

a. $\frac{1}{2} \times \frac{1}{3} =$ ▢ **b.** $\frac{1}{3} \times \frac{1}{3} =$ ▢ **c.** $\frac{1}{2} \times \frac{1}{2} =$ ▢ **d.** $\frac{1}{3} \times \frac{1}{4} =$ ▢

e. $\frac{1}{2} \times \frac{1}{4} =$ ▢ **f.** $\frac{1}{2} \times \frac{2}{4} =$ ▢ **g.** $\frac{1}{2} \times \frac{1}{5} =$ ▢ **h.** $\frac{1}{4} \times \frac{2}{4} =$ ▢

To multiply a fraction times a fraction we may use these steps:

1) Multiply the numerators.
2) Multiply the denominators.
3) Express the fraction in lowest terms.

$$\frac{5}{8} \times \frac{4}{6} = \frac{5 \times 4}{8 \times 6} = \frac{20}{48} = \frac{5}{12}$$

Find each product. Express each product in lowest terms.

1. $\frac{1}{2} \times \frac{3}{4} =$ ■
2. $\frac{2}{3} \times \frac{2}{2} =$ ■
3. $\frac{4}{4} \times \frac{5}{5} =$ ■
4. $\frac{1}{2} \times \frac{4}{5} =$ ■
5. $\frac{3}{4} \times \frac{4}{5} =$ ■
6. $\frac{3}{5} \times \frac{5}{6} =$ ■
7. $\frac{2}{3} \times \frac{9}{2} =$ ■
8. $\frac{8}{7} \times \frac{7}{8} =$ ■
9. $\frac{1}{3} \times 9 =$ ■
10. $\frac{4}{6} \times \frac{3}{2} =$ ■
11. $\frac{2}{4} \times \frac{4}{3} =$ ■
12. $\frac{1}{4} \times 2 =$ ■
13. $\frac{2}{3} \times \frac{3}{7} =$ ■
14. $15 \times \frac{1}{5} =$ ■
15. $\frac{1}{3} \times \frac{7}{8} =$ ■

16. At 9 A.M. a 36-foot pole casts a shadow $\frac{7}{8}$ the height of the pole. How long is the shadow?

17. At 5 P.M. the 36-foot pole casts a shadow $\frac{3}{2}$ the height of the pole. How long is the shadow?

18. Jill bought $\frac{3}{8}$ lb. of candy that cost 32¢ per pound. How much did the candy cost?

19. Al drank $\frac{3}{4}$ qt. of milk. Pete drank $\frac{2}{3}$ as much as Al. How much milk did Pete drink?

20. Cathy walked $\frac{7}{8}$ mi. Jean walked $\frac{1}{3}$ the distance Cathy walked. How far did Jean walk?

21. A recipe for cookies calls for $\frac{3}{4}$ cup of sugar. Maxine wishes to make only $\frac{1}{2}$ this number of cookies. How much sugar should she use?

Multiplicative Inverse

Find each product.

1. $\frac{1}{2} \times 2 = \square$
2. $\frac{2}{3} \times 3 = \square$
3. $\frac{1}{4} \times 4 = \square$
4. $\frac{1}{3} \times 3 = \square$
5. $\frac{1}{6} \times 6 = \square$
6. $\frac{2}{3} \times \frac{3}{2} = \square$
7. $\frac{3}{7} \times \frac{2}{3} = \square$
8. $\frac{3}{4} \times \frac{2}{3} = \square$
9. $\frac{8}{9} \times 9 = \square$
10. $\frac{6}{5} \times \frac{5}{6} = \square$
11. $\frac{3}{5} \times 25 = \square$
12. $\frac{4}{7} \times \frac{7}{4} = \square$

Complete each problem.

13. $\frac{1}{4} \times \square = 1$
14. $\frac{1}{3} \times \square = 1$
15. $2 \times \square = 1$
16. $\frac{2}{5} \times \square = 1$
17. $\frac{3}{7} \times \square = 1$
18. $\frac{4}{5} \times \square = 1$
19. $3 \times \square = 1$
20. $\frac{1}{6} \times \square = 1$
21. $8 \times \square = 1$

Pairs of numbers like $\frac{1}{4}$ and 4 or $\frac{2}{3}$ and $\frac{3}{2}$ are **multiplicative inverses.**
The product of any number and its multiplicative inverse is one.
The multiplicative inverse of 4 is $\frac{1}{4}$. $4 \times \frac{1}{4} = 1$
The multiplicative inverse of $\frac{2}{3}$ is $\frac{3}{2}$. $\frac{2}{3} \times \frac{3}{2} = 1$
Zero has no multiplicative inverse.

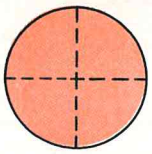
Give the multiplicative inverse of each number.

22. 6
23. $\frac{7}{3}$
24. $\frac{4}{5}$
25. $\frac{6}{5}$
26. 2
27. $\frac{1}{3}$
28. $\frac{7}{8}$
29. $\frac{9}{4}$
30. 8
31. $\frac{7}{16}$
32. $\frac{3}{5}$
33. 1

34. Which number has no multiplicative inverse?
35. Which number is its own multiplicative inverse?

Division of Fractions

We can divide by a whole number. We wish to divide a pie into 4 pieces and find the size of each piece. We divide by 4.

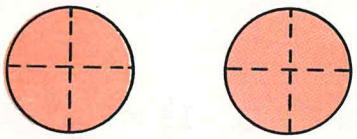


$$1 \div 4 = \frac{1}{4}$$

To divide by 4 gives the same result as multiplying by the multiplicative inverse of 4.

$$1 \div 4 = 1 \times \frac{1}{4} = \frac{1}{4}$$

We can divide by a fraction. We wish to divide 2 pies into fourths and to find the number of pieces we have. We divide by $\frac{1}{4}$.



$$2 \div \frac{1}{4} = 8$$

To divide by $\frac{1}{4}$ gives the same result as multiplying by the multiplicative inverse of $\frac{1}{4}$.

$$2 \div \frac{1}{4} = 2 \times 4 = 8$$

Copy and complete each problem.

1. $3 \div \frac{1}{5} = 3 \times \square = \square$
2. $6 \div \frac{2}{3} = 6 \times \square = \square$
3. $8 \div \frac{7}{4} = 8 \times \square = \square$
4. $6 \div \frac{2}{5} = 6 \times \square = \square$
5. $4 \div \frac{1}{7} = 4 \times \square = \square$
6. $\frac{7}{8} \div \frac{1}{2} = \frac{7}{8} \times \square = \square$
7. $\frac{4}{5} \div \frac{4}{5} = \frac{4}{5} \times \square = \square$
8. $\frac{9}{10} \div 3 = \frac{9}{10} \times \square = \square$
9. $\frac{8}{9} \div 4 = \frac{8}{9} \times \square = \square$
10. $\frac{4}{5} \div \frac{1}{5} = \frac{4}{5} \times \square = \square$
11. $2 \div \frac{2}{3} = 2 \times \square = \square$
12. $\frac{7}{9} \div 3 = \frac{7}{9} \times \square = \square$
13. $\frac{3}{10} \div \frac{4}{5} = \frac{3}{10} \times \square = \square$
14. $8 \div \frac{2}{5} = 8 \times \square = \square$

Find each quotient.

1. $8 \div \frac{1}{4} =$ ■
2. $4 \div \frac{2}{5} =$ ■
3. $\frac{2}{3} \div \frac{2}{3} =$ ■
4. $\frac{5}{6} \div \frac{1}{2} =$ ■
5. $\frac{4}{5} \div 8 =$ ■
6. $\frac{2}{5} \div 1 =$ ■
7. $\frac{7}{8} \div \frac{1}{4} =$ ■
8. $8 \div \frac{2}{3} =$ ■
9. $6 \div \frac{3}{4} =$ ■
10. $\frac{7}{8} \div \frac{2}{5} =$ ■
11. $\frac{8}{9} \div 4 =$ ■
12. $\frac{9}{10} \div \frac{1}{5} =$ ■

We can multiply and divide mixed numbers.

$$1\tfrac{1}{4} \times 2\tfrac{1}{2} = \tfrac{5}{4} \times \tfrac{5}{2} = \tfrac{5 \times 5}{4 \times 2} = \tfrac{25}{8} = 3\tfrac{1}{8}$$

$$1\tfrac{1}{5} \div 1\tfrac{2}{3} = \tfrac{6}{5} \div \tfrac{5}{3} = \tfrac{6}{5} \times \tfrac{3}{5} = \tfrac{18}{25}$$

Find each product.

13. $1\frac{1}{2} \times \frac{2}{5} =$ ■
14. $1\frac{3}{4} \times 1\frac{1}{3} =$ ■
15. $4\frac{2}{3} \times 9 =$ ■
16. $2\frac{1}{2} \times 1\frac{1}{2} =$ ■
17. $3 \times 4\frac{1}{2} =$ ■
18. $3\frac{1}{2} \times \frac{3}{7} =$ ■
19. $1\frac{2}{3} \times 1\frac{1}{5} =$ ■
20. $2\frac{1}{3} \times 1\frac{1}{2} =$ ■
21. $7\frac{1}{2} \times 1\frac{2}{5} =$ ■

Find each quotient.

22. $8\frac{1}{3} \div \frac{5}{6} =$ ■
23. $4\frac{1}{5} \div 1\frac{3}{7} =$ ■
24. $1\frac{2}{3} \div 1\frac{1}{2} =$ ■
25. $3\frac{1}{2} \div 2 =$ ■
26. $8 \div \frac{2}{3} =$ ■
27. $4\frac{1}{2} \div 1\frac{1}{4} =$ ■
28. $2\frac{2}{5} \div 1\frac{1}{5} =$ ■
29. $6 \div 1\frac{1}{2} =$ ■
30. $3\frac{1}{3} \div 2 =$ ■

31. Charles has 40 feet of wire. He uses $1\frac{1}{4}$ feet for each puzzle he is making.
    a. How many puzzles can he make?
    b. Will he have any wire left?
    c. How much wire does he need to make 20 puzzles?

1. Carol bought 6 pieces of ribbon each $1\frac{3}{4}$ yards in length. How many yards of ribbon did she buy?

2. Dan has a piece of lumber 6 feet long. He needs some pieces $1\frac{1}{4}$ feet long. How many pieces can he cut from the 6-foot piece?

3. Mr. Alexander plowed $7\frac{4}{5}$ acres in one day. On the next day he plowed $2\frac{3}{5}$ acres. How many acres did he plow all together?

4. The workmen have painted white lines on $5\frac{7}{10}$ miles of the highway. They must paint another $3\frac{5}{10}$ miles to finish the highway. How many miles will they have painted?

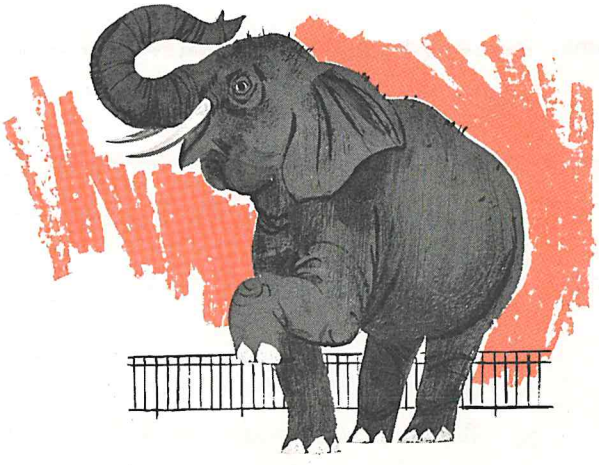
5. Mrs. Parker has 4 pounds of hamburger. If she makes hamburger patties that weigh $\frac{1}{4}$ of a pound each, how many patties can she make?

6. Mary has a horse that is 14 hands high. If each hand is $\frac{1}{3}$ of a foot long, what is the horse's height in feet?

7. If Mrs. Murphy pays \$1.00 a pound for a certain kind of steak, how much must she pay for $1\frac{3}{4}$ pounds of that steak?

8. Mr. Walsh is $5\frac{5}{6}$ feet tall. How many inches tall is Mr. Walsh?

9. Jumbo, the elephant in the zoo, weighs $5\frac{1}{4}$ tons.
   a. A ton is the same weight as 2000 pounds. How many pounds does Jumbo weigh?
   b. A new baby elephant weighs about 200 pounds. Will 40 baby elephants weigh as much as Jumbo weighs?



Feedback

Find each sum or difference.

1. $\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline \end{array}$

2. $\begin{array}{r} 4\frac{1}{3} \\ +3\frac{5}{12} \\ \hline \end{array}$

3. $\begin{array}{r} 6\frac{7}{5} \\ -2\frac{1}{5} \\ \hline \end{array}$

4. $\begin{array}{r} 8\frac{5}{7} \\ +9\frac{5}{14} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{5}{6} \\ +\frac{2}{9} \\ \hline \end{array}$

6. $\begin{array}{r} 8 \\ -4\frac{2}{5} \\ \hline \end{array}$

7. $\begin{array}{r} 8\frac{1}{3} \\ -3\frac{3}{4} \\ \hline \end{array}$

8. $\begin{array}{r} 7\frac{2}{5} \\ +4\frac{1}{3} \\ \hline \end{array}$

Complete each problem.

9. $\frac{2}{3} \times 8 =$ ■

10. $\frac{3}{5} \times \frac{4}{9} =$ ■

11. $\frac{7}{8} \div \frac{1}{2} =$ ■

12. $8 \div \frac{2}{3} =$ ■

13. $\frac{3}{4} + \frac{4}{5} =$ ■

14. $3 \times \frac{4}{5} =$ ■

15. $2 \div \frac{1}{4} =$ ■

16. $\frac{3}{4} \times \frac{8}{9} =$ ■

17. $\frac{3}{5} - \frac{3}{10} =$ ■

18. $\frac{9}{10} + \frac{2}{10} =$ ■

19. $1\frac{1}{4} \times \frac{3}{10} =$ ■

20. $1\frac{2}{3} \div 2\frac{1}{2} =$ ■

Choose $=$ or $\neq$.

21. $\frac{5}{10}$ ● $\frac{50}{100}$

22. $\frac{4}{4}$ ● 1

23. $\frac{3}{5}$ ● $\frac{6}{10}$

24. 23 ● $2\frac{3}{10}$

25. 1 ● $\frac{1}{1}$

26. $\frac{18}{20}$ ● $\frac{4}{5}$

27. Jean colored $1\frac{1}{3}$ dozen eggs and Tena colored $\frac{5}{6}$ dozen. How many dozen eggs did they color?

28. Dick is collecting stamps. He has 6 pages in his book that are $\frac{3}{4}$ filled. How many full pages would he need for his stamps?

29. Mrs. Valencia has 6 yards of toweling. She uses $\frac{2}{3}$ yard for each towel she makes.
    a. How many towels can she make?
    b. Will she have any material left?

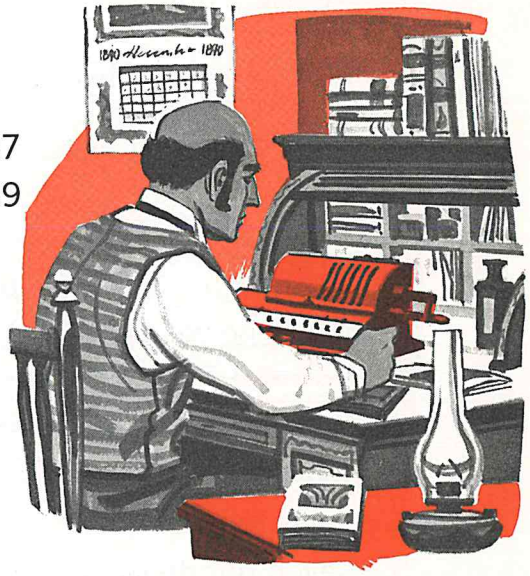
Feedback

Write in columns and then add.

1. 23,700 + 31,268 + 79,345
2. 19,367 + 23,499 + 64,646
3. 76,127 + 54,798 + 14,302
4. 81,349 + 43,027 + 60,004

Write in columns and then subtract.

5. 73,106 − 21,769
6. 45,342 − 37,731
7. 84,430 − 68,257
8. 38,006 − 29,739



9. Compute each power.

a. 10^1	e. 10^{10}	i. 3^3
b. 10^3	f. 10^5	j. 2^5
c. 10^6	g. 3^2	k. 6^2
d. 10^8	h. 2^4	l. 8^2

Choose $=$ or $\neq$.

10. 6 + 18 ● 18 + 6
11. 4 − 6 ● 6 − 4
12. 2 ÷ 4 ● 4 ÷ 2
13. 9 × (6 + 5) ● (9 × 6) + (9 × 5)
14. (8 − 4) − 2 ● 8 − (4 − 2)
15. (6 × 5) × 7 ● 6 × (5 × 7)

16. Pete has 256 marbles. Max has 4. Max wins $\frac{1}{4}$ of Pete's marbles. How many marbles does Max then have? How many marbles does Pete have left?

17. Write a mathematical sentence for each English sentence and find the answer.
    a. Jack has 3 more stamps than Fred.
    b. Together they have 77 stamps.
    c. How many stamps does each boy have?

Part 16 Decimal Fractions

We can count by tenths.

Instead of showing one tenth as $\frac{1}{10}$, we can show it as .1.
We use a dot called a **decimal point.**

We write: .5 instead of $\frac{5}{10}$ — We read: five tenths

.05 instead of $\frac{5}{100}$ — five hundredths

.5 and .05 are **decimal fractions.** We often call them **decimals.**
Decimal fractions are those fractions whose denominators are 10 or a higher power of 10 such as 100 or 1000.

Give a fraction name and a decimal name for each number.

1. four tenths
2. eight tenths
3. seven hundredths
4. one tenth
5. one and seventeen hundredths
6. one and nine tenths
7. two and three tenths
8. four and two tenths

Give each of these decimal fractions in fraction form.

9. .8 = $\frac{8}{10}$
10. .4 =
11. .6 =
12. .9 =
13. .2 =
14. .71 =
15. .03 =
16. .01 =

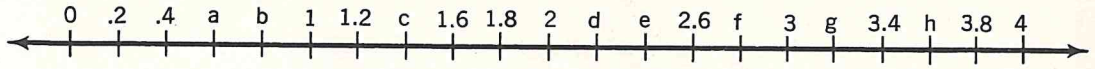
Give each of these fractions as a decimal fraction.

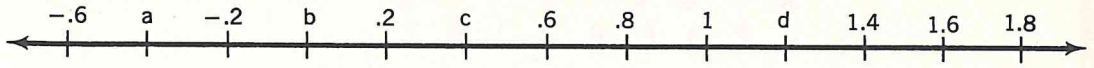
17. $\frac{3}{10}$ = .3
18. $\frac{9}{10}$ =
19. $\frac{1}{10}$ =
20. $\frac{7}{10}$ =
21. $\frac{6}{10}$ =
22. $\frac{8}{10}$ =
23. $\frac{2}{100}$ =
24. $\frac{4}{100}$ =

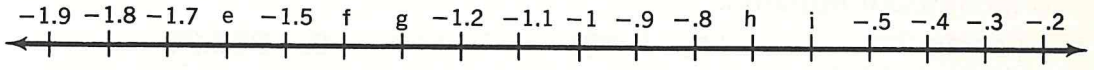
Give each of these decimals in English words.

25. .4
26. .1
27. .19
28. 1.8

Tell what number is represented by each letter.

1. 

2. 

3. 

Complete each pattern.

4. .8 | .9 | | 1.1 | | |

5. .2 | .4 | .6 | | | |

6. 1.4 | 1.6 | 1.8 | | | |

7. 3.5 | 3.7 | | | | 4.5 |

Choose < or = or >.

8. 0 ● .6
9. .4 ● .2
10. 1.3 ● 2
11. 1.2 ● 1.3
12. .5 ● 1
13. −.5 ● 0
14. .7 ● −.9
15. .4 ● −.4
16. −3.5 ● −2.0

17. Copy and complete the table below.

+	.01	.02	.03	.04	.05	.06			.09
.10	.11	.12					.17		
.20								.28	
				.34					
						.46			
.50									
									.69
		.72							
.80							.87		
			.93						

We use decimals to name more place values.

millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones	tenths	hundredths
4	2	9	6	8	3	1	4	2

Read aloud each numeral.

1. 146.32
2. 37.16
3. 7024.2
4. 865.38
5. 5.421
6. 96.81
7. 85.09
8. 1000.01
9. 268.25
10. 4316.8
11. 921.243
12. 5019.24

Write in decimal notation.

13. $\frac{34}{100} =$ ▢
14. $\frac{2}{10} =$ ▢
15. $\frac{3}{100} =$ ▢
16. $\frac{46}{100} =$ ▢
17. $11\frac{8}{10} =$ ▢
18. $7\frac{14}{100} =$ ▢
19. $3\frac{6}{10} =$ ▢
20. $14\frac{2}{100} =$ ▢
21. $90\frac{13}{100} =$ ▢
22. $17\frac{8}{100} =$ ▢
23. $9\frac{91}{100} =$ ▢
24. $10\frac{1}{100} =$ ▢

Give a decimal for each sum.

25. $4 + \frac{2}{10} + \frac{3}{100} =$ ▢
26. $6 + \frac{3}{10} + \frac{5}{100} =$ ▢
27. $8 + \frac{0}{10} + \frac{4}{100} + \frac{5}{1000} =$ ▢
28. $7 + \frac{2}{10} + \frac{0}{100} + \frac{4}{1000} =$ ▢
29. $9 + \frac{8}{100} =$ ▢
30. $3 + \frac{1}{10} + \frac{6}{100} =$ ▢
31. $5 + \frac{4}{10} + \frac{7}{1000} =$ ▢
32. $2 + \frac{5}{1000} =$ ▢

Express each number in expanded notation.

33. $8.34 = 8 + \frac{3}{10} + \frac{4}{100}$
34. $9.845 =$ ▢
35. $5.186 =$ ▢
36. $2.75 =$ ▢
37. $3.194 =$ ▢
38. $4.672 =$ ▢

Addition of Decimals

We can add decimals.

$.7 + .5 = 1.2$

$\frac{7}{10} + \frac{5}{10} = \frac{12}{10} = 1\frac{2}{10}$

We read: Seven tenths plus five tenths equals one and two tenths.

Show each problem on a number line. Then copy and complete each problem.

1. $.4 + .2 =$ ■
2. $.8 + .1 =$ ■
3. $.8 + .4 =$ ■
4. $.5 + .5 =$ ■
5. $.4 + .9 =$ ■
6. $.7 + .8 =$ ■

We can also write addition problems another way.

$$\begin{array}{r} .4 \\ +\ .2 \\ \hline .6 \end{array}$$

We keep the tenths in a column. We put the decimal points in a straight line to help us do this.

We know that 10 tenths equal 1 whole. We use this fact when we add. When we add mixed numbers we first add the fractions. Then we add the whole numbers.

Step 1

$$\begin{array}{r} 2\frac{8}{10} \\ +\ 5\frac{4}{10} \\ \hline \frac{12}{10} = 1\frac{2}{10} \end{array}$$

Step 2

$$\begin{array}{r} {\scriptstyle 1} \\ 2\frac{8}{10} \\ +\ 5\frac{4}{10} \\ \hline 8\frac{2}{10} \end{array}$$

When we add decimal fractions we first add the digits in the right-hand column.

Step 1

$$\begin{array}{r} {\scriptstyle 1} \\ 2.8 \\ +\ 5.4 \\ \hline 2 \end{array}$$

Step 2

$$\begin{array}{r} {\scriptstyle 1} \\ 2.8 \\ +\ 5.4 \\ \hline 8.2 \end{array}$$

Find each sum.

7. $\begin{array}{r} .6 \\ +.7 \\ \hline \end{array}$
8. $\begin{array}{r} .7 \\ +.7 \\ \hline \end{array}$
9. $\begin{array}{r} .8 \\ +.9 \\ \hline \end{array}$
10. $\begin{array}{r} 1.6 \\ +2.3 \\ \hline \end{array}$
11. $\begin{array}{r} 4.7 \\ +3.5 \\ \hline \end{array}$

Find each sum.

1. a. $\frac{8}{10} + \frac{3}{10}$ b. $.8 + .3$

2. a. $\frac{9}{100} + \frac{6}{100}$ b. $.09 + .06$

3. a. $1\frac{5}{10} + 2\frac{3}{10}$ b. $1.5 + 2.3$

4. a. $1\frac{8}{10} + 3\frac{9}{10}$ b. $1.8 + 3.9$

Find each sum.

5. $.6 + .6$	6. $.9 + .9$	7. $.5 + .6$	8. $.7 + .3$	9. $.09 + .05$
10. $.8 + .7$	11. $.9 + .6$	12. $.9 + .9$	13. $.3 + .9$	14. $.81 + .64$
15. $1.5 + 3.2$	16. $1.9 + .7$	17. $2.4 + 3.8$	18. $1.3 + 2.7$	19. $3.42 + 2.23$

Write in columns. Find each sum.

20. $5.1 + 1.9 =$ ▪
21. $4.3 + 2.8 =$ ▪
22. $1.8 + 2.9 =$ ▪
23. $4.5 + 6.04 =$ ▪
24. $8.06 + 2.3 =$ ▪
25. $9.65 + 3.42 =$ ▪

26. Copy and complete each problem.
   a. ▪ tenths = 1 one
   b. ▪ ones = 1 ten
   c. ▪ tens = 1 hundred
   d. ▪ hundreds = 1 thousand

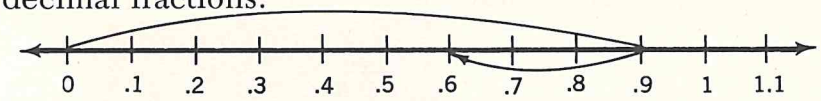
27. When we add, if we have more than 9 tenths, we regroup ▪ of the tenths as one.

28. The first place to the right of the decimal point is the ▪ place.

29. The second place to the right of the decimal point is the ▪ place.

We can subtract decimal fractions.

$.9 - .3 = .6$



Show each problem on a number line. Then copy and complete each problem.

1. $.8 - .4 =$ ■
2. $.7 - .2 =$ ■
3. $.9 - .3 =$ ■
4. $1.1 - .4 =$ ■
5. $1.6 - .9 =$ ■
6. $1.4 - .8 =$ ■

Find each difference.

7. $\begin{array}{r} .9 \\ -.2 \\ \hline \end{array}$
8. $\begin{array}{r} .8 \\ -.7 \\ \hline \end{array}$
9. $\begin{array}{r} .5 \\ -.1 \\ \hline \end{array}$
10. $\begin{array}{r} .34 \\ -.34 \\ \hline \end{array}$

11. $\begin{array}{r} .9 \\ -.6 \\ \hline \end{array}$
12. $\begin{array}{r} .8 \\ -.3 \\ \hline \end{array}$
13. $\begin{array}{r} .9 \\ -.4 \\ \hline \end{array}$
14. $\begin{array}{r} .85 \\ -.28 \\ \hline \end{array}$

We sometimes need to regroup 1 whole as 10 tenths or 1 tenth as 10 hundredths when we subtract.

$$\begin{array}{r} {\scriptstyle 2\ 12} \\ 3.2 \\ -1.6 \\ \hline 1.6 \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 12} \\ 4.52 \\ -1.38 \\ \hline 3.14 \end{array}$$

Copy and complete each problem.

15. $\begin{array}{r} 1.3 \\ -\ .1 \\ \hline \end{array}$
16. $\begin{array}{r} 1.9 \\ -\ .9 \\ \hline \end{array}$
17. $\begin{array}{r} 1.43 \\ -\ .81 \\ \hline \end{array}$
18. $\begin{array}{r} 1.70 \\ -\ .93 \\ \hline \end{array}$

19. $\begin{array}{r} 1.5 \\ -\ .8 \\ \hline \end{array}$
20. $\begin{array}{r} 2.3 \\ -1.4 \\ \hline \end{array}$
21. $\begin{array}{r} 4.54 \\ -2.72 \\ \hline \end{array}$
22. $\begin{array}{r} 5.10 \\ -4.68 \\ \hline \end{array}$

Write in columns. Find each difference.

23. $9.4 - 3.8 =$ ■
24. $4.25 - 1.24 =$ ■
25. $7.92 - 4.36 =$ ■
26. $3.16 - 2.50 =$ ■
27. $7.6 - 2.43 =$ ■
28. $3.7 - 1.85 =$ ■

Find each sum or difference. Then write each equation using decimals.

$$\frac{36}{100} - \frac{15}{100} = \frac{21}{100}$$
$$.36 - .15 = .21$$

1. $\frac{18}{100} + \frac{6}{100} =$ ▢
2. $\frac{17}{100} - \frac{7}{100} =$ ▢
3. $\frac{3}{100} - \frac{2}{100} =$ ▢
4. $\frac{35}{100} + \frac{13}{100} =$ ▢
5. $\frac{7}{10} - \frac{2}{10} =$ ▢
6. $\frac{31}{100} - \frac{8}{100} =$ ▢
7. $\frac{63}{100} + \frac{14}{100} =$ ▢
8. $\frac{9}{100} + \frac{7}{100} =$ ▢
9. $\frac{37}{100} - \frac{16}{100} =$ ▢
10. $\frac{9}{10} + \frac{4}{10} =$ ▢
11. $\frac{43}{100} - \frac{18}{100} =$ ▢
12. $\frac{15}{100} + \frac{4}{100} =$ ▢

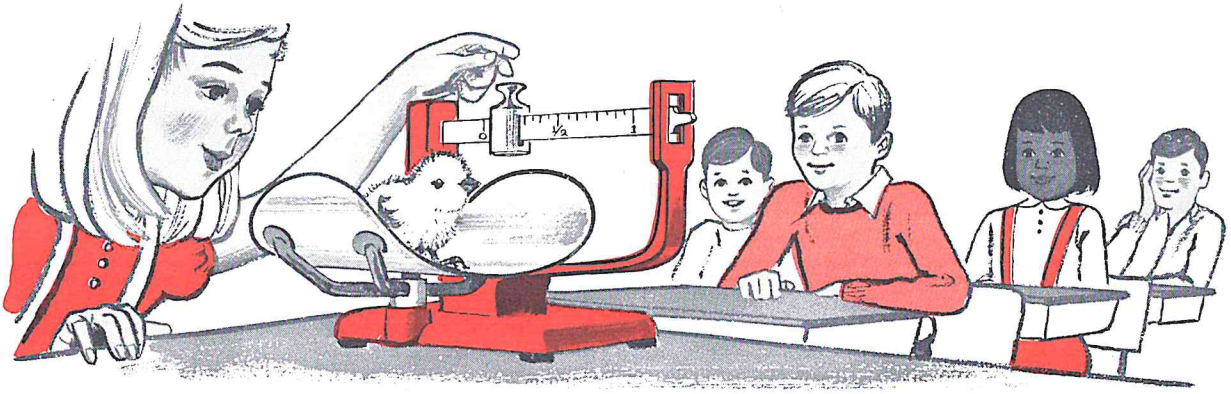
Choose < or = or >. Check by changing to fractions.

13. .9 ▢ .4
14. .16 ▢ .25
15. .4 ▢ .40
16. .01 ▢ .1
17. .6 ▢ .06
18. .40 ▢ .60
19. .3 ▢ .28
20. .7 ▢ .70
21. .92 ▢ .9

22. Karen walked .3 of a mile on Monday and .6 of a mile on Tuesday. Did she walk a mile all together?

23. Lance recorded the rainfall for two weeks. The total rainfall for the first week was .68 inch and for the second week .39 inch. How much more rain fell the first week?

24. Wendy's class performed an experiment growing baby chicks. On Monday Wendy's chick weighed .12 pound. On Friday the chick weighed .18 pound. How much more did the chick weigh Friday?



Money and Decimals

When we work with money we use decimals.

$$\begin{array}{r} \$1.25 \\ 3.48 \\ +\ 2.73 \\ \hline \$7.46 \end{array} \qquad \begin{array}{r} \$7.00 \\ -\ 2.35 \\ \hline \$4.65 \end{array} \qquad \begin{array}{r} \$2.83 \\ \times\ \ 3 \\ \hline \$8.49 \end{array} \qquad 4\overline{)\$8.64}\ \ \$2.16$$

Copy and complete each problem.

1. $\begin{array}{r} \$4.85 \\ 3.96 \\ 7.54 \\ \hline \end{array}$
2. $\begin{array}{r} \$7.95 \\ .89 \\ 6.38 \\ \hline \end{array}$
3. $\begin{array}{r} \$18.72 \\ 12.79 \\ 53.46 \\ \hline \end{array}$
4. $\begin{array}{r} \$18.59 \\ 6.37 \\ 39.43 \\ \hline \end{array}$

5. $\begin{array}{r} \$10.82 \\ -\ 4.68 \\ \hline \end{array}$
6. $\begin{array}{r} \$8.74 \\ -\ 3.69 \\ \hline \end{array}$
7. $\begin{array}{r} \$9.63 \\ -\ 2.87 \\ \hline \end{array}$
8. $\begin{array}{r} \$10.00 \\ -\ 4.59 \\ \hline \end{array}$

9. $\begin{array}{r} \$4.95 \\ \times\ \ 4 \\ \hline \end{array}$
10. $\begin{array}{r} \$8.67 \\ \times\ \ 7 \\ \hline \end{array}$
11. $\begin{array}{r} \$6.54 \\ \times\ \ 9 \\ \hline \end{array}$
12. $\begin{array}{r} \$4.07 \\ \times\ \ 8 \\ \hline \end{array}$

13. $3\overline{)\$9.63}$
14. $2\overline{)\$8.62}$
15. $4\overline{)\$4.08}$
16. $6\overline{)\$12.24}$

17. Jane had ten dollars. She spent \$3.46 at one store and \$4.16 at another store. How much money did she have left?

18. Carol has \$2.54, Tressie has \$4.38, and Margie has \$3.27. How much money do they have together?

19. Linda has saved \$14.83 for a Mother's Day present. If she spends \$7.98 for flowers, how much money does she have left?

20. Maria saves \$1.30 every week.
    a. How much will she save in 5 weeks?
    b. How much will she save in 8 weeks?

The Metric System

The metric system is a system of measure widely used in scientific work. The different units are multiples of 10, so it is easy to convert from one unit to another. The units for measuring distance are the kilometer (km.), meter (m.), decimeter (dm.), centimeter (cm.), and millimeter (mm.).

$1 \text{ m.} = 10 \text{ dm.}$
$= 100 \text{ cm.}$
$= 1000 \text{ mm.}$

$3 \text{ cm.} = 30 \text{ mm.}$

$1 \text{ mm.} = \frac{1}{10} \text{ cm.}$
$= \frac{1}{100} \text{ dm.}$
$= \frac{1}{1000} \text{ m.}$

$1 \text{ km.} = 1000 \text{ m.}$ or $\frac{1}{1000} \text{ km.} = 1 \text{ m.}$

1. Arrange by size from largest to smallest.
   1 cm. 1 km. 1 dm. 1 m. 1 mm.

Complete each problem.

2. 1 m. = ▢ cm.
3. 1 m. = ▢ mm.
4. 1 m. = ▢ dm.
5. 1 cm. = ▢ mm.
6. 1 mm. = ▢ cm.
7. 1 mm. = ▢ dm.
8. 1 mm. = ▢ m.
9. 1 km. = ▢ cm.

Complete each problem.

$2 \text{ m.} = (2 \times 100) \text{ cm.}$
$= 200 \text{ cm.}$

$18 \text{ mm.} = 18 \div 10 \text{ cm.}$
$= \frac{18}{10} \text{ cm.}$

10. 3 m. = ▢ dm.
11. 5 m. = ▢ cm.
12. 14 m. = ▢ cm.
13. 16 m. = ▢ mm.
14. 24 mm. = ▢ cm.
15. 73 mm. = ▢ dm.
16. 107 mm. = ▢ m.
17. 275 mm. = ▢ dm.
18. 600 mm. = ▢ m.

Choose $<$ or $=$ or $>$.

1. 80 cm. ● 8 dm.
2. 70 mm. ● 5 cm.
3. 91 cm. ● $\frac{9}{10}$ m.
4. 1 m. ● 35 dm.
5. 30 cm. ● $\frac{3}{10}$ m.
6. 9 m. ● 90 mm.
7. $\frac{1}{50}$ m. ● 2 cm.
8. 4 cm. ● 35 mm.
9. $\frac{3}{10}$ m. ● 300 mm.

10. Each side of a square measures 4 dm.
    a. What is its perimeter in dm.?
    b. What is its perimeter in cm.?

11. Nancy's camera uses 8-mm. film. Evelyn's uses 16-mm. film. By how many mm. do the two films differ?

12. One rectangle measures 8 cm. by 4 cm. Another measures $7\frac{1}{2}$ cm. by 5 cm.
    a. Which has the longer perimeter?
    b. How many cm. longer is it?

13. The level of mercury in a thermometer rose from 645 mm. to 665 mm.
    a. What was the rise in mm.?
    b. What was the rise in cm.?

14. A track runner ran a 1200-meter race in 3 min. What was the average number of meters he ran per minute?

15. Each side of a pentagon measures 23 cm.
    a. What is its perimeter in cm.?
    b. What is its perimeter in dm.?

16. The length of a rectangle is $5\frac{1}{2}$ cm. Its width is $\frac{1}{2}$ cm. What is its perimeter in cm.?

17. One rectangle measures 50 mm. by 15 mm. Another measures 4 cm. by $3\frac{1}{2}$ cm.
    a. Which rectangle has the longer perimeter?
    b. How many mm. longer is it?

Feedback

Choose < or = or >.

1. .8 ○ 1
2. .4 ○ .04
3. 8 ○ .8
4. 1.8 ○ 1.81
5. 2.1 ○ 2.01
6. 1.6 ○ 1.60
7. 1 ○ .92
8. .8 ○ .80
9. 1.5 ○ 1.50

Give each decimal in fraction form.

10. .8 = ▢
11. .04 = ▢
12. .95 = ▢
13. .9 = ▢
14. .4 = ▢
15. .01 = ▢

Give a decimal for each sum.

16. $6 + \frac{2}{10} + \frac{4}{100} =$ ▢
17. $8 + \frac{5}{100} =$ ▢
18. $2 + \frac{3}{100} + \frac{4}{100} =$ ▢
19. $9 + \frac{6}{10} + \frac{8}{100} + \frac{9}{1000} =$ ▢
20. $6 + \frac{2}{100} + \frac{7}{1000} =$ ▢
21. $7 + \frac{3}{10} + \frac{4}{1000} =$ ▢

Find each sum or difference.

22. 2.6 + 3.8 + 4.9 = ▢
23. 9.85 + 2.16 + 6.25 = ▢
24. 5.8 + 9.36 + 4.8 = ▢
25. 9.6 − 3.8 = ▢
26. 9.46 − 2.87 = ▢
27. 7.4 − 2.94 = ▢

28. Mildred saved $4.85 and Dorothy saved $4.13.
    a. How much did they save together.
    b. How much more do they need to buy a lamp for their room if the lamp costs ten dollars?

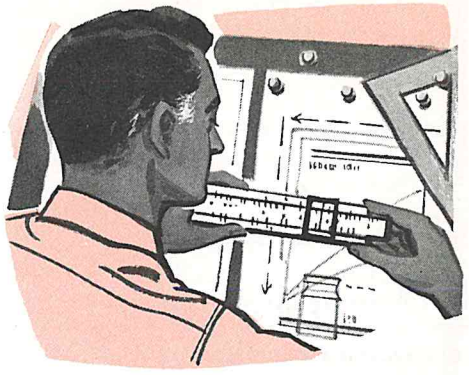
29. Bert bought 5 plane kits. How much did he spend if each kit cost $2.85?

30. Each side of a square measures 10 cm.
    a. What is its perimeter in mm.?
    b. What is its perimeter in dm.?

Feedback

Complete each problem, if n is a whole number.

1. $\{n: n < 5\} =$ ■
2. $\{n: 8 < n < 12\} =$ ■
3. $\{n: n < 4\} \cap \{n: 1 < n < 5\} =$ ■
4. $\{n: n < 3\} \cup \{n: 1 < n < 3\} =$ ■
5. $\{n: n < 6\} - \{n: 1 < n < 5\} =$ ■



Find each sum or difference.

6. $2\frac{1}{2} + 3\frac{7}{8}$
7. $6\frac{1}{6} + 2\frac{2}{3}$
8. $6\frac{4}{9} - 4\frac{1}{3}$
9. $8\frac{9}{14} - 5\frac{3}{7}$

10. Which of these numbers are prime numbers?

a. 2	c. 5	e. 23	g. 31	i. 37
b. 4	d. 19	f. 25	h. 33	j. 43

11. Complete each problem.

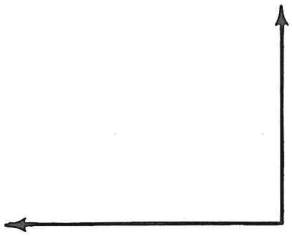
a. $\frac{4}{9} \times \frac{1}{2} =$ ■
b. $1\frac{2}{5} \times 3\frac{2}{3} =$ ■
c. $\frac{5}{7} \times \frac{4}{5} =$ ■
d. $\frac{7}{8} \times \frac{1}{2} =$ ■

Find each quotient.

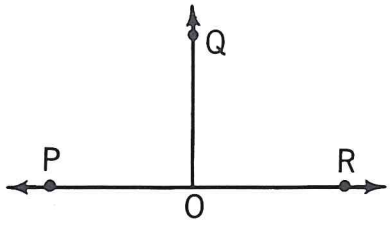
12. $90\overline{)63489}$
13. $30\overline{)4286}$
14. $60\overline{)9824}$
15. $60\overline{)592}$
16. $80\overline{)9829}$
17. $30\overline{)2004}$
18. $70\overline{)32594}$
19. $60\overline{)4872}$
20. $50\overline{)68248}$

Part 17 More Geometry

Right Angles

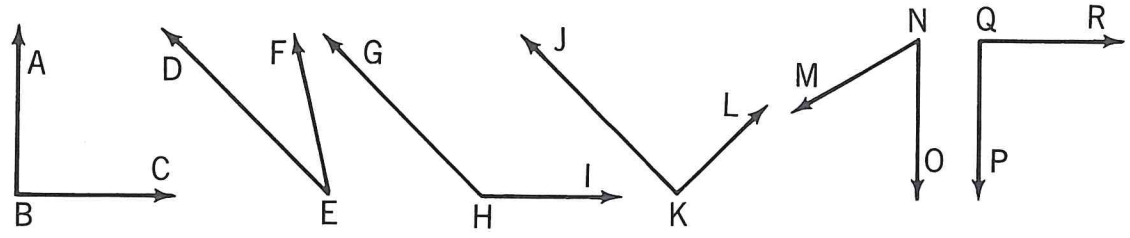


A right angle has a particular size.



∠ POQ is a right angle.
∠ QOR is a right angle.
$\overleftrightarrow{PR}$ is a straight line through O.

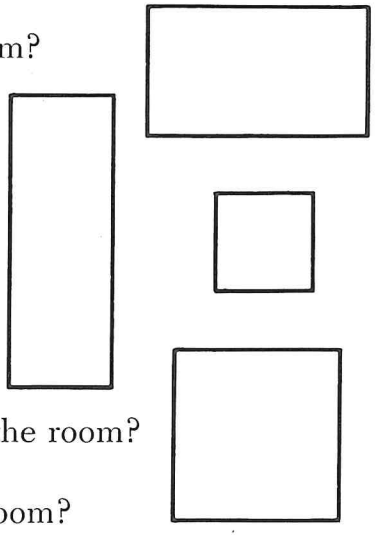
1. Which are right angles?



2. Where can you see some right angles in the room?

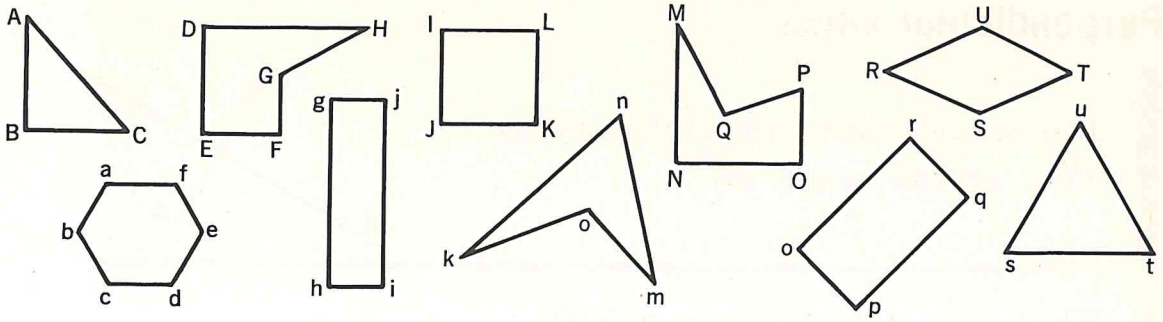
3. a. A rectangle is a quadrilateral whose 4 interior angles are each a ▪ angle.
   b. The opposite sides of a rectangle are ▪.

4. A square is a rectangle whose 4 sides are ▪.



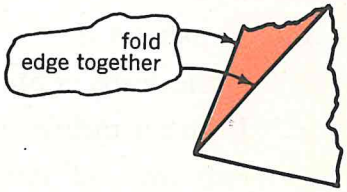
5. Where can you see some rectangular shapes in the room?

6. Where can you see some square shapes in the room?

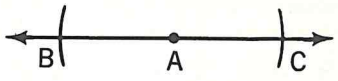


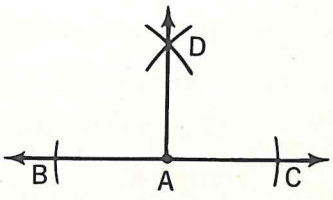
1. **a.** Name the right angles in these figures.
   b. Which of these figures are quadrilaterals?
   c. Which of these figures are rectangles?
   d. How many interior angles of a rectangle are right angles?
   e. Which of these figures are squares?

2. **a.** Cut or fold a piece of paper along a line.
   b. Fold the parts of the cut or folded edge together.
   c. You now have a model of a right angle. Use it as a measure to see if the cover of your book has four right angles.



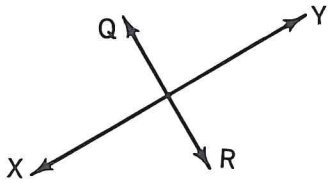
3. Construct a right angle at a given point on a given line.
   a. Draw a line and mark point A on it.
   b. Construct 2 small arcs of a circle with center A, intersecting the line in points B and C.
   c. With center B, construct an arc above A.
   d. Using the same radius, but with C as a center, construct another arc intersecting the first arc in D.
   e. Draw the ray from A through D. Angles DAB and DAC are both right angles.



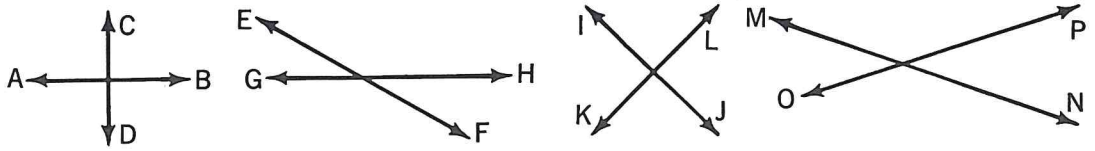


Perpendicular Lines

Perpendicular lines meet at right angles.
$\overleftrightarrow{XY}$ is perpendicular to $\overleftrightarrow{QR}$

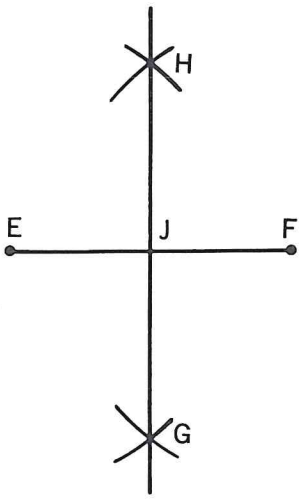


1. Which pairs of lines are perpendicular?



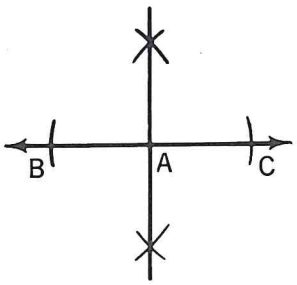
2. Construct the **perpendicular bisector** of a segment.

   a. Draw $\overline{EF}$.
   b. Construct above and below $\overline{EF}$ two arcs of a circle with center E.
   c. Using a radius of the same length construct two arcs of another circle with center F, intersecting the first 2 arcs in G and H.
   d. Draw $\overleftrightarrow{GH}$ intersecting $\overline{EF}$ in J.
   e. Is $\overline{EJ} \cong \overline{JF}$?
   f. Is $\overleftrightarrow{GH}$ perpendicular to $\overline{EF}$?

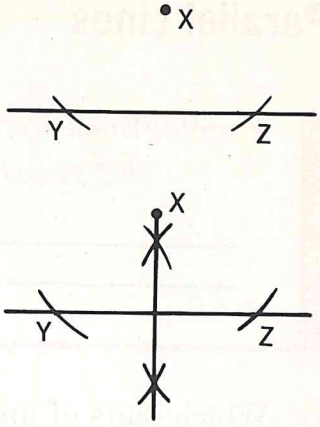


3. Construct a perpendicular to a line from a point on the line.

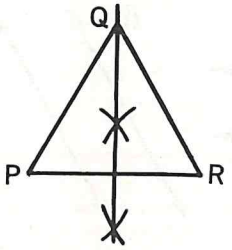
   a. Draw a line and mark point A on it.
   b. Construct 2 arcs of a circle with center A intersecting the line in B and C.
   c. Construct the perpendicular bisector of $\overline{BC}$. The perpendicular bisector passes through point A.



1. Construct a perpendicular to a line from a point not on the line.

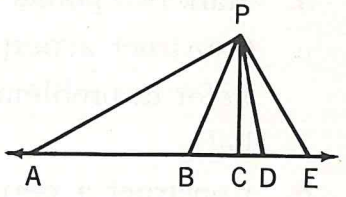


   a. Draw a line and mark a point X that is not on the line.
   b. Construct 2 small arcs of a circle with center X, intersecting the line in Y and Z.
   c. Construct the perpendicular bisector of YZ. This is the perpendicular passing through X.



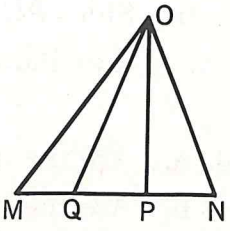
2. a. Construct an isosceles triangle PQR with base PR.
   b. Construct the perpendicular bisector of base PR.
   c. Does this bisector pass through Q?

3. a. Look at this figure. Is P on $\overleftrightarrow{AE}$?
   b. Which is longest, $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, $\overline{PD}$, or $\overline{PE}$?
   c. If angles PCA and PCE are right angles, which segment is perpendicular to $\overleftrightarrow{AE}$; $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, $\overline{PD}$, or $\overline{PE}$?
   d. The shortest path from a point to a line is along a perpendicular. Is $\overline{PC}$ the shortest path from P to $\overleftrightarrow{AE}$?



4. The perpendicular bisector of a line segment passes through the ■ of a segment.

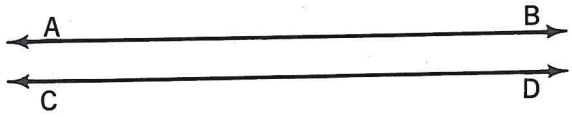
5. a. Which angles are right angles?
   b. Which line segments are perpendicular?
   c. The shortest path from O to $\overline{MN}$ is segment ■.



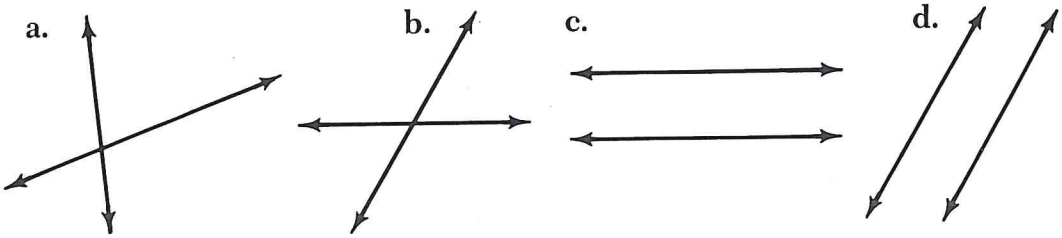
Parallel Lines

Parallel lines are lines in a plane that do not intersect.

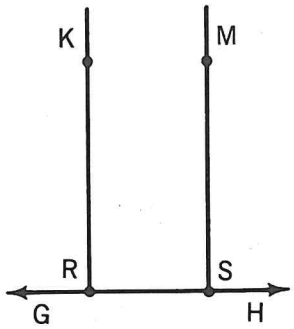
$\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$.



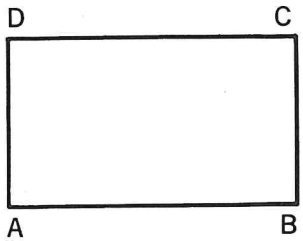
1. Which pairs of lines are parallel?



2. a. Draw $\overleftrightarrow{GH}$.
   b. Mark two points R and S on $\overleftrightarrow{GH}$.
   c. Construct a perpendicular RK to $\overleftrightarrow{GH}$ at R. Refer to problem 3 on page 268 if you need help.
   d. Construct a perpendicular SM to $\overleftrightarrow{GH}$ at S.
   e. Is $\overleftrightarrow{RK}$ parallel to $\overleftrightarrow{SM}$?



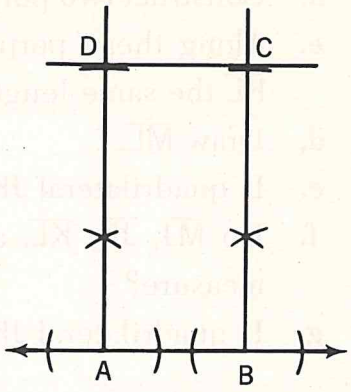
3. a. Sides AB and DC in rectangle ABCD are opposite sides of the rectangle. Are they parallel?
   b. Sides AD and BC are also opposite sides. Are they parallel?



4. a. Are the opposite sides of a square parallel?
   b. Are the opposite sides of quadrilaterals parallel?

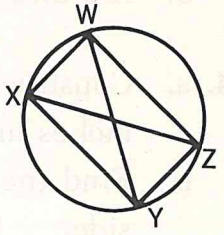
Constructing Rectangles

1. a. Draw $\overleftrightarrow{AB}$.
   b. Construct a perpendicular at A.
   c. Construct a perpendicular at B.
   d. Along the perpendiculars lay off segments AD and BC of the same length.
   e. Draw $\overline{DC}$.
   f. Use your compasses to measure $\overline{AD}$ and $\overline{BC}$. Are $\overline{AD}$ and $\overline{BC}$ congruent?
   g. Are $\overline{DC}$ and $\overline{AB}$ congruent?
   h. Is angle DAB a right angle?
   i. Is ABCD a rectangle?

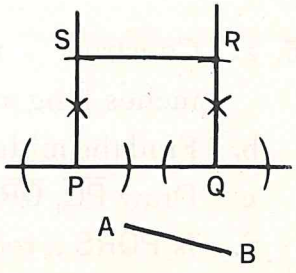


2. a. Construct a rectangle DEFG, and draw the diagonals DF and EG.
   b. Are $\overline{DF}$ and $\overline{EG}$ congruent?
   c. Are $\overline{DF}$ and $\overline{EG}$ perpendicular?

3. a. Construct a circle and draw 2 segments through the center of the circle.
   b. Label the points of intersection W, X, Y, and Z.
   c. Draw $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{ZW}$.
   d. Is quadrilateral WXYZ a rectangle?

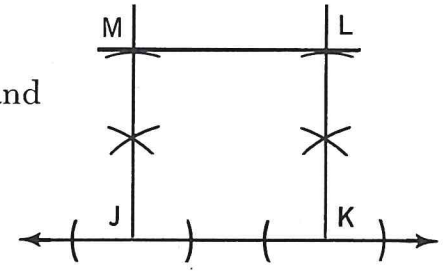


4. a. Construct a rectangle with sides of given lengths.
   b. Draw $\overline{PQ}$ and $\overline{AB}$ as shown.
   c. Extend $\overline{PQ}$, and construct perpendiculars at P and Q.
   d. Along the perpendiculars lay off $\overline{PS}$ and $\overline{QR}$ the same length as $\overline{AB}$. Draw $\overline{RS}$.



Constructing Squares

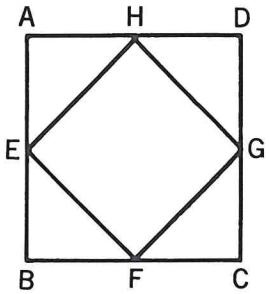
1. a. Draw $\overleftrightarrow{JK}$.
   b. Construct two perpendiculars at J and K.
   c. Along these perpendiculars lay off $\overline{JM}$ and $\overline{KL}$ the same length as $\overline{JK}$.
   d. Draw $\overline{ML}$.
   e. Is quadrilateral JKLM a rectangle?
   f. Do $\overline{MJ}$, $\overline{JK}$, $\overline{KL}$, and $\overline{ML}$ all have the same measure?
   g. Is quadrilateral JKLM a square?



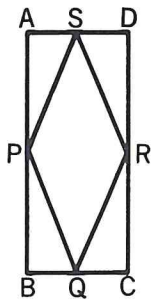
2. a. Construct a square ABCD, and draw diagonal AC.
   b. Is angle ACD ≅ angle ACB?

3. a. Construct a square EFGH, and draw diagonals EG and FH.
   b. Are these diagonals perpendicular?

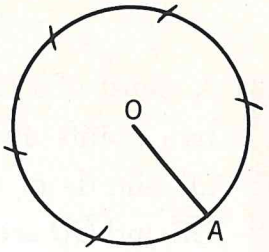
4. a. Construct a square ABCD with each side 4 inches in length.
   b. Find the midpoints E, F, G, and H of each side.
   c. Draw $\overline{EF}$, $\overline{FG}$, $\overline{GH}$, and $\overline{HE}$.
   d. Is EFGH a square?



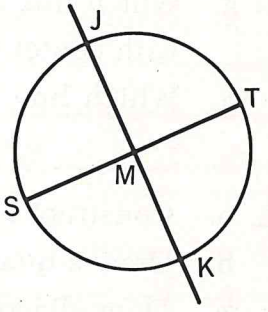
5. a. Construct a rectangle ABCD, with sides 2 inches long and 4 inches long.
   b. Find the midpoints P, Q, R, and S of the sides.
   c. Draw $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, and $\overline{SP}$.
   d. Is PQRS a rectangle?
   e. Is PQRS a square?



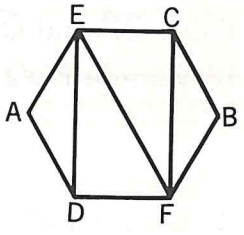
1. a. Construct a circle with center O.
   b. Mark a point A on the circle and draw radius OA.
   c. With radius OA start at A and construct arcs on the circle as shown.
   d. Label the points of intersection B, C, D, E, and F and draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$.
   e. A **hexagon** has 6 sides. Is figure ABCDEF a hexagon?



2. a. Construct a circle with center M.
   b. Draw through M a segment intersecting the circle in S and T.
   c. Construct the perpendicular bisector of ST.
   d. Label the points where the perpendicular bisector intersects the circle J and K.
   e. Draw $\overline{JS}$, $\overline{SK}$, $\overline{KT}$, and $\overline{TJ}$.
   f. Is JSKT a square?



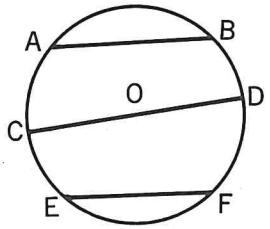
3. a. A polygon is any closed plane figure whose sides are line segments. Is a hexagon a polygon?
   b. Is a square a polygon?
   c. Is a pentagon a polygon?
   d. Is a quadrilateral a polygon?
   e. Is a triangle a polygon?
   f. List 10 polygons in the drawing at the right.



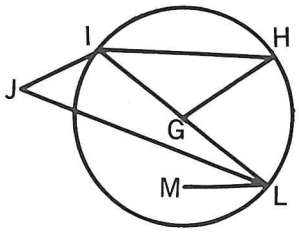
4. a. What is the perimeter of a pentagon whose sides are each 11 in. long?
   b. What is the perimeter of a hexagon whose sides are each $2\frac{1}{7}$ ft. long?
   c. What is the perimeter of a square with sides of length $\frac{3}{4}$ ft.?

Chords of a Circle

A **chord** of a circle is any line segment joining two points on the circle. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all chords of the given circle with center O. $\overline{OC}$ and $\overline{OD}$ are radii of the circle.



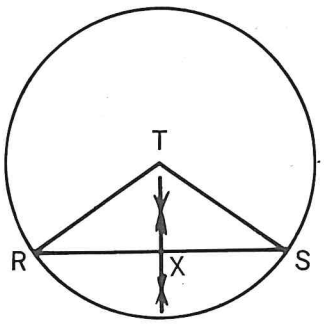
1. a. Which line segments are chords of the circle with center G?
   b. Which line segments are radii of the circle?



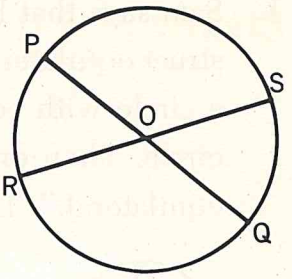
2. a. Construct a circle.
   b. Draw a quadrilateral ABCD with vertices on the circle.
   c. Draw diagonals AC and BD.
   d. Which of these line segments are chords of the circle?

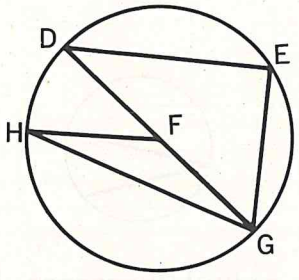
3. a. Draw a chord PQ of a circle with center O.
   b. Draw $\overline{OP}$ and $\overline{OQ}$.
   c. Is triangle POQ isosceles?

4. a. Draw a chord RS of a circle with center T.
   b. Construct the perpendicular bisector of chord RS. Label as X the midpoint of $\overline{RS}$.
   c. Does this bisector pass through center T?
   d. Draw $\overline{TR}$ and $\overline{TS}$.
   e. Is angle RXT $\cong$ angle SXT?



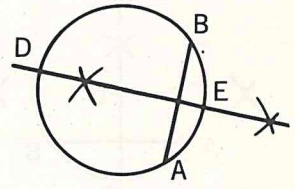
A diameter of a circle is any chord passing through the center of the circle.
Segments PQ and RS are both diameters of the given circle with center O.
Two diameters intersect in the center of the circle.

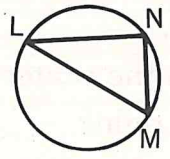




1. **a.** Which line segments are chords of the circle with center F?
   b. Which line segments are diameters of the circle?
   c. Which line segments are radii of the circle?
   d. Is angle DEG a right angle?

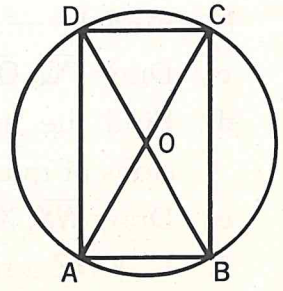
2. **a.** Draw a chord AB of a circle.
   b. Construct the perpendicular bisector of $\overline{AB}$, intersecting the circle in D and E.
   c. Is chord DE a diameter of the circle?



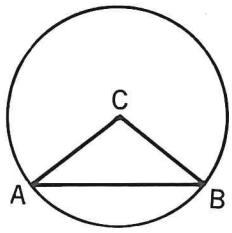


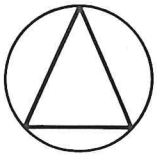
3. **a.** Draw a diameter LM of a circle.
   b. Draw two chords NL and NM.
   c. Is angle LNM a right angle?

4. **a.** Construct a circle with center O.
   b. Draw a chord AB which is not a diameter.
   c. Draw the diameter AC.
   d. Draw the diameter BD.
   e. Draw chord CD.
   f. Draw $\overline{AD}$ and $\overline{BC}$.
   g. Is ABCD a rectangle?



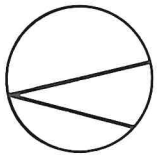
1. Sam says that he sees a new, easy way to construct equilateral triangles. He says, "Construct a circle with center C and any chord AB of the circle. Then draw $\overline{AC}$ and $\overline{BC}$. Triangle ABC is equilateral." Is Sam correct?

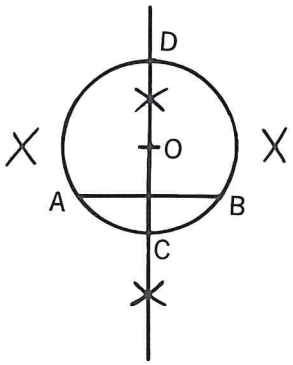




2. Joe says that every triangle whose vertices lie on a circle is isosceles. Can you draw a triangle that shows he is wrong?

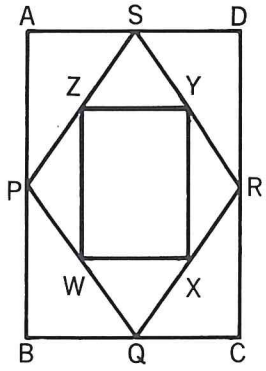
3. Joe also says that if two chords of a circle have a common endpoint, one must be a diameter of the circle. Can you draw two chords that show he is wrong?



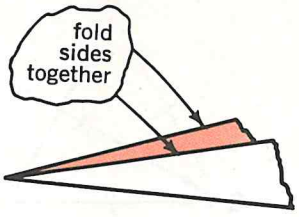


4. Debbie says she knows how to find the center of a circle. This is her method.
   a. Draw a chord AB of a circle.
   b. Construct the perpendicular bisector of the chord, intersecting the circle in C and D. $\overline{CD}$ is a diameter of the circle.
   c. Find the midpoint O of $\overline{CD}$.
   d. Debbie says that point O is the center of the circle. Do you think she is right?

5. a. Construct a rectangle ABCD with sides $2\frac{1}{2}$ inches long and 5 inches long.
   b. Find the midpoints P, Q, R, and S of the sides.
   c. Draw $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, and $\overline{SP}$.
   d. Find the midpoints W, X, Y, and Z of the sides of quadrilateral PQRS.
   e. Draw $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, and $\overline{ZW}$.
   f. Is WXYZ a rectangle?

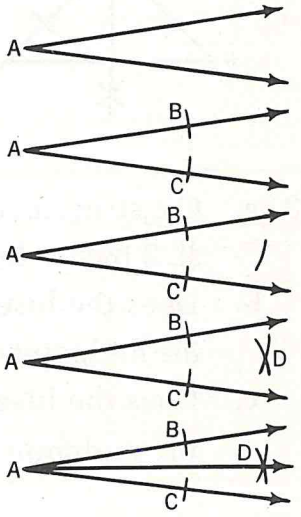


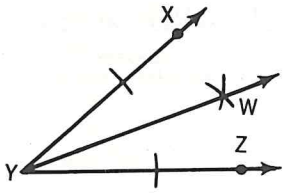
Bisecting Angles



1. **a.** On a sheet of paper draw an angle.
   b. Cut along the sides of the angle.
   c. Fold the sides together, and press along the line of fold.
   d. Unfold the paper. The line of fold **bisects** the angle.

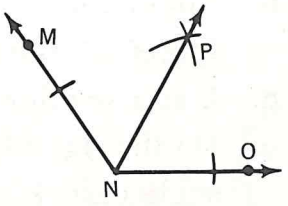
2. **a.** Draw an angle with vertex A.
   b. Using the same radius construct 2 small arcs of a circle with center A, intersecting the 2 rays in B and C.
   c. Between the rays construct an arc with center B.
   d. With center C and a radius of equal length, construct another arc intersecting the first arc at D.
   e. Draw the ray from A through D. Angles BAD and CAD have the same measure. $\overrightarrow{AD}$ bisects angle BAC.





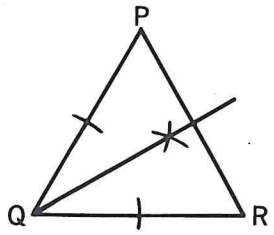
3. **a.** Draw angle XYZ.
   b. Bisect it with $\overrightarrow{YW}$.
   c. Show that angle XYW has the same measure as angle ZYW.

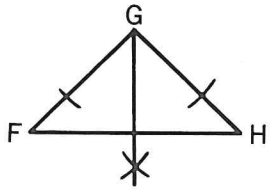
4. **a.** Draw angle MNO.
   b. Bisect it with $\overrightarrow{NP}$.
   c. Show that angle MNP has the same measure as angle PNO.



Feedback

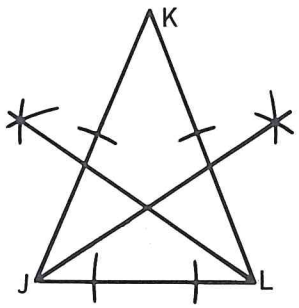
1. a. Construct an equilateral triangle PQR whose sides are 3 inches in length.
   b. Bisect interior angle PQR.
   c. Does the bisector of angle PQR pass through the midpoint of side PR?

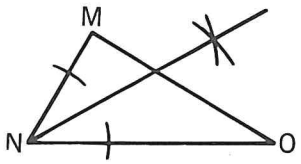




2. a. Construct an isosceles triangle FGH on base FH.
   b. Does the bisector of angle FGH pass through the midpoint of base FH?

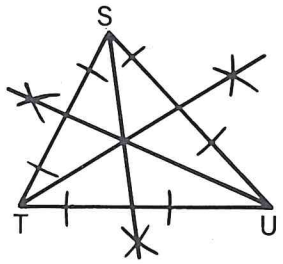
3. a. Construct an isosceles triangle JKL with base JL 3 inches long and sides 4 inches long.
   b. Does the bisector of angle JLK pass through the midpoint of opposite side JK?
   c. Does the bisector of angle LJK pass through the midpoint of opposite side LK?





4. a. Draw a triangle MNO that is not isosceles.
   b. Does the bisector of angle MNO pass through the midpoint of opposite side MO?

5. a. Draw a triangle STU that is not isosceles.
   b. Bisect interior angle STU.
   c. Bisect interior angle TUS.
   d. Bisect interior angle UST.
   e. Do the three bisectors of the angles of a triangle pass through one point?



Feedback

Find each amount.

1. $\begin{array}{r} \$46.84 \\ 3.57 \\ \underline{4.81} \end{array}$

2. $\begin{array}{r} \$6.78 \\ .99 \\ \underline{1.57} \end{array}$

3. $\begin{array}{r} \$17.45 \\ 3.94 \\ \underline{22.38} \end{array}$

4. $\begin{array}{r} \$16.52 \\ 7.18 \\ \underline{9.57} \end{array}$

Find each quotient.

5. $3\overline{)892}$

6. $83\overline{)9542}$

7. $19\overline{)485}$

8. $75\overline{)86326}$

9. Copy and complete the table below. Add these products to the table:
6×6 7×7 8×8 9×9 10×10.

Write each product as the sum of odd numbers, following the pattern below.

$1 \times 1 = 1$
$2 \times 2 = 4 = 1 + 3$
$3 \times 3 = 9 = 1 + 3 + 5$
$4 \times 4 = 16 = 1 + 3 + 5 + 7$
$5 \times 5 = 25 = 1 + 3 + 5 + 7 + \blacksquare$

Use the distributive law for multiplication to rewrite each problem.

10. $m \times (n + p)$

11. $9 \times (g + 6)$

12. $2 \times (f + g)$

Complete each problem.

13. $4\frac{1}{2} + 3\frac{1}{2} = \blacksquare$

14. $\frac{7}{8} - \frac{1}{4} = \blacksquare$

15. $4 \times \frac{2}{9} = \blacksquare$

16. $8\frac{3}{4} - 2\frac{1}{4} = \blacksquare$

17. $\frac{3}{5} + \frac{3}{10} = \blacksquare$

18. $\frac{8}{3} \div 2 = \blacksquare$

19. $\frac{1}{4} + \frac{3}{4} = \blacksquare$

20. $\frac{5}{12} - \frac{1}{3} = \blacksquare$

Part 18 Logic

Atomic and Molecular Sentences

These are atomic sentences.

It has stopped raining.	$y + 4 = 7$.
We are going sailing now.	$z = 0$.
We are going swimming.	$y < 9$.

We use connectives to form molecular sentences from atomic sentences. We use the connectives "**and,**" "**or,**" "**if . . ., then . . .,**" and "**not.**" The connectives are not part of the atomic sentences. Atomic sentences do not contain any connectives.

These are molecular sentences.

It has stopped raining **and** we are going sailing now.

We are going sailing now **or** we are going swimming.

If it has stopped raining, **then** we are going swimming.

We are **not** going home now.

$y + 4 = 7$ **or** $z = 0$.

If $y + 4 = 7$, **then** $y < 9$.

$y < 9$ **and** $z = 0$.

Write "A" for each atomic sentence or "M" for each molecular sentence. If the sentence is molecular, give the connective.

1. John is fishing and Susie is sleeping. M and
2. Stephen is the king or Matilda is the queen.
3. This figure is concave or it is a quadrilateral.
4. If you behave, then you may watch television.

5. $y + z = 9$.
6. $y + z = 9$ and $y - z = 1$.
7. $p = 2$ or p is odd.
8. If $g < h$, then $h > g$.

Conjunctions and Disjunctions

If the connective "and" is used in a molecular sentence, the sentence is a **conjunction.**

Conjunction: The door is white and the drapes are blue.

If the connective "or" is used in a molecular sentence, the sentence is a **disjunction.**

Disjunction: This figure is convex or it is concave.

Tell the type of each molecular sentence. Write "C" for conjunction and "D" for disjunction.

1. This dog is big and that dog is small.
2. Mary is reading and Beth is skating.
3. $n = 0$ or $n = 1$.
4. It will snow today or it will rain.
5. This picture is very old or it is a fake.
6. $n > 5$ and $n < 8$.
7. $A \subset B$ or $A = B$.
8. Jack draws and Tom paints.
9. $x = y$ and $y = z$.
10. p is prime or p is even.
11. $x + y = 3$ or $x + y > 3$.
12. The quadrilateral is convex and the pentagon is concave.
13. $C = \{\ \}$ or $C = \{0\}$.

14. a. Use these atomic sentences to make two conjunctions.

Helen is at home.	$x > z$.
$x < y$.	Billy is at school.
Father is at work.	$z = 0$.

b. Use any of the atomic sentences above to make two disjunctions.

Conditional Sentences

If the connective "if . . . , then . . ." is used in a molecular sentence, the sentence is a **conditional sentence.**

The two parts of a conditional sentence have names.

The sentence between "if" and "then" is the **antecedent.**

The sentence after "then" is the **consequent.**

Conditional: If I light the oven now, then it will be hot later.

Antecedent: I light the oven now.

Consequent: It will be hot later.

"If" and "then" are not a part of either the antecedent or the consequent.

Tell whether the antecedent or the consequent is underlined in each sentence.

Write "a" for antecedent and "c" for consequent.

1. If I stay away, then I shall miss supper.
2. If the triangle is symmetrical, then it is isosceles.
3. If $a = b$, then $2 \times a = 2 \times b$.
4. If A and B are disjoint, then $A \cap B = \{\ \}$.
5. If $4 \times 7 = 28$, then $28 \div 7 = 4$.
6. If my car starts, then I will be there in ten minutes.
7. If $8 \div m = 1$, then $m = 8$.
8. If the figure is a circle, then it is convex.
9. If the triangle has three lines of symmetry, then it is equilateral.
10. If $4 + y = 9$, then $y = 9 - 4$.
11. If $A = B$, then $A \cup B = A$.

12. Use these atomic sentences to make three conditional sentences.

Harold is sick.	$x = y$.
I go to camp.	$3x = 3y$.
I shall learn to swim.	$x > z$.

Denials

The connective "not" requires only one atomic sentence to make a molecular sentence. The molecular sentence is the **denial** of the atomic sentence.

Atomic sentence:	It is raining.
Molecular sentence:	It is not raining.

The molecular sentence "It is not raining" is the denial of "It is raining".

1. Which of these sentences are denials of an atomic sentence?
   a. 7 is equal to 3 + 4.
   b. 7 is not equal to 3 + 3.
   c. We do not define $x \div 0$.
   d. This figure is not convex.
   e. This figure is concave.
   f. I will not go home.

Make the denial of each atomic sentence by inserting "not". (Sometimes the English sentence requires a slight change as well:
"He reads it" becomes "He does not read it" or "He doesn't read it".
Also note that "$\neq$" is the denial of "$=$".)

2. The general surrenders.
3. A is a subset of B.
4. This is a pentagon.
5. $x + 6 = 7$.
6. That is a rose.
7. He likes me.
8. $A \cup \{1, 2\} = C$.
9. $\{\ \} \cup A = 1$.

True or false?

10. The sentence after "then" in a conditional sentence is the antecedent.
11. The connective in the denial of an atomic sentence is "not".
12. The results of combining atomic sentences with connectives are molecular sentences.
13. The connective in a disjunction is "and".
14. The result of joining two atomic sentences with "and" is a conjunction.

Drawing Conclusions

From given statements we can often **infer** other statements.
The given statements are the premises. The statement we infer is the conclusion.

Premise 1: If I drop this, then it falls.
Premise 2: I drop this.
Conclusion: It falls.

What conclusion can we infer from each pair of premises?

1. If $p + 8 = 29$, then $p = 29 - 8$.
   $p + 8 = 29$.
2. If I eat this, then I shall be sick.
   I eat this.
3. Martha drinks milk.
   If Martha drinks milk, then she will be healthy.
4. p is prime.
   If p is prime, then $p > 1$.
5. If $s + 3 = 7$, then $s = 4$.
   $s + 3 = 7$.
6. If $s = 4$, then $9 \times s = 36$.
   $s = 4$.

The conclusions inferred from the premises above use the "If-then" Rule.
To apply the "If-then" Rule one premise must be a conditional sentence and another premise must be the antecedent of the conditional sentence.

Use the "If-then" Rule to draw a conclusion from each pair of premises.

7. If this figure has four sides, then it has four vertices.
   This figure has four sides.
8. This figure has four sides.
   If this figure has four sides, then it is a quadrilateral.

Using the If-then Rule

We may use the "If-then" Rule more than once to draw a conclusion.

Premise 1:	If $5 \times q = 35$, then $q = 7$.	
Premise 2:	$5 \times q = 35$.	
Premise 3:	If $q = 7$, then $q \times q = 49$.	
First Conclusion:	$q = 7$.	from (2) and (1)
Second Conclusion:	$q \times q = 49$.	from (3) and (4)

On the right we show to what premises we apply the "If-then" Rule.

Use the "If-then" Rule twice to draw a conclusion from each set of premises.

1. If the Giants beat the Dodgers, then they win the pennant.
   If he is out, then the Giants beat the Dodgers.
   He is out.

2. If that is Orion, then this is Vega.
   If it is 9 P.M., then that is Orion.
   It is 9 P.M.

3. If this is a leap year, then tomorrow is February 29th.
   This year is 1968.
   If this year is 1968, then this is a leap year.

4. If point A is on the polygon, then point B is outside the polygon.
   If point B is outside the polygon, then point C is inside the polygon.
   Point A is on the polygon.

5. If n is prime, then $n > 1$.
   n is prime.
   If $n > 1$, then $n - 1 > 0$.

6. If $x + y = 0$, then $x < 0$.
   $z = 2$.
   If $z = 2$, then $x + y = 0$.

If-then-not Rule

Another logical rule is the "If-then-not" Rule.
Again we have a conditional sentence as a premise:

If the baby is a girl, then it is named "Sue".

The other premise is the denial of the consequent:

It is not named "Sue".

The conclusion is the denial of the antecedent:

The baby is not a girl.

Use the "If-then-not" Rule to draw a conclusion.

1. If the moon is full, I can see right across the yard.
   I can not see right across the yard.

2. If this is a square, then it has four sides of equal length.
   It does not have four sides of equal length.

3. If Catherine is at home, then Sheila is at home.
   Sheila is not at home.

4. You do not save rocks.
   If you are in a science class, then you save rocks.

5. t is not less than 4.
   If $t = 2$, then $t < 4$.

6. $p \neq 17$.
   If $p = 289$, then $p = 17$.

7. If $k = 5$, then $m = 8$.
   $m \neq 8$.

8. If $f = 45$, then $f > 43$.
   f is not greater than 43.

9. If $m = n$, then $n = 0$.
   $n \neq 0$.

10. If $m < y$, then $y < 4$.
    y is not less than 4.

We may also use the "If-then-not" Rule more than once to draw a conclusion.

Premise 1:	If $m = 28$, then $m \div 4 = 7$.	
Premise 2:	If m is even, then $m = 28$.	
Premise 3:	$m \div 4 \neq 7$.	
First Conclusion:	$m \neq 28$.	from (1) and (3)
Second Conclusion:	m is not even.	from (4) and (2)

Use the "If-then-not" Rule twice to draw a conclusion from each set of premises.

1. If John cries, then Sue cries.
   If Sue cries, then Mom is sad.
   Mom is not sad.

2. If $a = b$, then $c = d$.
   If $c = d$, then $d = 9$.
   $d \neq 9$.

3. If $p < q$, then $p < 17$.
   p is not less than 25.
   If $p < 17$, then $p < 25$.

4. If $x + y = 7$, then $x - y = 3$.
   If $x - y = 3$, then $x = 5$.
   $x \neq 5$.

5. If Mary is here, then Tom is gone.
   If Tom is gone, then Sam is sad.
   Sam is not sad.

6. If $b = 1$, then $c = 1$.
   If $a \times b = a$, then $b = 1$.
   $c \neq 1$.

Use the "If-then-not" Rule three times to draw a conclusion from each set of premises.

7. If this is the 26th, then I get paid.
   If I get paid, then I buy a coat.
   If I buy a coat, then I am warm.
   I am not warm.

8. If $a + b = a$, then $b = 0$.
   If $b = 0$, then $b < 8$.
   b is not less than 12.
   If $b < 8$, then $b < 12$.

The antecedent or consequent of a conditional sentence may be a molecular sentence.

Conditional sentence: If $z \neq 0$, then $x = 0$ and $y = 0$.
Antecedent: $z \neq 0$.
Consequent: $x = 0$ and $y = 0$.

Tell whether each underlined sentence is the antecedent or consequent and whether it is an atomic sentence or a molecular sentence. Write "a" for antecedent, "c" for consequent, "A" for atomic, and "M" for molecular.

1. If you do not start scoring, then I shall win the game.
2. If you start scoring, then I shall not win the game.
3. If my father comes or my mother calls, then I shall go home.
4. If John does not join, then Judy will not join.
5. If $m + n = 1$, then $m = 0$ or $n = 0$.

What is the denial of a sentence whose connective is "not"?

Premise 1: If $q \neq 0$, then $r + q = 5$.
Premise 2: $r + q \neq 5$.
Conclusion: $q = 0$.

We infer the denial of "$q \neq 0$," that is, "$q = 0$."

"$q \neq 0$" is the denial of "$q = 0$."
"$q = 0$" is the denial of "$q \neq 0$."

"It is raining" is the denial of "It is not raining".

Give the denial of each sentence.

6. $a + b = b - a$.
7. $a - b \neq b - a$.
8. That is not right.
9. p is not prime.
10. This is not the first time.
11. $x = y + 3$.
12. The barn did not burn down.
13. $x = y + 10$.
14. He does not lie to me.
15. $x + y \neq z - 3$.

Which rule is used to infer each conclusion? Write "IF" for the "If-then" Rule, and "IFN" for the "If-then-not" Rule.

1. Premise: If this is Friday, then my mother is at the doctor's.
Premise: My mother is not at the doctor's.
Conclusion: This is not Friday.

2. Premise: If Richard does not eat, then he loses his temper.
Premise: He does not lose his temper.
Conclusion: Richard does eat.

3. Premise: He does not pay me soon.
Premise: If he does not pay me soon, then I will be very angry.
Conclusion: I will be very angry.

4. Premise: There will be a marriage.
Premise: If you can't afford a carriage, then there won't be any marriage.
Conclusion: You can afford a carriage.

5. Premise: You can't afford a carriage.
Premise: If you can't afford a carriage, then there won't be any marriage.
Conclusion: There won't be any marriage.

6. Premise: If it has six sides, then it has six vertices.
Premise: If it has six vertices, then it is a hexagon.
Premise: It is not a hexagon.
First Conclusion: It does not have six vertices.
Second Conclusion: It does not have six sides.

Inferences about Conjunctions

We may combine two separate premises to form a conjunction.

Premise: Abby is fair.
Premise: Ted is not tall.
Conclusion: Abby is fair and Ted is not tall,
or: Ted is not tall and Abby is fair.

From each pair of premises draw a conclusion that is a conjunction.

1. This is an elm tree.
   That is not a weeping willow.

2. The sedge is withered.
   No birds sing.

3. $x + y = 8$.
   $x - y = 2$.

4. p is prime.
   $p < 17$.

We may give one of the statements from a conjunction as a conclusion.

Conjunction: Today is Friday and the sun is shining.
Conclusion 1: Today is Friday.
Conclusion 2: The sun is shining.

Draw two conclusions from each sentence.

5. He did not turn to the left and he did not turn to the right.
6. Light she was and her shoes were number nine.
7. First I ate dinner at home and then I ate another meal with my friend.
8. I went to grade school and Bob went to high school.
9. Mary sang and Doris played the piano.
10. $a = b$ and $c = d$.
11. $3 < y$ and $y < 10$.
12. The sky is blue and the grass is green.

Inferences about Disjunctions

From a disjunction and the denial of one part of the disjunction, we may infer the other part of the disjunction as a conclusion.

Premise: Roses are red or violets are blue.
Premise: Violets are not blue.
Conclusion: Roses are red.

We call this rule of inference the "Or" Rule.

Use the "Or" Rule to draw a conclusion from each pair of premises.

1. It is a quadrilateral or it is a pentagon.
   It is not a pentagon.

2. It is convex or it is concave.
   It is not convex.

3. Cinderella does not stay after midnight or the Prince sees her in rags.
   Cinderella does stay after midnight.

4. Terry was born on Saturday or he was born on Sunday.
   Terry was not born on Sunday.

5. You eat four large meals each day.
   You do not stay thin or you do not eat four large meals each day.

6. $a = b$ or $c = 4$.
   $c \neq 4$.

7. $m = n$ or $m > n$.
   $m \neq n$.

8. $y = 0$ or $z + y = 5$.
   $y \neq 0$.

9. $q = 0$ or $p \neq q$.
   $p = q$.

10. $a \neq c$ or $a > 6$.
    $a = c$.

11. $a \neq b$ or $g \neq h$.
    $g = h$.

Feedback

Write "A" for each atomic sentence and "M" for each molecular sentence. If the sentence is molecular, tell what type of sentence it is. You may write "C" for conjunction, "D" for disjunction, "Cond." for conditional, and "De" for the denial of an atomic sentence.

1. Some gave them white bread and some gave them brown.
2. The public is not informed on this issue.
3. The weather has been very pleasant.
4. Sarah wins the game and Donna loses.
5. $(a + b) + c = a + (b + c)$.
6. $a - (b - c) \neq (a - b) - c$.
7. $m > n$ and $n < p$.
8. $m = 6$ or $m = 0$.
9. If $a = b$, then $a = 6$.
10. $a(b + c) = ab + ac$.

Draw a conclusion from the given premises.

11. If the figure has three sides, it is a triangle.
    The figure has three sides.
12. Herb will go to town or he will sleep.
    He will not sleep.
13. Mary is sick or Sheila is sad.
    Mary is not sick.
14. If the figure is not a quadrilateral, then it is not a rectangle.
    It is a rectangle.
15. $p = 7$ or $q = 6$.
    $q \neq 6$.
16. If $a = 4$, then $b = 5$.
    $a = 4$.
17. $d \neq 6$ or $e = 6$.
    $e \neq 6$.
18. If $m \neq a$, then $b = c$.
    $b \neq c$.

Feedback

Find each sum or difference.

1. $8\frac{1}{8} + 2\frac{3}{4}$

2. $7 - 2\frac{1}{5}$

3. $4\frac{2}{3} + 7\frac{3}{4}$

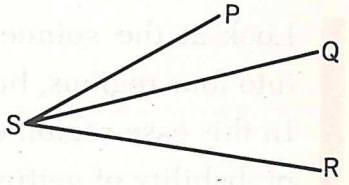
4. $9\frac{1}{2} - 2\frac{5}{8}$

Solve each equation.

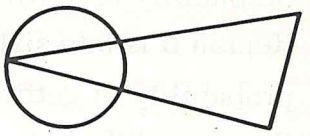
5. $7 + 4 = x$
6. $8 + 9 = a$
7. $3 - 2 = b$
8. $7 \times 8 = y$
9. $12 \times 11 = q$
10. $2 \div 9 = r$
11. $j + 7 = 19$
12. $k \div 8 = 7$
13. $a + 3 = 21$
14. $f \div 5 = 7$
15. $h \div 9 = 12$
16. $n - 4 = 0$

17. How many vertices does a triangle have?
18. How many vertices does a quadrilateral have?
19. How many vertices does a pentagon have?

20. Segments SP, SQ, and SR have how many endpoints in common?



21. How many points of intersection do the two figures have?



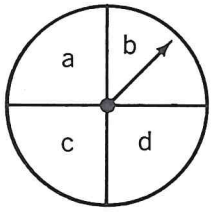
22. Mike has four doughnuts, Al has three, and Joe has half a dozen. How many do they have together?

23. Jean has fifty-four cents and Ann has thirty-two cents. How many more cents does Jean have than Ann?

24. A rancher bought 25 calves for $47.39 each. What did he have to pay all together for the calves?

Part 19 Probability

The probability of getting an a when spinning this spinner is $\frac{1}{4}$. One of four results can occur: a, b, c, or d.

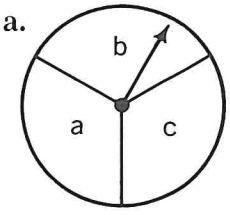


One of these four results is a.

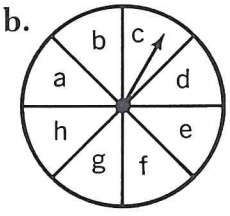
The probability of getting an a is one out of four chances or $\frac{1}{4}$.

1. What is the probability of getting an a when spinning each of the spinners?

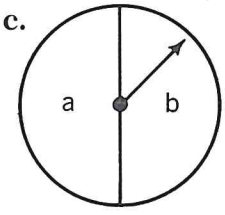
a.



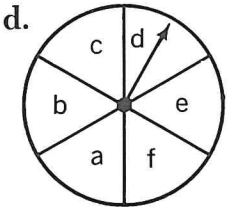
b.



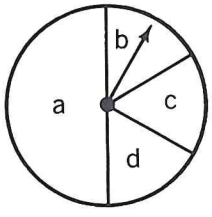
c.



d.



Look at the spinner shown on the right. The spinner is still divided into four regions, but now they are not equal in size.



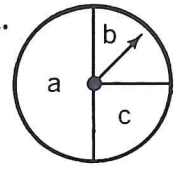
In this case region a is half of the spinner, so the probability of getting a is $\frac{1}{2}$.

Region b is one sixth of the spinner, so the probability of getting b is $\frac{1}{6}$.

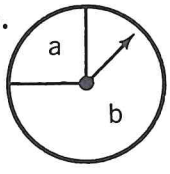
Because different results have different probabilities, we say that the results are not equally likely, or that the spinner is not symmetrical.

2. Which spinners shown below are symmetrical?

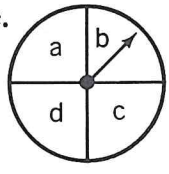
a.



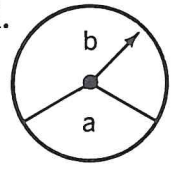
b.



c.



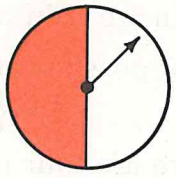
d.



3. What is the probability of getting b when spinning each of the spinners?

Outcomes and Events

1. Look at the two-color spinner. We may write r when the spinner lands on red, and w when the spinner lands on white.



   a. Suppose we spin this spinner twice. Complete the table to find what different outcomes there can be.

	First spin	Second spin
Outcome 1	r	r
Outcome 2	r	w
Outcome 3	w	
Outcome 4	w	w

   b. We may think of these four possible outcomes as a set, X. Copy and complete the set notation.

$X = \{rr, rw, \_\_\_, ww\}$

2. When we spin the spinner twice, various events can happen. We may think of these events as sets of some of the outcomes in X.

   a. Let the event A be the event of at least one r. Which possible outcomes have at least one spin landing on red?

   b. Look at the table for ways A may occur: rr, rw, and wr.

   A is a subset of X and $A = \{rr, \_\_\_, \_\_\_\}$.

3. Pair the events and subsets of X.

The event B of at least one w.

The event C of exactly one r.

The event D of two w's.

The event E of two r's.

The event F of the two spins landing on the same color.

The event G of at least one r.

a. $\{rw, wr\}$

b. $\{ww\}$

c. $\{rw, wr, ww\}$

d. $\{ww, rr\}$

e. $\{rr\}$

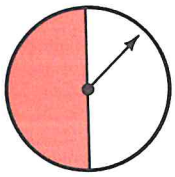
f. $\{rw, wr, rr\}$

Probability of Outcomes and Events

When we spin our two-color spinner twice, the set of possible outcomes is X.

$$X = \{rr, rw, wr, ww\}.$$

There are four possible outcomes. One outcome is as likely to occur as another because the spinner is symmetrical. Each outcome has 1 out of 4 chances. The probability of each outcome is $\frac{1}{4}$.



r = landing on red

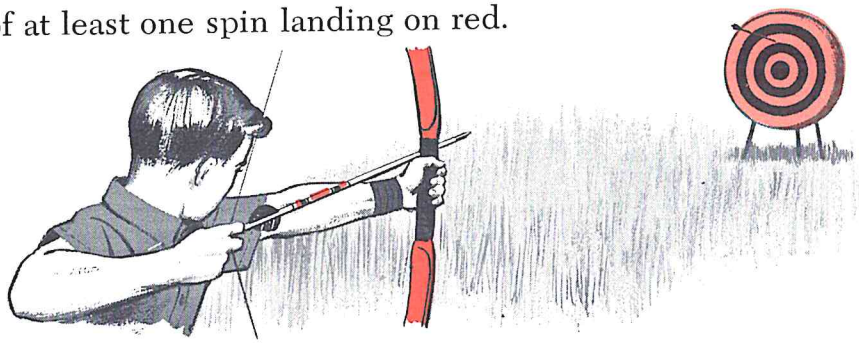
w = landing on white

The probability of an event is just the sum of the probabilities of the possible outcomes that will make the event occur.

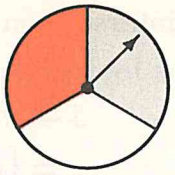
If $A = \{rr, rw, wr\}$,

then the probability of $A = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$.

1. What is the probability of each of these events when we spin the two-color spinner twice?
   - **a.** The event of exactly one spin landing on red.
   - **b.** The event of both spins landing on white.
   - **c.** The event of at least one spin landing on white.
   - **d.** The event of exactly one spin landing on white.
   - **e.** The event of both spins landing on the same color.
   - **f.** The event of the two spins landing on different colors.
   - **g.** The event of both spins landing on red.
   - **h.** The event of at least one spin landing on red.



1. Look at this three-color spinner. We spin it twice.



g = landing on gray

r = landing on red

w = landing on white

   a. The set X of possible outcomes has nine members. List them.

$$X = \{gg, gr, \_, \_, \_, \_, \_, \_, \_\}$$

   b. One outcome (is, is not) as likely as another because the spinner is symmetrical.

   c. Each possible outcome of the two spins has 1 out of ■ chances.

   d. The probability of each outcome of two spins is ■.

2. a. When we spin the three-color spinner twice, describe each of the following events as a subset of X.

The event Y of landing on gray or white on the second spin.

$$Y = \{gg, gw, rg, rw, wg, ww\}$$

The event A of at least one spin landing on red.

The event B of at least one spin landing on white.

The event C of exactly one spin landing on gray.

The event D of the second spin landing on white.

The event E of the first spin landing on gray.

The event F of the first spin landing on red or white.

The event G of both spins landing on the same color.

The event H of neither spin landing on gray.

The event I of the two spins landing on different colors.

The event J of the first spin landing on red.

The event K of the second spin landing on gray.

The event L of the first spin landing on red or white or gray.

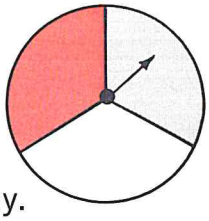
The event M of the second spin landing on red or white.

   b. What is the probability of each event listed above?

$$\text{probability of } Y = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

The intersection or union of two events is also an event.

Let us look again at the three-color spinner.



J = the event of the first spin landing on red.

$= \{rg, rr, rw\}$

K = the event of the second spin landing on gray.

$= \{gg, rg, wg\}$

Then: $J \cap K = \{rg, rr, rw\} \cap \{gg, rg, wg\}$

$= \{rg\}$

= the event of the first spin landing on red and the second spin landing on gray.

And: $J \cup K = \{rg, rr, rw\} \cup \{gg, rg, wg\}$

$= \{rg, rr, rw, gg, wg\}$

= the event of the first spin landing on red or the second spin landing on gray.

Suppose the three-color spinner shown above is spun twice. Describe in terms of intersection of sets the following events. Find the answer set.

1. The event of the first spin landing on gray and the second spin landing on white.
2. The event of at least one spin landing on red and neither spin landing on gray.

Describe in terms of union of sets the following events. Find the answer set.

3. The event of the first spin landing on red or the second spin landing on white.
4. The event of at least one spin landing on red or at least one spin landing on white.

Feedback

Complete each problem.

1. $\begin{array}{r} \$4.68 \\ +\ 3.78 \\ \hline \end{array}$

2. $\begin{array}{r} \$16.38 \\ \times\ \ 5 \\ \hline \end{array}$

3. $\begin{array}{r} \$10.00 \\ -\ 4.97 \\ \hline \end{array}$

4. $6\overline{)\$19.32}$

5. $\begin{array}{r} 18\frac{1}{3} \\ +\ 9\frac{3}{4} \\ \hline \end{array}$

6. $\begin{array}{r} 15\frac{1}{4} \\ -\ 7\frac{3}{8} \\ \hline \end{array}$

7. $\begin{array}{r} 908 \\ \times\ 45 \\ \hline \end{array}$

8. $28\overline{)4687}$

Perform the operations. Then choose $<$ or $=$ or $>$.

9. $3^2 \bigcirc 2^3$
10. $6^2 \bigcirc 2^2 \times 3^2$
11. $2^5 \bigcirc 3^3$
12. $2^2 \bigcirc 3^3 - 5^2$
13. $4^3 \bigcirc 4^2 + 4$
14. $7^2 \bigcirc 1 + (2^4 \times 3)$
15. $2^4 \bigcirc 2^3 + 2$
16. $5^2 \bigcirc 5^3 - 5$
17. $6^2 \bigcirc 6^2 \times 0$
18. $3^2 + 4^2 \bigcirc 5^2$
19. $6^2 + 8^2 \bigcirc 10^2$
20. $2^2 + 2^2 \bigcirc 2^4$
21. $4^2 \bigcirc (3 \times 5) + 1$
22. $7^2 \bigcirc (6 \times 8) + 1$

23. Express each number as a product of prime factors.

a. 6	c. 14	e. 42	g. 27
b. 9	d. 25	f. 33	h. 24

24. Find the least common multiple of 12 and 54.

25. Ed has $5\frac{1}{3}$ yd. of fishline, Al has $16\frac{1}{2}$ ft., and Si has 192 in. Which boy has the most fishline?

26. The area of a rectangle is 48 square units. The length is 8 units. What is the width?

27. Beth gave 5 pencils to each of 3 friends. How many pencils did she give away?

Part 20 Graphs and Functions

Sets and Rules

We can sometimes list members of a set if we are given a rule.

Rule: Give the set of all whole numbers n such that n is greater than 1 and less than 6.

$$\{n: 1 < n < 6\} = \{2, 3, 4, 5\}$$

Given the set of whole numbers, list the members of each set.

1. $\{n: 7 < n < 12\}$
2. $\{n: 19 < n < 24\}$
3. $\{n: 983 < n < 989\}$
4. $\{n: 258 < n < 264\}$
5. $\{n: 417 < n < 420\}$
6. $\{n: 5 < n < 10\}$

We can sometimes find the rule if we are given the set.

$\{2, 4, 6, 8\}$ **Rule:** Give the even numbers between 0 and 10.

$\{3, 5, 7\}$ **Rule:** Give the odd numbers between 2 and 8.

$\{3, 8, 50\}$ We do not know the rule.

$\{\text{Ann, Pete, Bob}\}$ We do not know the rule.

For which sets can you find rules? Give the rules.

7. $\{10, 12, 14, 16, 18, 20\}$
8. $\{\text{Joyce, a, b, 7, } \frac{1}{2}\text{, September}\}$
9. $\{136, 137, 138, 139, 140, 141\}$
10. $\{\text{Penny, Mark, Jim, Betsy}\}$
11. $\{7, 9, 11, 13, 15, 17, 19\}$
12. $\{\text{January, February, March}\}$
13. $\{2, 4, 6, 8, 10, 12, 14\}$
14. $\{6, 9, 12, 15, 18\}$
15. $\{0, 5, 10, 15, 20\}$
16. $\{0, 1, 2, 3, 4, 5\}$

Ordered Pairs and Rules

We can sometimes list a set of **ordered pairs** if we are given a rule. The things in the ordered pairs need not be numbers.

Rule: Pair Hawaii, Iowa, Maine, and Virginia with the number of representatives each has in the House of Representatives.

{(Hawaii, 2), (Iowa, 7), (Maine, 2), (Virginia, 10)}

Rule: Pair the numbers 0, 3, and 8 with the number plus 4.

{(0, 4), (3, 7), (8, 12)}

first number — first number plus 4

List the set of ordered pairs given by each rule.

1. Pair each number with 5 times the number: 0, 5, 10, 11, and 12.
2. Start with January and 1 and pair each month with a whole number.
3. Pair each number with the number plus $\frac{1}{2}$: 0, $\frac{1}{2}$, 2, $4\frac{1}{2}$, 5, 8, and $9\frac{1}{2}$.
4. Pair each number with the number divided by 8: 72, 40, 64, 48.

We can sometimes find the rule if we are given a set of ordered pairs.

{(1, 0), (4, 3), (7, 6), (8, 7)} **Rule:** Subtract 1 from 1, 4, 7, and 8.

{(3, 6), (A, 6), (May, 6)} **Rule:** Pair 3, A, and May with 6.

{(A, B), (D, E), (M, N)} **Rule:** Give the letter of the alphabet which follows: A, D, M.

Give the rule for each set of ordered pairs.

5. {(April, May), (June, July)}
6. {(D, C), (P, O), (S, R), (Y, X)}
7. {(0, 3), (4, 3), (6, 3), (8, 3)}
8. {(4, 8), (5, 9), (6, 10)}
9. {(6, 4), (7, 5), (8, 6)}
10. {(3, 12), (5, 20), (7, 28)}

Sometimes we do not know what rule gave the set of ordered pairs.

$\{(A, 10), (B, 0), (C, 13), (D, 347)\}$

$\{(3, 7), (19, 20), (\frac{1}{2}, -1)\}$

For which sets of ordered pairs can you find the rules? Give them.

1. $\{(April, 3), (May, 7)\}$
2. $\{(\frac{1}{2}, 1), (\frac{2}{3}, Mars), (0, 3)\}$
3. $\{(1, 2), (2, 4), (4, 8), (5, 10)\}$
4. $\{(4, 9), (5, 10), (6, 11)\}$
5. $\{(0, 2), (2, 4), (4, 6), (6, 8), (8, 10)\}$
6. $\{(Joe, 3), (Pat, 5), (Andy, \frac{3}{4})\}$
7. $\{(1, 9), (2, 9), (June, 9), (\frac{1}{2}, 9)\}$
8. $\{(6, 3), (12, 6), (24, 12)\}$

The same first member is sometimes in more than one of the ordered pairs in a set.

$\{(7, 4), (9, 1), (9, 2)\}$ — same first member

$\{(3, 3), (4, 3), (3, 4)\}$ — same first member

In which sets are there two ordered pairs with the same first member?

9. $\{(3, 6), (4, 7), (2, 5), (3, 10)\}$
10. $\{(March, April), (April, May)\}$
11. $\{(1, 1), (0, 0), (1, 2), (2, 4)\}$
12. $\{(2, 6), (3, 8), (4, 10)\}$
13. $\{(Ed, 7), (Al, 10), (Betty, 3)\}$
14. $\{(10, 5), (5, 6), (3, 1), (10, 4)\}$
15. $\{(1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3), (\frac{1}{4}, 4), (1, 2)\}$
16. $\{(4, 2), (5, 3), (4, 4)\}$

Give a rule for finding the second member. Complete each problem.

17. $\{(1, 10), (2, 20), (3, 30), (4, \square), (5, \square)\}$
18. $\{(1, -1), (2, -2), (3, -3), (4, \square), (5, \square)\}$
19. $\{(1, 3), (2, 6), (3, 9), (4, \square), (5, \square)\}$

Functions

A **function** is a set of ordered pairs in which no two ordered pairs have the same first member.

$\{(1, 2), (2, 3), (4, 7)\}$ is a function.

$\{(1, 2), (1, 3), (\frac{1}{2}, 4)\}$ is not a function

These two pairs have the same first member.

Which of the sets are functions?

1. $\{(1, 1), (1, 2), (1, 3)\}$
2. $\{(\frac{1}{2}, 1), (\frac{2}{3}, 2), (\frac{1}{4}, \text{Jane})\}$
3. $\{(1, 2), (1, 3), (1, 4), (1, 5)\}$
4. $\{(\text{Kay}, \text{Amy}), (\text{Amy}, \text{Kay})\}$
5. $\{(\frac{1}{2}, \frac{2}{4}), (\frac{1}{3}, \frac{3}{9}), (\frac{1}{4}, \frac{4}{16})\}$
6. $\{(1, 4), (2, 4), (3, 4)\}$
7. $\{(1, -7), (7, -7), (8, -8), (9, -9)\}$
8. $\{(\frac{1}{4}, 0), (-3, 9), (7, 6), (\frac{1}{2}, 1)\}$
9. $\{(3, 2), (4, 3), (4, 2), (5, 4)\}$
10. $\{(\text{Mercury}, 1), (\text{Venus}, 2), (\text{Earth}, 3)\}$

List the set of ordered pairs given by each rule and tell which sets of ordered pairs are functions.

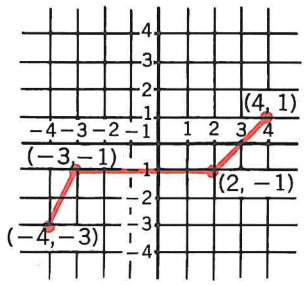
Rule: Pair 0 with 1 and 2. $\{(0, 1), (0, 2)\}$ is not a function.

Rule: Pair 3, 4, and 5 with 3. $\{(3, 3), (4, 3), (5, 3)\}$ is a function.

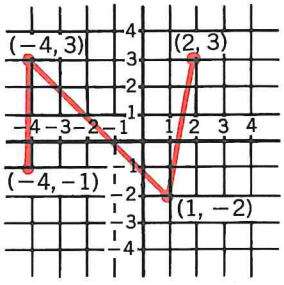
11. Pair each of these numbers with itself: 1, 2, 3, 4, 5.
12. Pair each number with 3 plus the number: 3, 7, 9, 11.
13. Pair 2 with 1, 3, and 5.
14. Pair these states with their capitals: Wyoming, Iowa, Kansas, Georgia.
15. Pair $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ with 1.
16. Pair each number with the number minus itself: $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}, \frac{4}{5}, \frac{6}{7}$.
17. Pair each month with its correct season: December, July, October, April.
18. Pair each number with the number divided by 2: 0, 1, 2, 3, 4, 5, 6.

Graphs of Sets of Ordered Pairs

We can make a graph of a set of ordered pairs by plotting the points. If any two points determine a vertical segment, the set of ordered pairs is not a function.



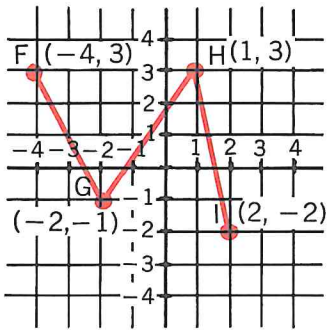
This graph represents a set of ordered pairs that is a function.



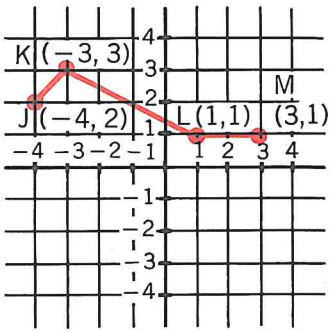
This graph represents a set of ordered pairs that is not a function.

Which of these graphs represent functions?

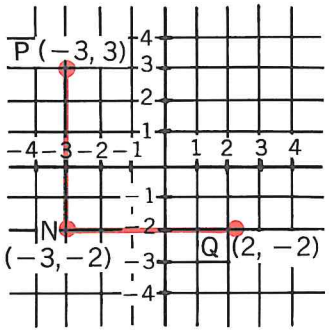
1.



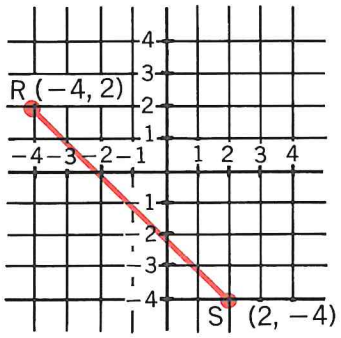
2.



3.



4.



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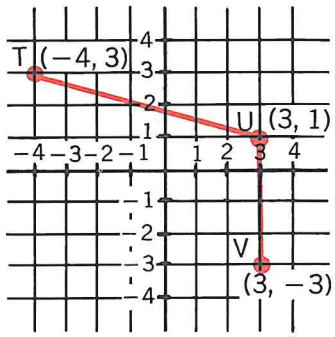

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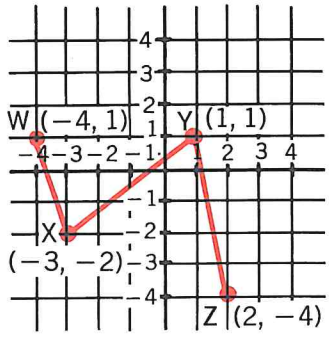


Using Tables

Instead of listing ordered pairs: $(-7, -2)$ $(-1, 3)$ $(0, 6)$ $(2, -1)$ $(1, 1)$

We can write them in a table:

x	−7	−1	0	2	1
y	−2	3	6	−1	1

Row x gives the horizontal coordinates.
Row y gives the vertical coordinates.

Copy and complete each table, using the given set of ordered pairs. Notice that the horizontal coordinate always is placed in the first row.

1.

g	−7	−4	0		
h	−1			6	0

$\{(-7, -1), (-4, 3), (0, 7), (2, 6), (3, 0)\}$

2.

j	−4				4
k		2			

$\{(-4, 3), (-3, 2), (-2, 4), (1, -3), (4, 1)\}$

3.

v	−8	−7			
w				0	

$\{(-8, 1), (-7, -5), (-1, 3), (0, 0), (2, 4)\}$

List the set of ordered pairs given in each table.

4.

r	−9	−1	0	5	6	8
s	3	7	4	5	2	−4

5.

m	4	4	4	4	4	4
n	−7	−4	−1	3	6	9

6.

e	−7	−4	−1	2	5	8
f	−6	−3	0	3	6	9

7.

c	−8	−7	−2	0	3	9
d	−6	−6	−6	−6	−6	−6

8. For Exercises 4-7 draw a coordinate system and plot the points.
   - **a.** Which sets of ordered pairs determine at least one vertical segment?
   - **b.** Which sets of ordered pairs are functions?
   - **c.** Which sets of ordered pairs plot on straight lines?
   - **d.** Which sets of ordered pairs plot on vertical lines?
   - **e.** Which sets of ordered pairs plot on horizontal lines?

Some rules may be written either as English sentences or as mathematical sentences.

English sentence	Mathematical sentence
Rule: Multiply each number by 2.	$n = 2 \times m$
Rule: Subtract 4 from each number.	$y = x - 4$
Rule: Add 1 to each number.	$q = p + 1$

Match the rules written as mathematical sentences with those written as English sentences.

1. To convert pints to quarts, divide the number of pints by 2.
2. To find out how old someone will be in 1 year, add 1 to his present age.
3. To convert feet to inches, multiply the number of feet by 12.
4. If we add zero to any number, we get the same number.
5. The perimeter of a square is 4 times the length of a side.
6. To find out how old someone was 5 years ago, subtract 5 from his present age.

A. $p = 4 \times s$

B. $i = 12 \times f$

C. $y = a + 1$

D. $z = z + 0$

E. $q = p \div 2$

F. $k = j - 5$

Match each rule with a table. Copy and complete each table.

7.

r	0	2	4	6	9	11	12
s	0	6	12	18	27		

8.

r	−3	−2	−1	0	1	2	3
s		−2	−1	0	1	2	

9.

r	0	2	3	5	7	8	9
s	0	4	6	10	14		

A. $s = r$

B. $s = 3 \times r$

C. $s = 2 \times r$

Match each table with a rule. Copy and complete each table.

1.

p	0	1	2	3	4	5	6
q	1	2	3	4			7

2.

p	0	1	2	3	4	7	9
q	−1	0		2	3	6	

3.

p	−2	−1	0	1	2	5	7
q	0	1	2	3	4		

A. $q = p - 1$

B. $q = p + 1$

C. $q = p + 2$

If all the vertical coordinates in a table are the same, the function is **constant.** No matter what the horizontal coordinate is, the vertical coordinate is always the same. We describe a constant function with a rule using only one letter or variable.

Rule: $q = -5$

p	−10	−6	−1	0	3	7	9
q	−5	−5	−5	−5	−5	−5	−5

Which tables show constant functions? Write the rule used to make each table, and then copy and complete the table.

4.

r	−8	−6	0	2	4	6	8
s	3	3	3		3	3	

5.

t	−8	−4	0	1	3	5	9
w	−10	−6	−2	−1	1	3	

6.

x	−10	−7	−2	−1	1	2	3
y	−1	−1	−1	−1			−1

7.

a	−4	−2	5	6	7	9
b	−9	−7	0	1	2	

8.

m	−10	−2	3	5	6	9
n	2			2	2	2

9.

c	−5	−3	0	2	4	7
d	1	3		8	10	

10. For Exercises 4-9, draw a coordinate system and plot the points.

11. Which sets of ordered pairs in Exercises 4-9 are functions?

Given a rule, we can make a table of ordered pairs.

We first choose some horizontal coordinates.

We then find the corresponding vertical coordinates.

Rule: $y = x - 3$

We wish to find ordered pairs (x, y).

Suppose we choose 4, 2, and 1 as horizontal coordinates.

We find the corresponding vertical coordinates this way.

If $x = 4$, then $y = 4 - 3 = 1$

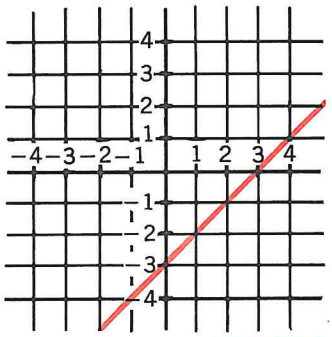
If $x = 2$, then $y = 2 - 3 = -1$

If $x = 1$, then $y = 1 - 3 = -2$

Our table looks like this:

x	4	2	1
y	1	−1	−2

Our graph looks like this:



For each rule, make a table of values using the three horizontal coordinates given.

1. $y = x + 3$

x	0	2	3
y			

2. $q = p + 2$

p	1	3	5
q			

3. $y = 4$

x	0	2	4
y			

4. $k = j + 4$

j	−6	−4	0
k			

5. $y = x + 5$

x	−3	0	3
y			

6. $s = r - 3$

r	1	2	3
s			

7. $v = u - 2$

u	−3	−2	−1
v			

8. $e = d + 1$

d	−3	−1	1
e			

9. $z = y$

y	−7	0	7
z			

10. $n = 2m$

m	0	1	2
n			

For each rule make a table of values. Choose three numbers as horizontal coordinates.

1. $y = x - 6$

x	4	5	6
y			

2. $y = x$

x	−6	−5	−4
y			

3. $y = x + 4$

x	−4	0	4
y			

4. $y = 3$

x			
y			

5. $y = 2x$

x			
y			

6. $y = x - 1$

x			
y			

7. $y = 0$

x			
y			

8. $y = 3x$

x			
y			

9. $y = x + 6$

x			
y			

10. $y = \frac{1}{2}x$

x			
y			

11. For Exercises 1-10 draw a coordinate system and plot the ordered pairs in your table.

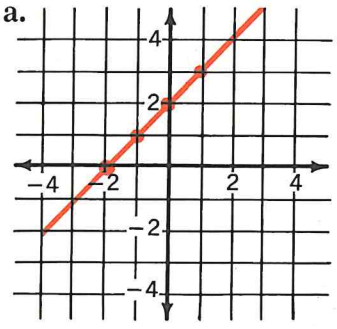
12. Do the points in each set lie in a straight line?

A linear function determines a straight line when plotted.

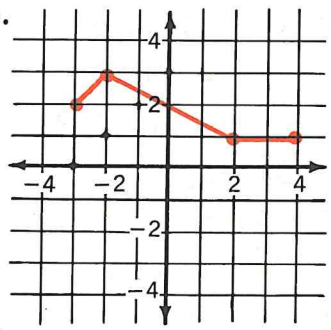
13. Which rules in Exercises 1-10 describe functions?

14. Which of these graphs are graphs of linear functions?

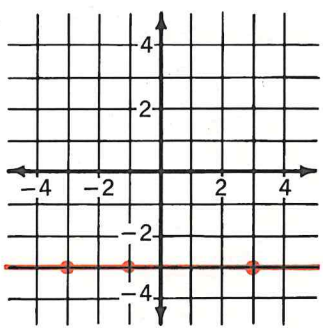
a.



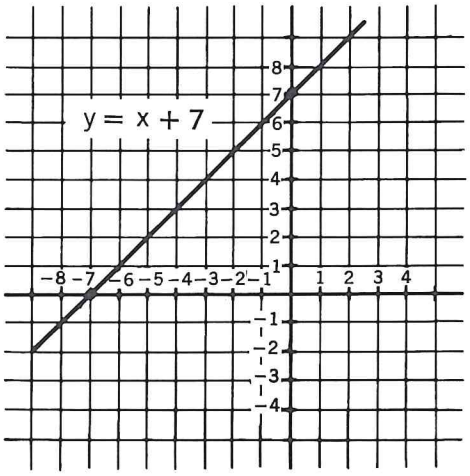
b.



c.



1. Which of these points lie on the graph of $y = x + 7$?



a.	(−8, −2)	h.	(0, 6)
b.	(−7, 0)	i.	(−5, 2)
c.	(−6, −1)	j.	(1, 8)
d.	(−4, 3)	k.	(0, 0)
e.	(−3, 2)	l.	(−3, 4)
f.	(−1, 6)	m.	(−7, 7)
g.	(0, 5)	n.	(−3, 3)

We can tell if a point will lie on the graph of a function without using a graph. If a point is on the graph, its coordinates must satisfy the rule.

Does (2, 3) satisfy $y = x + 1$? Yes, because $3 = 2 + 1$.
$y = 3$ $x = 2$

Does (0, 4) satisfy $q = p + 3$? No, because $4 \neq 0 + 3$.
$q = 4$ $p = 0$

Does (4, 6) satisfy $y = 7$? No, because $6 \neq 7$.
$y = 6$

2. Which points lie on each graph?

a.	$b = a + 3$	(−3, 0) (2, 5) (1, 5)
b.	$n = m + 4$	(−2, 6) (3, 7) (4, 4)
c.	$z = t - 1$	(4, 3) (6, 5) (9, 8)
d.	$g = f - 3$	(2, −1) (6, 2) (8, 4)
e.	$q = -7$	(9, −7) (4, 6) (3, 8)
f.	$d = c + 1$	(4, 5) (−2, −1) (8, 10)
g.	$r = s - 5$	(3, −1) (5, 0) (7, 3)
h.	$h = k$	(3, 2) (3, 4) (3, 3)
i.	$u = 3$	(4, 3) (8, 3) (−6, 3)

Feedback

Solve each equation.

1. $3 \div 7 = s$
2. $17 - 12 = t$
3. $6 - 6 = p$
4. $m - 8 = 5$
5. $x - 3 = 11$
6. $g \times 5 = 50$
7. $19 + r = 23$
8. $31 - n = 26$
9. $43 - t = 37$

Find each quotient.

10. $6\overline{)4925}$
11. $29\overline{)9384}$
12. $43\overline{)8721}$
13. $19\overline{)4852}$

14. Express each number as a product of prime factors.
    - a. 10
    - b. 12
    - c. 8
    - d. 15
    - e. 20
    - f. 21
    - g. 44
    - h. 48

15. If each pear tree produces an average of 359 pears, how many pears will an orchard of 87 pear trees produce?

16. Pete, Scott, and Mark are working on a science project about frogs. Pete collected 283 frogs' eggs, Scott collected 146, and Mark collected 302. How many frogs' eggs did they collect all together?

Write a mathematical sentence for each English sentence and answer each question.

17. a. Sue is 2 years older than Fay.
    b. Together their ages total 14.
    c. How old is each girl?

18. a. Harry has 3 times as many stamps as Fred.
    b. Together they have 80 stamps.
    c. How many stamps does each boy have?

Part 21 More About Sets

Sometimes we use a capital letter to stand for a set. This saves time and space when we write the answer set.

On this page we will use these capital letters to stand for these sets.

$A = \{\ \}$ $\quad E = \{Al, Ed\}$

$B = \{Al\}$ $\quad F = \{Al, Ann\}$

$C = \{Ed\}$ $\quad G = \{Ed, Ann\}$

$D = \{Ann\}$ $\quad H = \{Al, Ed, Ann\}$

Using these set descriptions and the corresponding capital letters we solve the problem

$$\{Ed, Ann\} \cup \{Al\} = \{Al, Ann, Ed\}$$

by writing:

$$G \cup B = H$$

1. Using the capital letters given above, copy and solve each problem.

a. $\{Al, Ed\} - \{Ed\} = B$

b. $\{Ann\} \cup \{Al, Ed\} =$

c. $\{Ed\} \cup \{Al, Ed\} =$

d. $\{Al, Ed, Ann\} \cap \{Al\} =$

e. $\{Al, Ann\} - \{Al, Ed, Ann\} =$

f. $\{Al, Ed\} \cap \{Ed, Ann\} =$

2. Find each answer set.

a. $A \cup B = \{Al\}$

b. $H - E =$

c. $G \cap C =$

d. $D \cap F =$

e. $C - G =$

f. $E \cup D =$

g. $H \cup G =$

h. $A \cap D =$

i. $E \cap B =$

j. $G \cup D =$

k. $F - A =$

l. $E - G =$

1. Each of the letters names a set. Solve each equation as in the example.

$\{\text{April, May}\} \cap \{\text{May}\} = T$

$T = \{\text{May}\}$

a. $\{1, 2, 4, 5\} \cup \{3, 0\} = Y$

b. $\{a, b, c, d\} \cap \{c, d, e\} = M$

c. $\{\text{Tim, Pete}\} - \{\text{Mike, Pete}\} = X$

d. $\{\text{Bill, Sam}\} \cup \{\text{Sam, Bill}\} = G$

e. $\{10, 20, 30\} \cap \{30\} = Q$

2. Use the capital letters given and solve each problem.

$A = \{\ \}$ $\quad$ $D = \{v, y\}$ $\quad$ $G = \{v, y, z\}$

$B = \{z\}$ $\quad$ $E = \{x, y\}$ $\quad$ $H = \{x, y, z\}$

$C = \{y\}$ $\quad$ $F = \{x, z\}$ $\quad$ $I = \{v, x, y, z\}$

a. $C \cup F =$ H

b. $E \cap H =$ ▢

c. $B \cup D =$ ▢

d. $G - B =$ ▢

e. $B - G =$ ▢

f. $C - I =$ ▢

g. $D \cap E =$ ▢

h. $H - C =$ ▢

i. $D \cup F =$ ▢

j. $A - E =$ ▢

k. $G \cup H =$ ▢

l. $B \cap I =$ ▢

m. $F \cap G =$ ▢

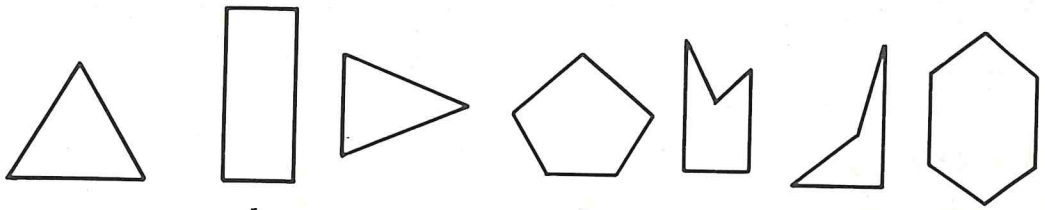
n. $I - F =$ ▢

o. $G \cup A =$ ▢

3. a. What is the set T of triangles?

b. What is the set Q of quadrilaterals?

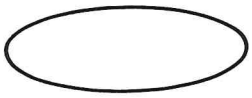
c. What is the set P of pentagons?

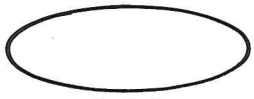


a. **b.** **c.** **d.** **e.** **f.** **g.**

Intersection of Sets

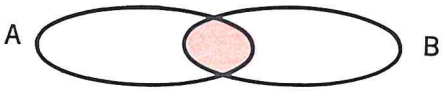
We can use diagrams to show sets.





We can show with diagrams the intersection of sets.

$A \cap B$



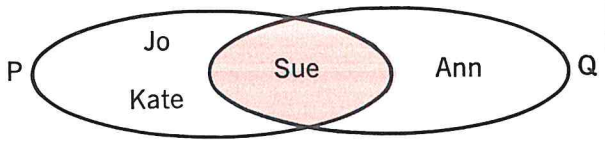
The shaded area shows the part common to both sets.

Find the intersection of each pair of sets. Draw a diagram for each problem.

$P = \{Jo, Kate, Sue\}$

$Q = \{Sue, Ann\}$

$P \cap Q = \{Sue\}$



The shaded part represents $\{Sue\}$.

1. $M = \{Paris, London\}$
   $N = \{Paris\}$

2. $R = \{n: 1 < n < 6\}$
   $S = \{4, 5, 6\}$

3. $X = \{a, b, c\}$
   $Y = \{a, b\}$

4. $J = \{Karen\}$
   $K = \{Jean, Mary\}$

5. $E = \{Kim, Eve\}$
   $F = \{Kate, Kim\}$

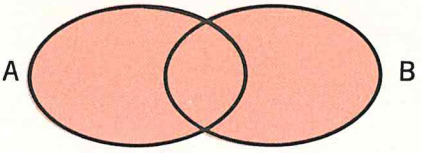
6. $K = \{3, 4, 5\}$
   $L = \{n: 4 < n < 8\}$

7. $G = \{1, 2, 3, 4, 5\}$
   $H = \{n: 2 < n < 5\}$

8. $U = \{a, b, c, d, e\}$
   $V = \{d, e, f, g\}$

Union of Sets

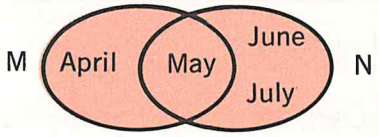
We can shade diagrams to show union of sets.

$A \cup B$ 

The shaded area shows the union of the two sets.

Find the union of each pair of sets. Draw a diagram for each problem.

$M = \{April, May\}$
$N = \{May, June, July\}$
$M \cup N = \{April, May, June, July\}$



The shaded area represents $\{April, May, June, July\}$.

1. $W = \{Al, Ed\}$
   $Z = \{Vi, Jo\}$

2. $X = \{Ned, Ted\}$
   $Y = \{Ted, Fred\}$

3. $R = \{n: 2 < n < 6\}$
   $S = \{5, 6, 7\}$

4. $P = \{m, n\}$
   $Q = \{y, z, w\}$

5. $T = \{1, 2, 3, 4\}$
   $G = \{\ \}$

6. $C = \{s, t, x, z\}$
   $D = \{r, s, t\}$

Using diagrams, show the union of sets and the intersection of sets for each problem. Tell what the shaded area represents.

7. $T = \{0, 2, 4, 6\}$
   $S = \{1, 3, 5\}$

8. $V = \{1, 2\}$
   $W = \{n: 0 < n < 4\}$

9. $K = \{a, b, c\}$
   $L = \{c, a, b\}$

10. $F = \{h, j\}$
    $G = \{s, t\}$

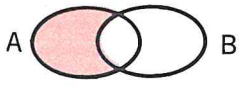
11. $B = \{Ed, Al\}$
    $C = \{Al, Di, Vi\}$

12. $D = \{Karen\}$
    $E = \{Jo, Sue\}$

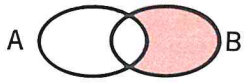
Difference of Sets

We can shade diagrams to show the difference of sets.

The shaded area shows the things left in A after B is taken away.



A − B



B − A

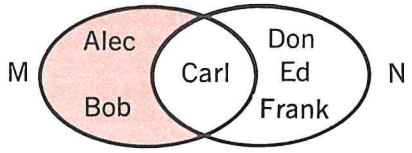
The shaded area shows the things left in B after A is taken away.

Find the difference of each pair of sets. Draw a diagram for each problem.

$M = \{\text{Alec, Bob, Carl}\}$

$N = \{\text{Carl, Don, Ed, Frank}\}$

$M - N = \{\text{Alec, Bob}\}$



The shaded area represents $\{\text{Alec, Bob}\}$.

1. $R = \{\text{Rex, Bob, Roy}\}$
   $S = \{\text{Ray, Don, Tod}\}$
   $R - S =$ ▒

2. $X = \{3, 4, 5, 6\}$
   $Y = \{5, 6, 7, 8\}$
   $X - Y =$ ▒

3. $P = \{n: n < 4\}$
   $Q = \{0, 1, 2, 3\}$
   $P - Q =$ ▒

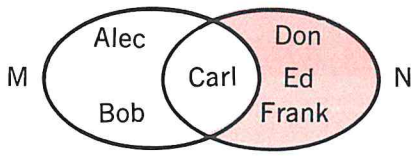
4. $F = \{\ \}$
   $G = \{\text{Jean, Joan}\}$
   $F - G =$ ▒

Using the sets in Exercises 1-4 above, find each answer set.

$M = \{\text{Alec, Bob, Carl}\}$

$N = \{\text{Carl, Don, Ed, Frank}\}$

$N - M = \{\text{Don, Ed, Frank}\}$



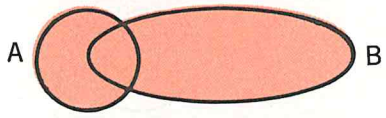
The shaded area represents $\{\text{Don, Ed, Frank}\}$.

5. $S - R =$ ▒
6. $Y - X =$ ▒
7. $Q - P =$ ▒
8. $G - F =$ ▒

For each pair of sets, perform these operations: union, intersection, taking the second set from the first set, and taking the first set from the second set. Make a diagram to show each operation and tell what the shaded area represents.

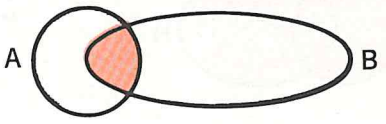
$A = \{a, b, c\}$
$B = \{b, c, d, e\}$

$A \cup B = \{a, b, c, d, e\}$



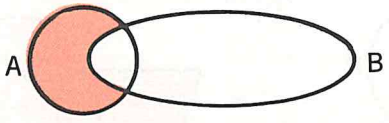
The shaded area represents $\{a, b, c, d, e\}$.

$A \cap B = \{b, c\}$



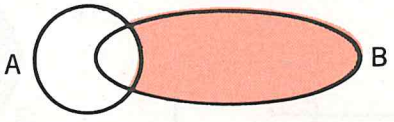
The shaded area represents $\{b, c\}$.

$A - B = \{a\}$



The shaded area represents $\{a\}$.

$B - A = \{d, e\}$



The shaded area represents $\{d, e\}$.

1. $M = \{Amy, Ann\}$
   $N = \{Ann, Alex\}$

2. $C = \{Tim, Ted\}$
   $D = \{Ted, Tod, Tom\}$

3. $G = \{m, n, o\}$
   $H = \{n, o, p, q\}$

4. $L = \{4, 5, 6, 7, 8\}$
   $K = \{4, 6, 8\}$

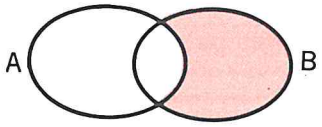
5. $V = \{\ \}$
   $W = \{a, b, c, d\}$

6. $R = \{0, 4, 8\}$
   $S = \{n: 3 < n < 5\}$

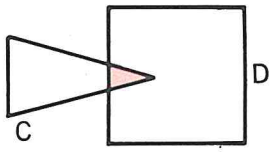
7. $H = \{o, p, q, r, s\}$
   $K = \{o, p\}$

8. $E = \{n: n < 6\}$
   $F = \{6, 7, 8\}$

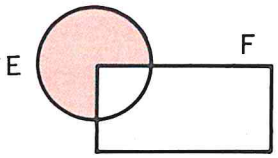
Choose $-$ or $\cup$ or $\cap$.

1. 

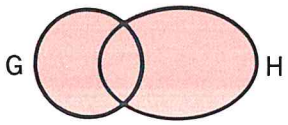
B ○ A

2. 

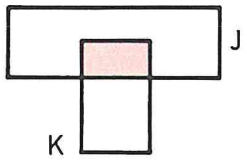
C ○ D

3. 

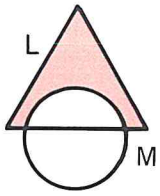
E ○ F

4. 

G ○ H

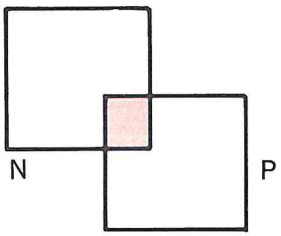
5. 

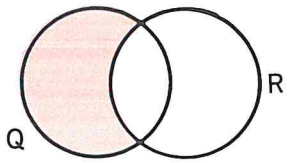
J ○ K

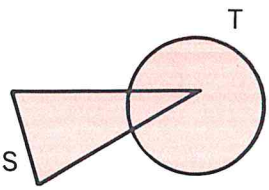
6. 

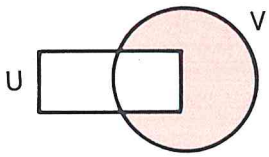
L ○ M

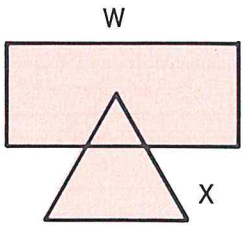
Tell what the shaded area in each diagram represents.

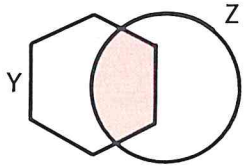
7. 

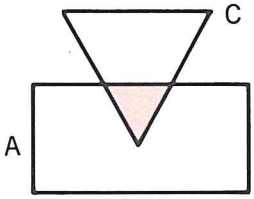
8. 

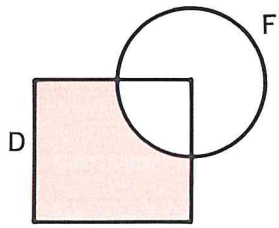
9. 

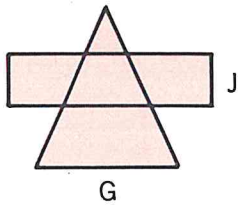
10. 

11. 

12. 

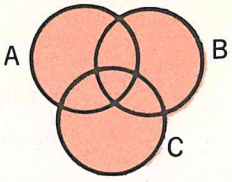
13. 

14. 

15. 

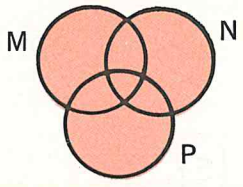
We can shade diagrams to show the union of more than two sets.

$A \cup B \cup C$



The shaded area represents $A \cup B \cup C$.

Copy each problem and make a diagram for it. Tell what the union of the three sets is and what the shaded area represents.



$M = \{4, 5\}$ $N = \{5, 6\}$ $P = \{3, 6, 7\}$

$M \cup N \cup P = \{3, 4, 5, 6, 7\}$

The shaded area represents $\{3, 4, 5, 6, 7\}$.

1. $P = \{3, 4, 5\}$
   $Q = \{0, 1, 3\}$
   $R = \{3, 5\}$

2. $R = \{\text{Dot, Eva}\}$
   $S = \{\text{Eva, May}\}$
   $T = \{\text{Bob, Ben}\}$

3. $D = \{n: n < 3\}$
   $E = \{0, 1, 2, 3\}$
   $F = \{4\}$

4. $X = \{d, e, f, g\}$
   $Y = \{f, g, h\}$
   $Z = \{d, e, f\}$

5. $V = \{4, 5, a, b\}$
   $W = \{a\}$
   $X = \{b, a, c\}$

6. $J = \{5, 6, 8\}$
   $K = \{n: 9 < n < 12\}$
   $L = \{8, 9, 10\}$

Each letter stands for a set. Find each answer set.

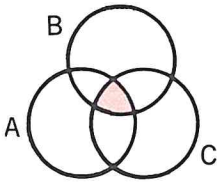
$\{4, 5\} \cup \{6\} \cup \{5, 6\} = A$

$A = \{4, 5, 6\}$

7. $\{3, 2, 1\} \cup \{\ \} \cup \{5, 4, 3\} = M$
8. $\{a, b, c\} \cup \{c\} \cup \{b, a, c\} = N$
9. $\{0, 1\} \cup \{4, 2, 1\} \cup \{4, 5\} = Y$

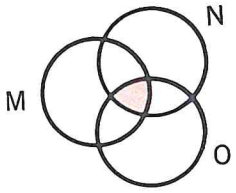
We can shade diagrams to show the intersection of more than two sets.

$A \cap B \cap C$



The shaded area represents the part common to all three sets.

Copy each problem and make a diagram for it. Tell what the intersection of the three sets is and what the shaded area represents.



$M = \{4, 5\}$ $N = \{5, 6\}$ $O = \{3, 5, 7\}$

$M \cap N \cap O = \{5\}$

The shaded area represents $\{5\}$.

1. $R = \{5, 6, 8\}$
   $P = \{5, 4, 3\}$
   $Q = \{1, 3, 5\}$

2. $V = \{Jo, Di, Vi\}$
   $X = \{Jo\}$
   $Y = \{Vi, Jo\}$

3. $J = \{10, 9, 8\}$
   $K = \{6, 8, 10\}$
   $L = \{8, 10, 12\}$

4. $S = \{a, b, c\}$
   $N = \{c, d, e\}$
   $Q = \{d, e, f\}$

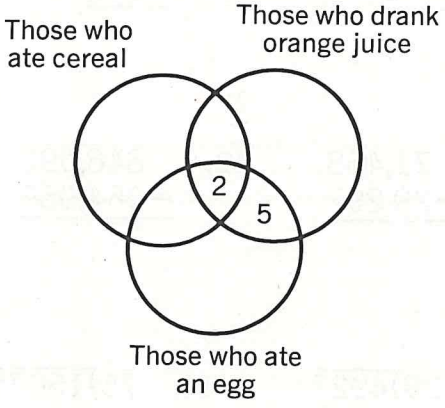
Each letter stands for a set. Find each answer set.

5. $\{3, 4, 5\} \cap \{4, 8, 12\} \cap \{2, 5, 6\} = A$
6. $\{a, b, c\} \cap \{\ \} \cap \{a, c, g\} = B$
7. $\{Pat, Don\} \cap \{Don, Tim\} \cap \{Jerry, Don\} = C$
8. $\{1, 3, 5\} \cap \{3, 5\} \cap \{1, 3, 5, 7, 9\} = D$

The following facts are known about the children in Miss Dean's class.

16 children ate cereal for breakfast.
15 children drank orange juice for breakfast.
13 children ate an egg for breakfast.
9 children ate cereal and drank orange juice.
7 children drank orange juice and ate an egg.
6 children ate cereal and ate an egg.
2 children ate cereal, drank orange juice, and ate an egg.

We can show this information using a diagram.

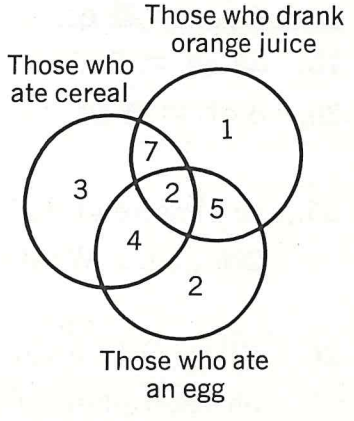


We know that only 2 children ate cereal, drank orange juice, and ate an egg. We show the "2" first.

We know that 7 children drank orange juice and ate an egg. We subtract 2 from 7 to find that there were 5 children who drank orange juice and ate an egg, but who did not eat cereal.

Study the completed diagram at the right before you answer the following questions.

1. How many children in Miss Dean's class ate only cereal for breakfast?
2. How many children drank only orange juice for breakfast?
3. How many children ate only an egg for breakfast?
4. How many children were in Miss Dean's class?
5. How many children did not eat cereal?
6. How many children did not drink orange juice?
7. How many children did not eat an egg?



Final Feedback

Find each amount.

1. $\begin{array}{r} \$16 \\ 28 \\ 95 \\ \hline \end{array}$

2. $\begin{array}{r} \$846 \\ 395 \\ 276 \\ \hline \end{array}$

3. $\begin{array}{r} \$14.28 \\ 38.09 \\ 54.46 \\ \hline \end{array}$

4. $\begin{array}{r} \$152{,}869 \\ 395{,}402 \\ 408{,}679 \\ \hline \end{array}$

Find each product.

5. $\begin{array}{r} 648 \\ \times\ 9 \\ \hline \end{array}$

6. $\begin{array}{r} 785 \\ \times\ 26 \\ \hline \end{array}$

7. $\begin{array}{r} 9064 \\ \times\ 8 \\ \hline \end{array}$

8. $\begin{array}{r} 938 \\ \times 250 \\ \hline \end{array}$

Find each difference.

9. $\begin{array}{r} 964 \\ -385 \\ \hline \end{array}$

10. $\begin{array}{r} 9047 \\ -2839 \\ \hline \end{array}$

11. $\begin{array}{r} 71{,}468 \\ -29{,}297 \\ \hline \end{array}$

12. $\begin{array}{r} 846{,}092 \\ -384{,}969 \\ \hline \end{array}$

Find each quotient.

13. $8\overline{)3452}$

14. $60\overline{)7213}$

15. $29\overline{)4927}$

16. $75\overline{)16075}$

Complete each problem.

17. 18 ft. = ■ yd.
18. 4 gal. = ■ qt.
19. 48 oz. = ■ lb.
20. 8 qt. = ■ pt.
21. 3600 sec. = ■ min.
22. 6 yd. 4 ft. = ■ ft.
23. 2 yd. 5 in. = ■ in.
24. 24 ft. = ■ yd.

25. Art has read 315 pages, Ben 237 pages, Carol 274 pages and Debbie 208 pages. What is the total number of pages read by the four children?

26. Ellen puts seventy-two cookies on four plates. How many cookies are on each plate, if each plate has the same number of cookies on it?

Final Feedback

Choose $\cap$, $\cup$, $-$, $\subset$, or $\in$.

1. $\{2, 7, 10\} \bullet \{6, 7, 8, 9\} = \{7\}$
2. $\{2, 7, 10\} \bullet \{6, 7, 8, 9\} = \{2, 10\}$
3. $\{7\} \bullet \{6, 7, 8, 9\}$
4. $7 \bullet \{6, 7, 8, 9\}$
5. $\{2, 7, 10\} \bullet \{6, 7, 8, 9\} = \{2, 6, 7, 8, 9, 10\}$

6. Express each number as a product of prime factors.

a. 22	c. 26	e. 40	g. 45
b. 51	d. 16	f. 39	h. 18

7. Find the least common multiple of 18 and 24.

8. $9\frac{1}{3} + 4\frac{5}{6}$

9. $8\frac{3}{4} + 2\frac{2}{3}$

10. $8\frac{3}{5} - 2\frac{3}{10}$

11. $7\frac{1}{2} - 3\frac{5}{8}$

Complete each problem.

12. $1\frac{2}{3} \times \frac{3}{8} = \blacksquare$
13. $4\frac{1}{2} \div 1\frac{1}{2} = \blacksquare$
14. $7\frac{1}{3} \times 1\frac{1}{11} = \blacksquare$
15. $.8 + .4 + .9 = \blacksquare$
16. $.6 + .24 + .18 = \blacksquare$
17. $4.92 + 2.84 = \blacksquare$

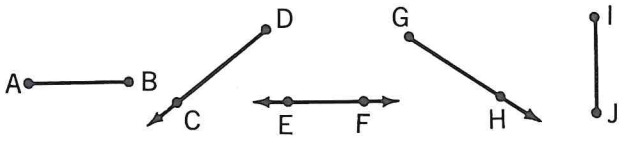
Find a mathematical sentence for each English sentence.

18. Marcia has 3 more stamps than Dora.
19. Mark has the same number of stamps as Don.
20. Marla has fewer stamps than Dot.

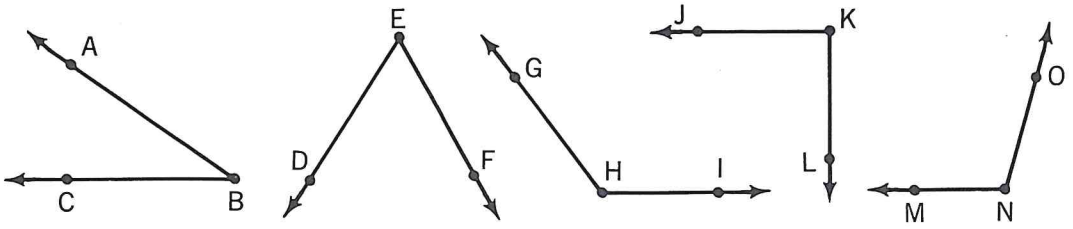
A. $m = d$
B. $m = d + 3$
C. $m < d$

Final Feedback

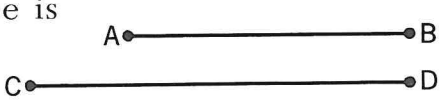
1. **a.** Which are lines?
   b. Which are rays?
   c. Which are line segments?



2. List the angles from smallest to largest.

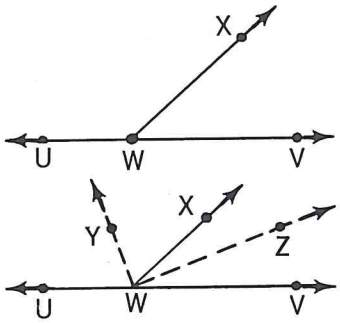


3. Construct an isosceles triangle whose base is equal in length to $\overline{AB}$ and whose sides are equal in length to $\overline{CD}$.



4. Draw a circle, and a diameter BC of the circle. Mark point A on the circle. Draw chords BA and CA. What kind of angle is BAC?

5. **a.** Draw a line UV and mark point W between U and V.
   b. Draw some ray WX.
   c. Bisect angle UWX by ray WY.
   d. Bisect angle XWV by ray WZ.
   e. Is angle YWZ a right angle? It should be.



6. **a.** Draw a line segment MN.
   b. Bisect line segment MN.

Final Feedback

1. Plot these ordered pairs. Label each point.

A (0, −3)	C (−1, 4)	E (2, 0)
B (4, 2)	D (−3, −3)	F (3, −2)

2. Which sets of ordered pairs are functions?

   a. $\{(-8, 4), (-7, 6), (-3, 10), (0, 7), (4, 6), (8, 7)\}$

   b. $\{(2, 4), (-5, 4), (2, -3), (0, 4), (-2, -3)\}$

   c. $\{(-8, 2), (-6, 2), (-3, 2), (2, 2), (5, 2), (9, 2)\}$

Match each rule with the set of ordered pairs which satisfy the rule.

3.	$q = p + 3$	A.	$\{(-8, -10), (-4, -6), (-1, -3), (3, 1), (6, 4), (8, 6)\}$
4.	$k = -j$	B.	$\{(-10, -7), (-6, -3), (-1, 2), (0, 3), (2, 5), (4, 7)\}$
5.	$b = a + 1$	C.	$\{(-8, -7), (-3, -2), (0, 1), (1, 2), (3, 4)\}$
6.	$s = -2 + r$	D.	$\{(-9, 9), (-7, 7), (-3, 3), (0, 0), (4, -4), (6, -6)\}$

What rule is used to infer each conclusion? Write "IF" for the "If-then" Rule or "IFN" for the "If-then-not" Rule.

7. Premise: If it is a dog, then it is an animal.
   Premise: It is not an animal.
   Conclusion: It is not a dog.

8. Premise: If you can go, then I shall go with you.
   Premise: You can go.
   Conclusion: I shall go with you.

Lists of Symbols

$\{1, 2\}$	the set whose members are 1 and 2
$\{n: n < 1\}$	the set of all n such that n is less than 1
$\{\ \}$	the empty set
$\cup$	union of sets symbol
$\cap$	intersection of sets symbol
$-$	difference of sets symbol
$\subset$	subset symbol
$\in$	is an element of
$=$	is equal to
$\neq$	is not equal to
$\approx$	is equivalent to
$>$	is greater than
$<$	is less than
$\overline{AB}$	line segment AB
$\overleftrightarrow{AB}$	line AB
$\overrightarrow{AB}$	ray AB
$\angle ABC$	angle ABC
$\cong$	is congruent to

Tables of Measure

LENGTH

12 inches	1 foot
36 inches	1 yard
3 feet	1 yard
$16\frac{1}{2}$ feet	1 rod
$5\frac{1}{2}$ yards	1 rod
5280 feet	1 mile
1760 yards	1 mile
320 rods	1 mile

LIQUID

3 teaspoons	1 tablespoon
2 tablespoons	1 fluid ounce
8 fluid ounces	1 cup
2 cups	1 pint
2 pints	1 quart
4 quarts	1 gallon

TIME

60 seconds	1 minute
60 minutes	1 hour
24 hours	1 day
7 days	1 week
52 weeks	1 year
365 days	1 year
366 days	1 leap year
12 months	1 year
10 years	1 decade
100 years	1 century
1000 years	1 millenium

WEIGHT

16 ounces	1 pound
100 pounds	1 hundredweight
2000 pounds	1 ton

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